VWO
B
DEEL 4

GETAL & RUIMTE

UITWERKINGEN

4

NOORDHOFF UITGEVERS

GETAL& RUIMTE

Uitwerkingen vwo B deel 4

ELFDE EDITIE, 2017

J.H. Dijkhuis
C.J. Admiraal
J.A. Verbeek
G. de Jong
H.J. Houwing
J.D. Kuis
F. ten Klooster
S.K.A. de Waal
J. van Braak
J.H.M. Liesting-Maas
M. Wieringa
M.L.M. van Maarseveen
R.D. Hiele
J.E. Romkes
M. Haneveld
S. Voets
I. Cornelisse

Noordhoff Uitgevers Groningen

Inhoud

13 Limieten en asymptoten

Voorkennis Limieten

Bladzijde 9

1 **a** De perforatie is het punt (2, -2).

b De functie f bestaat voor $x = 5$, dus f is continu in 5.

c $f(4) = \dfrac{16 - 24 + 8}{4 - 2} = 0$, dus de functie bestaat voor $x = 4$, dus f is continu in 4.

2 **a** $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5} = \lim\limits_{x \to 5} \dfrac{(x + 5)(x - 5)}{x - 5} = \lim\limits_{x \to 5} (x + 5) = 10$

b $\lim\limits_{x \to 6} \dfrac{x^2 - 10x + 24}{x - 6} = \lim\limits_{x \to 6} \dfrac{(x - 4)(x - 6)}{x - 6} = \lim\limits_{x \to 6} (x - 4) = 2$

c $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 + 1} = \dfrac{1^2 - 1}{1^2 + 1} = \dfrac{0}{1} = 0$

d $\lim\limits_{x \to 0} \dfrac{x^2 + 2x}{x^2 + x} = \lim\limits_{x \to 0} \dfrac{x(x + 2)}{x(x + 1)} = \lim\limits_{x \to 0} \dfrac{x + 2}{x + 1} = \dfrac{2}{1} = 2$

Bladzijde 11

3 **a** $\lim\limits_{x \to \infty} \dfrac{5 - 2x}{3x + 1} = \lim\limits_{x \to \infty} \dfrac{\frac{5}{x} - 2}{3 + \frac{1}{x}} = \dfrac{0 - 2}{3 + 0} = -\frac{2}{3}$

b $\lim\limits_{x \to -\infty} \dfrac{4x}{3 - x} = \lim\limits_{x \to -\infty} \dfrac{4}{\frac{3}{x} - 1} = \dfrac{4}{0 - 1} = -4$

c $\lim\limits_{x \to \infty} \dfrac{|3 - 2x|}{x + 1} = \lim\limits_{x \to \infty} \dfrac{-(3 - 2x)}{x + 1} = \lim\limits_{x \to \infty} \dfrac{-3 + 2x}{x + 1} = \lim\limits_{x \to \infty} \dfrac{-\frac{3}{x} + 2}{1 + \frac{1}{x}} = \dfrac{0 + 2}{1 + 0} = 2$

d $\lim\limits_{x \to -\infty} \dfrac{x + 3}{|2x + 1|} = \lim\limits_{x \to -\infty} \dfrac{x + 3}{-(2x + 1)} = \lim\limits_{x \to -\infty} \dfrac{x + 3}{-2x - 1} = \lim\limits_{x \to -\infty} \dfrac{1 + \frac{3}{x}}{-2 - \frac{1}{x}} = \dfrac{1 + 0}{-2 - 0} = -\frac{1}{2}$

e $\lim\limits_{x \to \infty} \dfrac{|x - 1| + x}{|4 - 3x| + x} = \lim\limits_{x \to \infty} \dfrac{x - 1 + x}{-(4 - 3x) + x} = \lim\limits_{x \to \infty} \dfrac{x - 1 + x}{-4 + 3x + x} = \lim\limits_{x \to \infty} \dfrac{2x - 1}{4x - 4} = \lim\limits_{x \to \infty} \dfrac{2 - \frac{1}{x}}{4 - \frac{4}{x}} = \dfrac{2 - 0}{4 - 0} = \frac{1}{2}$

f $\lim\limits_{x \to -\infty} \dfrac{|2x| - x + 1}{|3 - 4x|} = \lim\limits_{x \to -\infty} \dfrac{-2x - x + 1}{3 - 4x} = \lim\limits_{x \to -\infty} \dfrac{-3x + 1}{3 - 4x} = \lim\limits_{x \to -\infty} \dfrac{-3 + \frac{1}{x}}{\frac{3}{x} - 4} = \dfrac{-3 + 0}{0 - 4} = \frac{3}{4}$

4 **a** verticale asymptoot:
$|2x + 5| = 0 \wedge 5 - 3x \neq 0$ geeft $x = -2\frac{1}{2}$
Dus de verticale asymptoot is de lijn $x = -2\frac{1}{2}$.

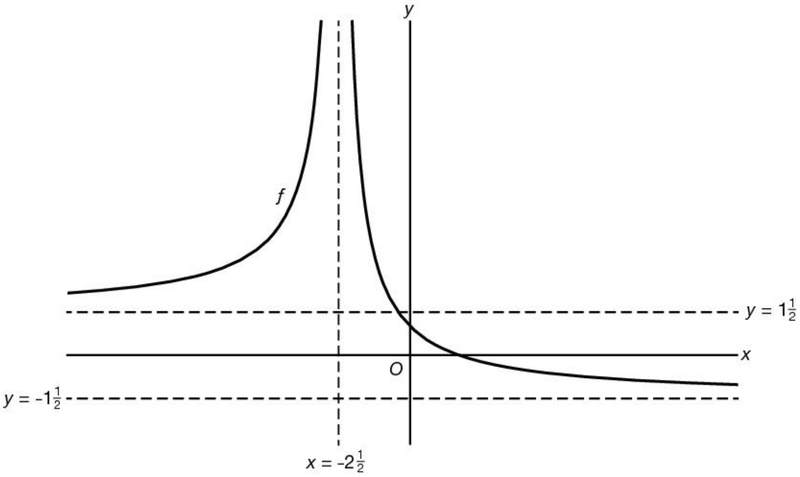
$$f(x) = \frac{5 - 3x}{|2x + 5|} = \begin{cases} \dfrac{5 - 3x}{2x + 5} & \text{voor } 2x + 5 > 0 \text{ ofwel } x > -2\frac{1}{2} \\ \dfrac{5 - 3x}{-2x - 5} & \text{voor } 2x + 5 < 0 \text{ ofwel } x < -2\frac{1}{2} \end{cases}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{5-3x}{2x+5} = \lim_{x\to\infty} \frac{\frac{5}{x}-3}{2+\frac{5}{x}} = \frac{0-3}{2+0} = -1\tfrac{1}{2}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{5-3x}{-2x-5} = \lim_{x\to-\infty} \frac{\frac{5}{x}-3}{-2-\frac{5}{x}} = \frac{0-3}{-2-0} = 1\tfrac{1}{2}$$

Dus de horizontale asymptoten zijn de lijnen $y = -1\tfrac{1}{2}$ en $y = 1\tfrac{1}{2}$.

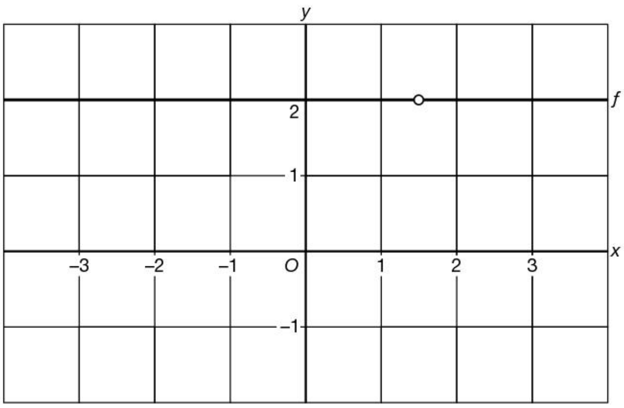
b



5 **a** $f(x) = \dfrac{4x-6}{2x-3} = \dfrac{2(2x-3)}{2x-3} = 2$ mits $x \neq 1\tfrac{1}{2}$

De grafiek van f is dus de lijn $y = 2$ zonder het punt $\left(1\tfrac{1}{2}, 2\right)$.

b



13.1 Evenredigheden en inverse functies

Bladzijde 12

1 **a** Bij de formule $y_1 = 12x$ geldt dat y met k wordt vermenigvuldigd als x met k wordt vermenigvuldigd.

b Bij de formule $y_2 = \dfrac{12}{x}$ geldt dat y door k wordt gedeeld als x met k wordt vermenigvuldigd.

Bladzijde 13

2 **a** y evenredig met $\dfrac{x}{4x+1}$, dus $y = a \cdot \dfrac{x}{4x+1}$.

$x = 1$ en $y = 0{,}6$ geeft $a \cdot \dfrac{1}{4+1} = 0{,}6$

$0{,}2a = 0{,}6$

$a = 3$

Dus $y = 3 \cdot \dfrac{x}{4x+1}$ ofwel $y = \dfrac{3x}{4x+1}$.

b $$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{3x}{4x+1} = \lim_{x\to\infty} \frac{3}{4+\frac{1}{x}} = \frac{3}{4+0} = \tfrac{3}{4}$$

3 **a** $\frac{2x}{x-1}$ omgekeerd evenredig met y, dus $\frac{2x}{x-1} = \frac{a}{y}$.

$x = 5$ en $y = 1{,}6$ geeft $\frac{2 \cdot 5}{5-1} = \frac{a}{1{,}6}$

$4a = 16$

$a = 4$

Dus $\frac{2x}{x-1} = \frac{4}{y}$

$2xy = 4x - 4$

$y = \frac{4x-4}{2x}$

$y = \frac{2x-2}{x}$

b $\lim\limits_{x \to \infty} y = \lim\limits_{x \to \infty} \frac{2x-2}{x} = \lim\limits_{x \to \infty} \frac{2 - \frac{2}{x}}{1} = \frac{2-0}{1} = 2$

4 y is omgekeerd evenredig met $2 + \frac{4}{|x|}$ geeft $y = \frac{a}{2 + \frac{4}{|x|}}$.

$x = -2$ en $y = 1$ geeft $1 = \frac{a}{2 + \frac{4}{|-2|}}$

$a = 2 + \frac{4}{2} = 2 + 2 = 4$

Dus $y = \frac{4}{2 + \frac{4}{|x|}}$.

$\lim\limits_{x \to -\infty} y = \lim\limits_{x \to -\infty} \frac{4}{2 + \frac{4}{|x|}} = \lim\limits_{x \to -\infty} \frac{4}{2 + \frac{4}{-x}} = \lim\limits_{x \to -\infty} \frac{4}{2 - \frac{4}{x}} = \frac{4}{2-0} = 2$

Bladzijde 14

5 **a** $L = \frac{a}{d^2}$, $d = 4$ en $L = 50$ $\Big\}$ $\frac{a}{4^2} = 50$

$a = 50 \cdot 4^2 = 800$

De formule is $L = \frac{800}{d^2}$.

b $d = 2$ geeft $L = \frac{800}{2^2} = 200$

De geluidssterkte is 200.

c $L = 20$ geeft $\frac{800}{d^2} = 20$

$20d^2 = 800$

$d^2 = 40$

$d = \sqrt{40} = 6{,}32...$

Dus op een afstand van 6,3 m.

d De afstand d_1 geeft $L_1 = \frac{800}{d_1^2}$.

$d_2 = 2d_1$, $L_2 = \frac{800}{d_2^2}$ $\Big\}$ $L_2 = \frac{800}{d_2^2} = \frac{800}{(2d_1)^2} = \frac{800}{4d_1^2} = \frac{1}{4} \cdot \frac{800}{d_1^2}$

Dus de geluidssterkte wordt 4 keer zo klein.

6 **a** $\left.\begin{array}{l} W_1 = av^2 \\ v = 720 \text{ en } W_1 = 120\,000 \end{array}\right\}$ $a \cdot 720^2 = 120\,000$

$a = \dfrac{120\,000}{720^2} = 0{,}231...$

Dus $W_1 = 0{,}23v^2$.

$\left.\begin{array}{l} W_i = \dfrac{b}{v^2} \\ v = 720 \text{ en } W_i = 300\,000 \end{array}\right\}$ $\dfrac{b}{720^2} = 300\,000$

$b = 300000 \cdot 720^2 = 1{,}555... \cdot 10^{11}$

Dus $W_i = \dfrac{1{,}56 \cdot 10^{11}}{v^2}$.

b $W = W_1 + W_i = 0{,}23v^2 + \dfrac{1{,}56 \cdot 10^{11}}{v^2}$

Voer in $y_1 = 0{,}23x^2 + \dfrac{1{,}56 \cdot 10^{11}}{x^2}$.

De optie minimum geeft $x \approx 908$ (en $y_1 \approx 378\,840$).

Dus bij een snelheid van 908 km/uur is de totale weerstand minimaal.

c $v = 908$ geeft $W_1 = 0{,}23 \cdot 908^2 \approx 189\,626$ N

$v = 908$ geeft $W_i = \dfrac{1{,}56 \cdot 10^{11}}{908^2} \approx 189\,214$ N

Dus bij minimale totale weerstand geldt $W_1 = W_i$.

Bladzijde 15

7 **a** In de grafiek is te zien dat bij elke y hoogstens één x hoort. Dus f heeft een inverse.

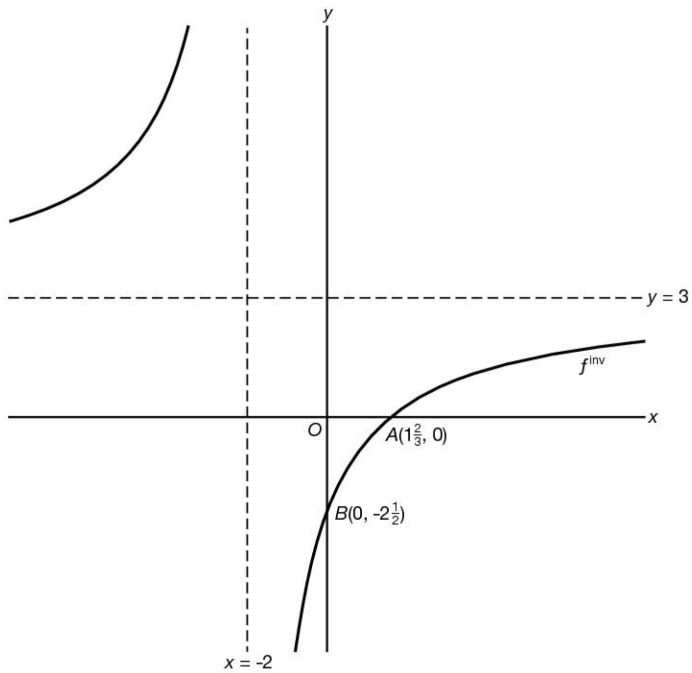
b De horizontale asymptoot van de grafiek van f is de lijn $y = -2$, dus de verticale asymptoot van de grafiek van f^{inv} is de lijn $x = -2$.

De verticale asymptoot van de grafiek van f is de lijn $x = 3$, dus de horizontale asymptoot van de grafiek van f^{inv} is de lijn $y = 3$.

c De grafiek van f snijdt de y-as in het punt $\left(0, 1\frac{2}{3}\right)$, dus de grafiek van f^{inv} snijdt de x-as in het punt $A\left(1\frac{2}{3}, 0\right)$.

De grafiek van f snijdt de x-as in het punt $\left(-2\frac{1}{2}, 0\right)$, dus de grafiek van f^{inv} snijdt de y-as in het punt $B\left(0, -2\frac{1}{2}\right)$.

d



e De grafieken van f en f^{inv} zijn elkaars spiegelbeeld in de lijn $y = x$.

Omdat de grafiek van f de lijn $y = x$ niet snijdt, snijdt de grafiek van f^{inv} ook niet de lijn $y = x$ en zullen de grafieken van f en f^{inv} elkaar dus ook niet snijden.

Bladzijde 16

8 **a** $f(x) = 3 - \dfrac{4}{x+2} = \dfrac{3(x+2)}{x+2} - \dfrac{4}{x+2} = \dfrac{3x+6-4}{x+2} = \dfrac{3x+2}{x+2}$

$$\lim_{x \to \infty} \frac{3x+2}{x+2} = \lim_{x \to \infty} \frac{3 + \frac{2}{x}}{1 + \frac{2}{x}} = \frac{3+0}{1+0} = 3$$

Dus de horizontale asymptoot van de grafiek van f is de lijn $y = 3$.
$x + 2 = 0$ geeft $x = -2$, dus de verticale asymptoot van de grafiek van f is de lijn $x = -2$.
Omdat de f en g elkaars inverse zijn, is de lijn $x = 3$ de verticale asymptoot van de grafiek van g en de lijn $y = -2$ de horizontale asymptoot van de grafiek van g.

b Het lijkt mij dat de vergelijking $f(x) = g(x)$ meer rekenwerk geeft dan de vergelijking $f(x) = x$ of de vergelijking $g(x) = x$.

Bladzijde 17

9 **a** Voor f geldt $y = \dfrac{2x}{x+3}$, dus voor f^{inv} geldt $x = \dfrac{2y}{y+3}$

$$\begin{aligned} xy + 3x &= 2y \\ xy - 2y &= -3x \\ (x-2)y &= -3x \\ y &= \frac{-3x}{x-2} \end{aligned}$$

Dus $f^{\text{inv}}(x) = \dfrac{-3x}{x-2}$.

b Voor f geldt $y = \dfrac{3x-1}{4x+5}$, dus voor f^{inv} geldt $x = \dfrac{3y-1}{4y+5}$

$$\begin{aligned} 4xy + 5x &= 3y - 1 \\ 4xy - 3y &= -5x - 1 \\ (4x-3)y &= -5x - 1 \\ y &= \frac{-5x-1}{4x-3} \end{aligned}$$

Dus $f^{\text{inv}} = \dfrac{-5x-1}{4x-3}$.

c Voor f geldt $y = \dfrac{ax+b}{cx+d}$, dus voor f^{inv} geldt $x = \dfrac{ay+b}{cy+d}$

$$\begin{aligned} cxy + dx &= ay + b \\ cxy - ay &= -dx + b \\ (cx-a)y &= -dx + b \\ y &= \frac{-dx+b}{cx-a} \end{aligned}$$

Dus $f^{\text{inv}} = \dfrac{-dx+b}{cx-a}$.

10 **a** $$\lim_{x \to \infty} \left(3 - \frac{3}{x+1}\right) = \lim_{x \to \infty} \left(3 - \frac{\frac{3}{x}}{1 + \frac{1}{x}}\right) = 3 - \frac{0}{1+0} = 3 - 0 = 3$$

Dus de horizontale asymptoot van de grafiek van f is de lijn $y = 3$.
$x + 1 = 0$ geeft $x = -1$, dus de verticale asymptoot van de grafiek van f is de lijn $x = -1$.

b $f(x) = 3 - \dfrac{3}{x+1} = 3 - 3(x+1)^{-1}$ geeft $f'(x) = 3(x+1)^{-2} = \dfrac{3}{(x+1)^2}$

Stel k: $y = ax + b$ met $a = f'(1) = \dfrac{3}{(1+1)^2} = \frac{3}{4}$.

$$\left.\begin{aligned} &y = \tfrac{3}{4}x + b \\ &A\left(1, 1\tfrac{1}{2}\right) \end{aligned}\right\} \begin{aligned} \tfrac{3}{4} \cdot 1 + b &= 1\tfrac{1}{2} \\ \tfrac{3}{4} + b &= 1\tfrac{1}{2} \\ b &= \tfrac{3}{4} \end{aligned}$$

Dus k: $y = \frac{3}{4}x + \frac{3}{4}$.

c De grafiek van f snijden met de lijn $y = x$ geeft $3 - \frac{3}{x+1} = x$

$$3 - x = \frac{3}{x+1}$$
$$3x + 3 - x^2 - x = 3$$
$$-x^2 + 2x = 0$$
$$-x(x-2) = 0$$
$$x = 0 \vee x = 2$$

Dus $B(2, 2)$.

Stel de raaklijn in B aan de grafiek van f is m: $y = ax + b$ met $a = f'(2) = \frac{3}{(2+1)^2} = \frac{1}{3}$.

$$\left.\begin{array}{l} y = \frac{1}{3}x + b \\ B(2, 2) \end{array}\right\} \frac{1}{3} \cdot 2 + b = 2$$
$$\frac{2}{3} + b = 2$$
$$b = 1\tfrac{1}{3}$$

Dus m: $y = \frac{1}{3}x + 1\frac{1}{3}$.

De lijn l is het spiegelbeeld van m in de lijn $y = x$, dus voor l geldt $x = \frac{1}{3}y + 1\frac{1}{3}$

$$-\tfrac{1}{3}y = -x + 1\tfrac{1}{3}$$
$$y = 3x - 4$$

Dus l: $y = 3x - 4$.

Alternatieve uitwerking

Na de berekening van $B(2, 2)$ kun je ook als volgt verdergaan.

Voor f geldt $y = 3 - \frac{3}{x+1} = \frac{3x+3}{x+1} - \frac{3}{x+1} = \frac{3x}{x+1}$, dus voor f^{inv} geldt $x = \frac{3y}{y+1}$

$$xy + x = 3y$$
$$xy - 3y = -x$$
$$(x-3)y = -x$$
$$y = \frac{-x}{x-3}$$

Dus $f^{\text{inv}}(x) = \frac{-x}{x-3}$ en dit geeft $f^{\text{inv}\prime}(x) = \frac{(x-3) \cdot -1 - -x \cdot 1}{(x-3)^2} = \frac{-x+3+x}{(x-3)^2} = \frac{3}{(x-3)^2}$.

Stel l: $y = ax + b$ met $a = f^{\text{inv}\prime}(2) = \frac{3}{(2-3)^2} = 3$

$$\left.\begin{array}{l} y = 3x + b \\ B(2, 2) \end{array}\right\} 3 \cdot 2 + b = 2$$
$$6 + b = 2$$
$$b = -4$$

Dus l: $y = 3x - 4$.

11 V is symmetrisch in de lijn $y = x$. Dus de oppervlakte van V is twee keer de oppervlakte van het vlakdeel dat wordt ingesloten door de grafiek van f en de lijn $y = x$.

$$O(V) = 2 \cdot \int_0^2 (f(x) - x)\,\mathrm{d}x = 2 \cdot \int_0^2 \left(3 - \frac{3}{x+1} - x\right)\mathrm{d}x = 2 \cdot \left[3x - 3\ln(x+1) - \tfrac{1}{2}x^2\right]_0^2$$
$$= 2(6 - 3\ln(3) - 2) - 2(0 - 3\ln(1) - 0) = 8 - 6\ln(3)$$

12 **a** $\lim_{x \to \infty} \frac{ax-2}{2x-5} = \lim_{x \to \infty} \frac{a - \frac{2}{x}}{2 - \frac{5}{x}} = \frac{a-0}{2-0} = \frac{1}{2}a$

Dus de horizontale asymptoot van de grafiek van f is de lijn $y = \frac{1}{2}a$.

$\frac{1}{2}a = 3$ geeft $a = 6$

b De verticale asymptoot van de grafiek van f^{inv} is de lijn $x = \frac{1}{2}a$.

$\frac{1}{2}a = 4$ geeft $a = 8$

c Voor g geldt $y = 2\frac{1}{2} + \dfrac{4}{x-2}$, dus voor g^{inv} geldt $x = 2\frac{1}{2} + \dfrac{4}{y-2} = \dfrac{2\frac{1}{2}y - 5 + 4}{y-2} = \dfrac{2\frac{1}{2}y - 1}{y-2}$

$$x = \frac{2\frac{1}{2}y - 1}{y-2} \text{ geeft } \begin{aligned} xy - 2x &= 2\tfrac{1}{2}y - 1 \\ xy - 2\tfrac{1}{2}y &= 2x - 1 \\ y\left(x - 2\tfrac{1}{2}\right) &= 2x - 1 \\ y = \frac{2x-1}{x-2\frac{1}{2}} &\text{ ofwel } y = \frac{4x-2}{2x-5} \end{aligned}$$

Dus $g^{\text{inv}}(x) = \dfrac{4x-2}{2x-5}$. En dit is gelijk aan $f(x)$ als $a = 4$.

d $\dfrac{ax-2}{2x-5} = x$
$2x^2 - 5x = ax - 2$
$2x^2 + (-a-5)x + 2 = 0$

geen oplossingen, dus $D < 0$

$$\begin{aligned} (-a-5)^2 - 4 \cdot 2 \cdot 2 &< 0 \\ (a+5)^2 &< 16 \\ -4 < a + 5 &< 4 \\ -9 < a &< -1 \end{aligned}$$

Dus voor $-9 < a < -1$.

13

$$\left.\begin{aligned} y &= \frac{ax+b}{x+1} \\ x_A &= 3, \text{ dus } A(3, 3) \end{aligned}\right\} \begin{aligned} 3 &= \frac{3a+b}{3+1} \\ 3a + b &= 12 \end{aligned}$$

$$\left.\begin{aligned} y &= \frac{ax+b}{x+1} \\ x_B &= 5, \text{ dus } B(5, 5) \end{aligned}\right\} \begin{aligned} 5 &= \frac{5a+b}{5+1} \\ 5a + b &= 30 \end{aligned}$$

$$\begin{cases} 3a + b = 12 \\ 5a + b = 30 \end{cases} -$$
$$-2a = -18$$

$$\left.\begin{aligned} a &= 9 \\ 3a + b &= 12 \end{aligned}\right\} \begin{aligned} 3 \cdot 9 + b &= 12 \\ 27 + b &= 12 \\ b &= -15 \end{aligned}$$

Dus $a = 9$ en $b = -15$.

13.2 Asymptoten bij gebroken functies

Bladzijde 19

14 **a** $f(10) \approx 2{,}308$
$f(100) \approx 2{,}939$
$f(1000) \approx 2{,}994$
Ik denk 3.

b $f(-10) \approx 3{,}462$
$f(-100) \approx 3{,}059$
$f(-1000) \approx 3{,}006$
Ik denk 3.

Bladzijde 21

15 $\lim\limits_{x \to \infty} g(x) = \lim\limits_{x \to \infty} \dfrac{2x^3 - 1}{x^2 - 1} = \lim\limits_{x \to \infty} \dfrac{2 - \frac{1}{x^3}}{\frac{1}{x} - \frac{1}{x^3}} = \dfrac{2-0}{0-0} = \dfrac{2}{0}$ en dit kan niet, want delen door nul is niet toegestaan.

Hetzelfde geldt voor $\lim\limits_{x \to -\infty} g(x)$.

16 **a** $\lim_{x \to \infty} \frac{3x^2 - x}{2 - x^2} = \lim_{x \to \infty} \frac{3 - \frac{1}{x}}{\frac{2}{x^2} - 1} = \frac{3 - 0}{0 - 1} = -3$

b $\lim_{x \to -\infty} \frac{5 - x^3}{2x^3} = \lim_{x \to -\infty} \frac{\frac{5}{x^3} - 1}{2} = \frac{0 - 1}{2} = -\frac{1}{2}$

c $\lim_{x \to \infty} \frac{6x^2}{2 - x^3} = \lim_{x \to \infty} \frac{\frac{6}{x}}{\frac{2}{x^3} - 1} = \frac{0}{0 - 1} = 0$

d $\lim_{x \to \infty} \frac{(2x - 3)^2}{x^2 + 1} = \lim_{x \to \infty} \frac{4x^2 - 12x + 9}{x^2 + 1} = \lim_{x \to \infty} \frac{4 - \frac{12}{x} + \frac{9}{x^2}}{1 + \frac{1}{x^2}} = \frac{4 - 0 + 0}{1 + 0} = 4$

17 **a** $\lim_{x \to -\infty} \frac{(4x - 1)^2}{x^3 + 4} = \lim_{x \to -\infty} \frac{16x^2 - 8x + 1}{x^3 + 4} = \lim_{x \to -\infty} \frac{\frac{16}{x} - \frac{8}{x^2} + \frac{1}{x^3}}{1 + \frac{4}{x^3}} = \frac{0 - 0 + 0}{1 + 0} = 0$

b $\lim_{x \to \infty} \frac{x(2x + 1)^2}{x^3 + 1} = \lim_{x \to \infty} \frac{x(4x^2 + 4x + 1)}{x^3 + 1} = \lim_{x \to \infty} \frac{4x^3 + 4x^2 + x}{x^3 + 1} = \lim_{x \to \infty} \frac{4 + \frac{4}{x} + \frac{1}{x^2}}{1 + \frac{1}{x^3}} = \lim_{x \to \infty} \frac{4 + 0 + 0}{1 + 0} = 4$

c $\lim_{x \to -\infty} \frac{|x^3 - 8|}{2x^3 + x} = \lim_{x \to -\infty} \frac{-(x^3 - 8)}{2x^3 + x} = \lim_{x \to -\infty} \frac{-x^3 + 8}{2x^3 + x} = \lim_{x \to -\infty} \frac{-1 + \frac{8}{x^3}}{2 + \frac{1}{x^2}} = \frac{-1 + 0}{2 + 0} = -\frac{1}{2}$

d $\lim_{x \to -\infty} \frac{|4x^3| - 10}{|x^3 - 1|} = \lim_{x \to -\infty} \frac{-4x^3 - 10}{-(x^3 - 1)} = \lim_{x \to -\infty} \frac{-4x^3 - 10}{-x^3 + 1} = \lim_{x \to -\infty} \frac{-4 - \frac{10}{x^3}}{-1 + \frac{1}{x^3}} = \frac{-4 - 0}{-1 + 0} = 4$

18 **a** $x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$x = -2 \vee x = 3$

Dus de verticale asymptoten zijn de lijnen $x = -2$ en $x = 3$.

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{4x^2 - 1}{x^2 - x - 6} = \lim_{x \to \infty} \frac{4 - \frac{1}{x^2}}{1 - \frac{1}{x} - \frac{6}{x^2}} = \frac{4 - 0}{1 - 0 - 0} = 4$

Dus de horizontale asymptoot is de lijn $y = 4$.

b $\frac{4x^2 - 1}{x^2 - x - 6} = 4$

$4x^2 - 1 = 4x^2 - 4x - 24$

$4x = -23$

$x = -5\frac{3}{4}$

Dus $A\left(-5\frac{3}{4}, 4\right)$.

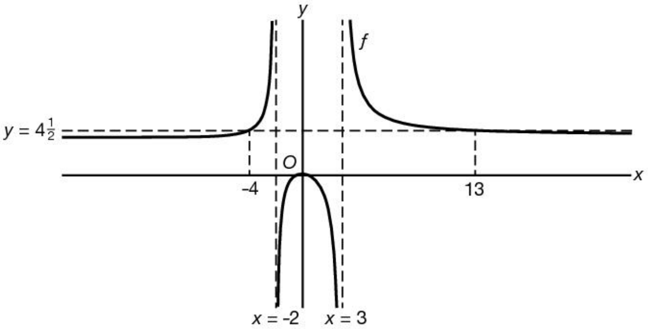
c $f(x) = 4\frac{1}{2}$ geeft $\dfrac{4x^2 - 1}{x^2 - x - 6} = \frac{9}{2}$

$9x^2 - 9x - 54 = 8x^2 - 2$

$x^2 - 9x - 52 = 0$

$(x + 4)(x - 13) = 0$

$x = -4 \vee x = 13$



$f(x) < 4\frac{1}{2}$ geeft $x < -4 \vee -2 < x < 3 \vee x > 13$

19 **a** $\lim\limits_{x \to \infty} \dfrac{|x^3|}{x^3 - 4x} = \lim\limits_{x \to \infty} \dfrac{x^3}{x^3 - 4x} = \lim\limits_{x \to \infty} \dfrac{1}{1 - \frac{4}{x^2}} = \dfrac{1}{1 - 0} = 1$, dus voor $x \to \infty$ is de lijn $y = 1$ horizontale asymptoot.

$\lim\limits_{x \to -\infty} \dfrac{|x^3|}{x^3 - 4x} = \lim\limits_{x \to -\infty} \dfrac{-x^3}{x^3 - 4x} = \lim\limits_{x \to -\infty} \dfrac{-1}{1 - \frac{4}{x^2}} = \dfrac{-1}{1 - 0} = -1$, dus voor $x \to -\infty$ is de lijn $y = -1$ horizontale asymptoot.

$x^3 - 4x = 0 \wedge |x^3| \neq 0$

$x(x^2 - 4) = 0 \wedge x^3 \neq 0$

$(x = 0 \vee x^2 = 4) \wedge x \neq 0$

$x = -2 \vee x = 2$

Dus de verticale asymptoten zijn de lijnen $x = -2$ en $x = 2$.

b $y = 1$ geeft $\left(x > 0 \wedge \dfrac{x^3}{x^3 - 4x} = 1\right) \vee \left(x < 0 \wedge \dfrac{-x^3}{x^3 - 4x} = 1\right)$

$(x > 0 \wedge x^3 = x^3 - 4x) \vee (x < 0 \wedge -x^3 = x^3 - 4x)$

$(x > 0 \wedge 4x = 0) \vee (x < 0 \wedge -2x^3 + 4x = 0)$

geen opl. $\quad x < 0 \wedge -2x(x^2 - 2) = 0$

$x < 0 \wedge (x = 0 \vee x^2 = 2)$

$x < 0 \wedge \left(x = 0 \vee x = -\sqrt{2} \vee x = \sqrt{2}\right)$

$x = -\sqrt{2}$

$y = -1$ geeft $\left(x > 0 \wedge \dfrac{x^3}{x^3 - 4x} = -1\right) \vee \left(x < 0 \wedge \dfrac{-x^3}{x^3 - 4x} = -1\right)$

$(x > 0 \wedge x^3 = -x^3 + 4x) \vee (x < 0 \wedge -x^3 = -x^3 - 4x)$

$(x > 0 \wedge 2x^3 - 4x = 0) \vee (x < 0 \wedge 4x = 0)$

$(x > 0 \wedge 2x(x^2 - 2) = 0) \vee (x < 0 \wedge x = 0)$

$x > 0 \wedge (x = 0 \vee x^2 = 2) \quad$ geen opl.

$x > 0 \wedge \left(x = 0 \vee x = -\sqrt{2} \vee x = \sqrt{2}\right)$

$x = \sqrt{2}$

Dus $A\left(-\sqrt{2}, 1\right)$ en $B\left(\sqrt{2}, -1\right)$.

Stel k: $y = ax + b$ met $a = \text{rc}_{AB} = \dfrac{-1 - 1}{\sqrt{2} - -\sqrt{2}} = \dfrac{-2}{2\sqrt{2}} = \dfrac{-1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$.

$\left.\begin{array}{l} y = -\frac{1}{2}x\sqrt{2} + b \\ \text{door } A\left(-\sqrt{2}, 1\right) \end{array}\right\} -\frac{1}{2} \cdot -\sqrt{2} \cdot \sqrt{2} + b = 1$

$b + 1 = 0$

$b = 0$

Dus k: $y = -\frac{1}{2}x\sqrt{2}$.

c $f(x) = 1\frac{1}{3}$ geeft $x > 0 \wedge \dfrac{x^3}{x^3 - 4x} = \frac{4}{3}$ $\vee\ x < 0 \wedge \dfrac{-x^3}{x^3 - 4x} = \frac{4}{3}$

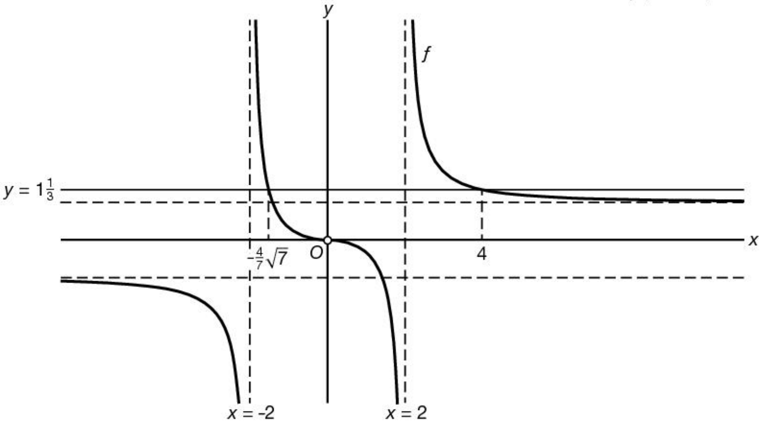
$x > 0 \wedge 4x^3 - 16x = 3x^3$ $\quad x < 0 \wedge 4x^3 - 16x = -3x^3$

$x > 0 \wedge x^3 - 16x = 0$ $\quad x < 0 \wedge 7x^3 - 16x = 0$

$x > 0 \wedge x(x^2 - 16) = 0$ $\quad x < 0 \wedge x(7x^2 - 16) = 0$

$x > 0 \wedge (x = 0 \vee x = -4 \vee x = 4)$ $\quad x < 0 \wedge \left(x = 0 \vee x = -\sqrt{\frac{16}{7}} \vee x = \sqrt{\frac{16}{7}}\right)$

$x = 4$ $\quad x = -\sqrt{\frac{16}{7}} = -\frac{4}{7}\sqrt{7}$



$f(x) > 1\frac{1}{3}$ geeft $-2 < x < -\frac{4}{7}\sqrt{7} \vee 2 < x < 4$

20 **a** $\left.\begin{array}{l} bx^2 - 18 = 0 \\ x = -3 \end{array}\right\}$ $9b - 18 = 0$, $9b = 18$, $b = 2$ $\quad \left.\begin{array}{l} bx^2 - 18 = 0 \\ x = 3 \end{array}\right\}$ $9b - 18 = 0$, $9b = 18$, $b = 2$

Dus $b = 2$.

$$\lim_{x\to\infty} \frac{ax^2 + 5}{bx^2 - 18} = \lim_{x\to\infty} \frac{a + \frac{5}{x^2}}{b - \frac{18}{x^2}} = \frac{a + 0}{b - 0} = \frac{a}{b}$$

$\left.\begin{array}{l} \frac{a}{b} = 6 \\ b = 2 \end{array}\right\}$ $\frac{a}{2} = 6$, $a = 12$

Dus $a = 12$ en $b = 2$.

b $\left.\begin{array}{l} ax^4 + bx^3 - 2 = 0 \\ x = -2 \end{array}\right\}$ $16a - 8b - 2 = 0$

$\left.\begin{array}{l} ax^4 + bx^3 - 2 = 0 \\ x = 2 \end{array}\right\}$ $16a + 8b - 2 = 0$

$$\begin{cases} 16a - 8b - 2 = 0 \\ 16a + 8b - 2 = 0 \end{cases} -$$

$-16b \quad = 0$

$\left.\begin{array}{l} b = 0 \\ 16a - 8b - 2 = 0 \end{array}\right\}$ $16a = 2$, $a = \frac{1}{8}$

$$\lim_{x\to\infty} \frac{5x^3 - 2x^2 + 1}{\frac{1}{8}x^4 - 2} = \lim_{x\to\infty} \frac{\frac{5}{x} - \frac{2}{x^2} + \frac{1}{x^4}}{\frac{1}{8} - \frac{2}{x^4}} = \frac{0 - 0 + 0}{\frac{1}{8} - 0} = 0$$, dus de lijn $y = 0$ is horizontale asymptoot klopt.

Dus $a = \frac{1}{8}$ en $b = 0$.

Bladzijde 22

21 **a** $$\lim_{x\to\infty} \frac{4}{x+2} = \lim_{x\to\infty} \frac{\frac{4}{x}}{1 + \frac{2}{x}} = \frac{0}{1+0} = 0 \text{ en } \lim_{x\to-\infty} \frac{4}{x+2} = \lim_{x\to-\infty} \frac{\frac{4}{x}}{1 + \frac{2}{x}} = \frac{0}{1+0} = 0$$

Dus de grafiek van f nadert de lijn $y = x + 3$ onbeperkt voor $x \to \infty$ en voor $x \to -\infty$.

b $$f(x) = x + 3 - \frac{4}{x+2} = \frac{x(x+2) + 3(x+2) - 4}{x+2} = \frac{x^2 + 2x + 3x + 6 - 4}{x+2} = \frac{x^2 + 5x + 2}{x+2}$$

Bladzijde 23

22 **a** verticale asymptoot:

$x - 1 = 0 \wedge 2x^2 + 4x - 5 \neq 0$

$x = 1 \wedge 2x^2 + 4x - 5 \neq 0$

$x = 1$

Dus de verticale asymptoot is de lijn $x = 1$.

scheve asymptoot:

$$f(x) = \frac{2x^2 + 4x - 5}{x - 1} = \frac{2x(x-1) + 2x + 4x - 5}{x - 1} = 2x + \frac{6x - 5}{x - 1} = 2x + \frac{6(x-1) + 6 - 5}{x - 1} = 2x + 6 + \frac{1}{x - 1}$$

$\lim\limits_{x \to \infty} \frac{1}{x-1} = 0$ en $\lim\limits_{x \to -\infty} \frac{1}{x-1} = 0$

Dus de scheve asymptoot is de lijn $y = 2x + 6$.

b verticale asymptoot:

$2x - 1 = 0 \wedge -x^2 + 3x - 2 \neq 0$

$2x = 1 \wedge -x^2 + 3x - 2 \neq 0$

$x = \frac{1}{2} \wedge -x^2 + 3x - 2 \neq 0$

$x = \frac{1}{2}$

Dus de verticale asymptoot is de lijn $x = \frac{1}{2}$.

scheve asymptoot:

$$g(x) = \frac{-x^2 + 3x - 2}{2x - 1} = \frac{-\frac{1}{2}x(2x-1) - \frac{1}{2}x + 3x - 2}{2x - 1} = -\frac{1}{2}x + \frac{2\frac{1}{2}x - 2}{2x - 1} = -\frac{1}{2}x + \frac{1\frac{1}{4}(2x-1) + 1\frac{1}{4} - 2}{2x - 1}$$

$$= -\frac{1}{2}x + 1\frac{1}{4} - \frac{\frac{3}{4}}{2x - 1}$$

$\lim\limits_{x \to \infty} \frac{\frac{3}{4}}{2x-1} = 0$ en $\lim\limits_{x \to -\infty} \frac{\frac{3}{4}}{2x-1} = 0$

Dus de scheve asymptoot is de lijn $y = -\frac{1}{2}x + 1\frac{1}{4}$.

c verticale asymptoten:

$x^2 - 9 = 0 \wedge x^3 + 3x^2 - 9x + 31 \neq 0$

$x^2 = 9 \wedge x^3 + 3x^2 - 9x + 31 \neq 0$

$(x = 3 \vee x = -3) \wedge x^3 + 3x^2 - 9x + 31 \neq 0$

$x = 3 \vee x = -3$

De verticale asymptoten zijn de lijnen $x = 3$ en $x = -3$.

scheve asymptoot:

$$h(x) = \frac{x^3 + 3x^2 - 9x + 31}{x^2 - 9} = \frac{x(x^2 - 9) + 9x + 3x^2 - 9x + 31}{x^2 - 9} = x + \frac{3x^2 + 31}{x^2 - 9} = x + \frac{3(x^2 - 9) + 27 + 31}{x^2 - 9} = x + 3 + \frac{58}{x^2 - 9}$$

$\lim\limits_{x \to \infty} \frac{58}{x^2 - 9} = 0$ en $\lim\limits_{x \to -\infty} \frac{58}{x^2 - 9} = 0$

De scheve asymptoot is de lijn $y = x + 3$.

d verticale asymptoot:

$x^2 - 1 = 0 \ \wedge \ x^3 - 1 \neq 0$

$x^2 = 1 \ \wedge \ x^3 \neq 1$

$(x = 1 \vee x = -1) \wedge x \neq 1$

$x = -1$

Dus de verticale asymptoot is de lijn $x = -1$.

scheve asymptoot:

$$j(x) = \frac{x^3 - 1}{x^2 - 1} = \frac{x(x^2 - 1) + x - 1}{x^2 - 1} = x + \frac{x - 1}{x^2 - 1}$$

$\lim\limits_{x \to \infty} \frac{x-1}{x^2-1} = \lim\limits_{x \to \infty} \frac{x-1}{(x+1)(x-1)} = \lim\limits_{x \to \infty} \frac{1}{x+1} = 0$ en $\lim\limits_{x \to -\infty} \frac{x-1}{x^2-1} = 0$

Dus de scheve asymptoot is de lijn $y = x$.

Bladzijde 24

23 **a** scheve asymptoot:

$$f(x) = \frac{x^2 - x - 2}{x + 2} = \frac{x(x+2) - 2x - x - 2}{x + 2} = x + \frac{-3x - 2}{x + 2} = x + \frac{-3(x+2) + 6 - 2}{x + 2} = x - 3 + \frac{4}{x + 2}$$

$\lim\limits_{x \to \infty} \frac{4}{x+2} = 0$ en $\lim\limits_{x \to -\infty} \frac{4}{x+2} = 0$, dus de scheve asymptoot is de lijn $y = x - 3$.

$x + 2 = 0$ geeft $x = -2$, dus de verticale asymptoot is de lijn $x = -2$.

b $f(x) = x - 3 + \frac{4}{x+2} = x - 3 + 4(x+2)^{-1}$ geeft $f'(x) = 1 - 4(x+2)^{-2} = 1 - \frac{4}{(x+2)^2}$

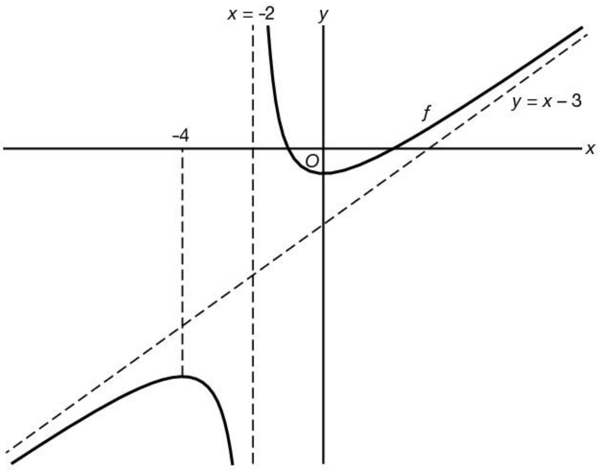
$f'(x) = 0$ geeft $1 - \frac{4}{(x+2)^2} = 0$

$\frac{4}{(x+2)^2} = 1$

$(x+2)^2 = 4$

$x + 2 = -2 \vee x + 2 = 2$

$x = -4 \vee x = 0$



max. is $f(-4) = -9$ en min. is $f(0) = -1$.

c $f(x) = 0$ geeft $x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1 \vee x = 2$

$f(x) \leq 0$ geeft $x < -2 \vee -1 \leq x \leq 2$

24 **a** verticale asymptoot:

$|x| = 0 \wedge x^2 + 3x + 2 \neq 0$

$x = 0$

Dus de verticale asymptoot is de lijn $x = 0$.

scheve asymptoten:

$\lim_{x\to\infty} \frac{x^2+3x+2}{|x|} = \lim_{x\to\infty} \frac{x^2+3x+2}{x} = \lim_{x\to\infty} \left(x + 3 + \frac{2}{x}\right)$

$\lim_{x\to\infty} \frac{2}{x} = 0$, dus de lijn $y = x + 3$ is scheve asymptoot.

$\lim_{x\to-\infty} \frac{x^2+3x+2}{|x|} = \lim_{x\to-\infty} \frac{x^2+3x+2}{-x} = \lim_{x\to-\infty} \left(-x - 3 - \frac{2}{x}\right)$

$\lim_{x\to-\infty} \frac{2}{x} = 0$, dus de lijn $y = -x - 3$ is scheve asymptoot.

b Voor $x > 0$ is $f(x) = \frac{x^2+3x+2}{|x|} = \frac{x^2+3x+2}{x} = x + 3 + \frac{2}{x} = x + 3 + 2x^{-1}$.

Dit geeft $f'(x) = 1 - 2x^{-2} = 1 - \frac{2}{x^2}$.

$f'(x) = 0$ geeft $1 - \frac{2}{x^2} = 0$

$\frac{2}{x^2} = 1$

$x^2 = 2$

$x = \sqrt{2} \vee x = -\sqrt{2}$

vold. vold. niet

Voor $x < 0$ is $f(x) = \dfrac{x^2 + 3x + 2}{|x|} = \dfrac{x^2 + 3x + 2}{-x} = -x - 3 - \dfrac{2}{x} = -x - 3 - 2x^{-1}$.

Dit geeft $f'(x) = -1 + 2x^{-2} = -1 + \dfrac{2}{x^2}$.

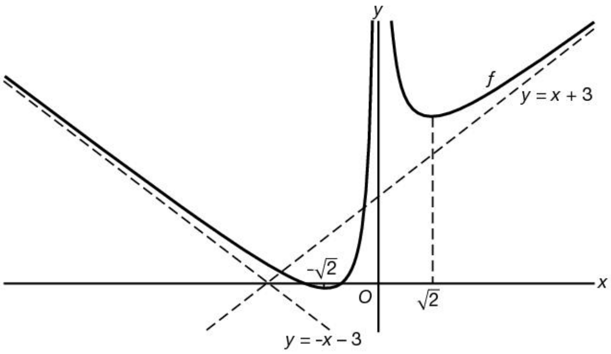
$f'(x) = 0$ geeft $-1 + \dfrac{2}{x^2} = 0$

$\dfrac{2}{x^2} = 1$

$x^2 = 2$

$x = \sqrt{2} \lor x = -\sqrt{2}$

vold. niet vold.



min. is $f(-\sqrt{2}) = \dfrac{4 - 3\sqrt{2}}{\sqrt{2}} = 2\sqrt{2} - 3$ en min. is $f(\sqrt{2}) = \dfrac{4 + 3\sqrt{2}}{\sqrt{2}} = 2\sqrt{2} + 3$

c Voor $x < 0$ geldt $f(x) = 6$ geeft $\dfrac{x^2 + 3x + 2}{-x} = 6$

$x^2 + 3x + 2 = -6x$

$x^2 + 9x + 2 = 0$

$D = 9^2 - 4 \cdot 1 \cdot 2 = 73$

$x = \dfrac{-9 - \sqrt{73}}{2} = -4\frac{1}{2} - \frac{1}{2}\sqrt{73} \lor x = -4\frac{1}{2} + \frac{1}{2}\sqrt{73}$

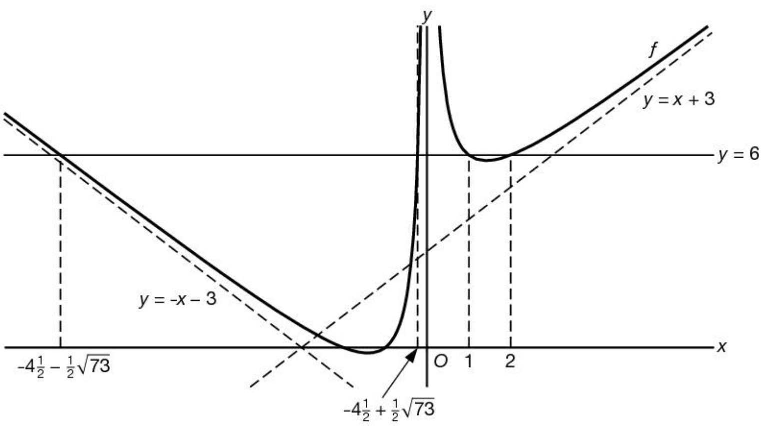
Voor $x > 0$ geldt $f(x) = 6$ geeft $\dfrac{x^2 + 3x + 2}{x} = 6$

$x^2 + 3x + 2 = 6x$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1 \lor x = 2$



$f(x) \leq 6$ geeft $-4\frac{1}{2} - \frac{1}{2}\sqrt{73} \leq x \leq -4\frac{1}{2} + \frac{1}{2}\sqrt{73} \lor 1 \leq x \leq 2$

25 **a** $1 - \sin(x) = 0 \land 1 + 2\sin(x) \neq 0$ geeft $\sin(x) = 1$

$x = \frac{1}{2}\pi + k \cdot 2\pi$

x op $[0, 2\pi]$ geeft als verticale asymptoot de lijn $x = \frac{1}{2}\pi$.

b $f(x) = 0$ geeft $1 + 2\sin(x) = 0$

$2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$

$x = -\frac{1}{6}\pi + k \cdot 2\pi \lor x = 1\frac{1}{6}\pi + k \cdot 2\pi$

x op $[0, 2\pi]$ geeft de nulpunten $1\frac{1}{6}\pi$ en $1\frac{5}{6}\pi$.

Bladzijde 25

26 verticale asymptoten:
$2\sin(x) + 1 = 0 \wedge \cos(x) \neq 0$
$2\sin(x) = -1 \wedge x \neq \frac{1}{2}\pi + k \cdot \pi$
$\sin(x) = -\frac{1}{2} \wedge x \neq \frac{1}{2}\pi + k \cdot \pi$
$\left(x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi\right) \wedge x \neq \frac{1}{2}\pi + k \cdot \pi$
De verticale asymptoten zijn de lijnen $x = 1\frac{5}{6}\pi$ en $x = 1\frac{1}{6}\pi$.
$f(x) = 0$ geeft $\cos(x) = 0$ ofwel $x = \frac{1}{2}\pi + k \cdot \pi$.
Dus $A\left(\frac{1}{2}\pi, 0\right)$.
$f(x) = \dfrac{\cos(x)}{2\sin(x) + 1}$ geeft

$$f'(x) = \frac{(2\sin(x) + 1) \cdot -\sin(x) - \cos(x) \cdot 2\cos(x)}{(2\sin(x) + 1)^2} = \frac{-2\sin^2(x) - \sin(x) - 2\cos^2(x)}{(2\sin(x) + 1)^2} = \frac{-2 - \sin(x)}{(2\sin(x) + 1)^2}$$

Stel k: $y = ax + b$ met $a = f'\left(\frac{1}{2}\pi\right) = \dfrac{-2 - 1}{(2 + 1)^2} = -\frac{1}{3}$.

$$\left.\begin{matrix} y = -\frac{1}{3}x + b \\ A\left(\frac{1}{2}\pi, 0\right) \end{matrix}\right\} -\frac{1}{3} \cdot \frac{1}{2}\pi + b = 0$$
$$b = \frac{1}{6}\pi$$

Dus k: $y = -\frac{1}{3}x + \frac{1}{6}\pi$.

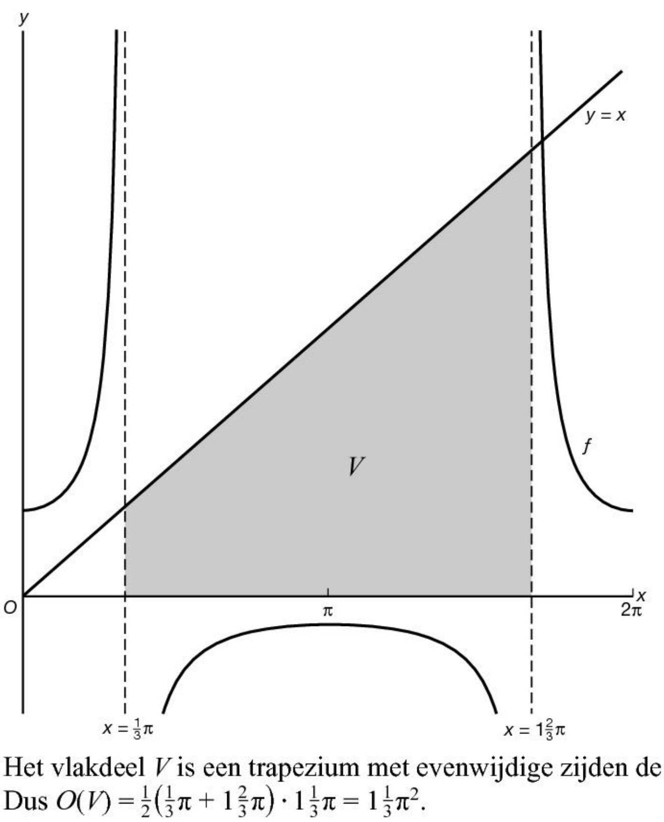
$x = 1\frac{1}{6}\pi$ geeft $y = -\frac{1}{3} \cdot 1\frac{1}{6}\pi + \frac{1}{6}\pi = -\frac{2}{9}\pi$

$x = 1\frac{5}{6}\pi$ geeft $y = -\frac{1}{3} \cdot 1\frac{5}{6}\pi + \frac{1}{6}\pi = -\frac{4}{9}\pi$

Dus $B\left(1\frac{1}{6}\pi, -\frac{2}{9}\pi\right)$ en $C\left(1\frac{5}{6}\pi, -\frac{4}{9}\pi\right)$.

Bladzijde 26

27 **a** verticale asymptoten:
$2\cos(x) - 1 = 0$
$2\cos(x) = 1$
$\cos(x) = \frac{1}{2}$
$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$
$x = [0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 1\frac{2}{3}\pi$
Dus de verticale asymptoten zijn de lijnen $x = \frac{1}{3}\pi$ en $x = 1\frac{2}{3}\pi$.



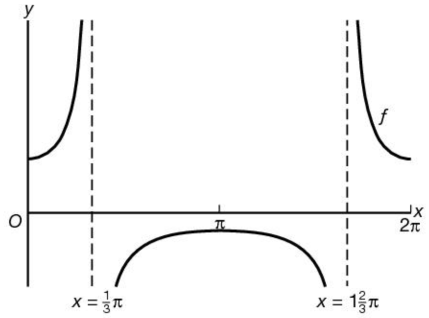
Het vlakdeel V is een trapezium met evenwijdige zijden de lijnen $x = \frac{1}{3}\pi$ en $x = 1\frac{2}{3}\pi$ en hoogte $1\frac{2}{3}\pi - \frac{1}{3}\pi = 1\frac{1}{3}\pi$.
Dus $O(V) = \frac{1}{2}\left(\frac{1}{3}\pi + 1\frac{2}{3}\pi\right) \cdot 1\frac{1}{3}\pi = 1\frac{1}{3}\pi^2$.

b $f(x)=\dfrac{1}{2\cos(x)-1}$ geeft $f'(x)=\dfrac{(2\cos(x)-1)\cdot 0-1\cdot -2\sin(x)}{(2\cos(x)-1)^2}=\dfrac{2\sin(x)}{(2\cos(x)-1)^2}$

$f'(x)=0$ geeft $2\sin(x)=0$

$x=k\cdot\pi$

x op $[0, 2\pi]$ geeft $x=0 \vee x=\pi \vee x=2\pi$.



min. is $f(0)=1$

max. is $f(\pi)=-\frac{1}{3}$

min. is $f(2\pi)=1$

Dus de vergelijking $f(x)=a$ heeft geen oplossing voor $-\frac{1}{3}<a<1$.

28 **a** $f(x)=\dfrac{\cos^2(x)}{\cos(x)+1}$ geeft $f'(x)=\dfrac{(\cos(x)+1)\cdot 2\cos(x)\cdot -\sin(x)-\cos^2(x)\cdot -\sin(x)}{(\cos(x)+1)^2}$

$$=\frac{-2\cos^2(x)\sin(x)-2\cos(x)\sin(x)+\cos^2(x)\sin(x)}{(\cos(x)+1)^2}$$

$$=\frac{-\cos^2(x)\sin(x)-2\cos(x)\sin(x)}{(\cos(x)+1)^2}$$

$$=\frac{-\cos(x)\sin(x)(\cos(x)-2)}{(\cos(x)+1)^2}$$

$f'(x)=0$ geeft $-\cos(x)\sin(x)(\cos(x)-2)=0 \wedge \cos(x)+1\neq 0$

$(\cos(x)=0 \vee \sin(x)=0 \vee \cos(x)=2) \wedge \cos(x)\neq -1$

$\left(x=\frac{1}{2}\pi+k\cdot\pi \vee x=k\cdot\pi\right) \wedge x\neq \pi+k\cdot 2\pi$

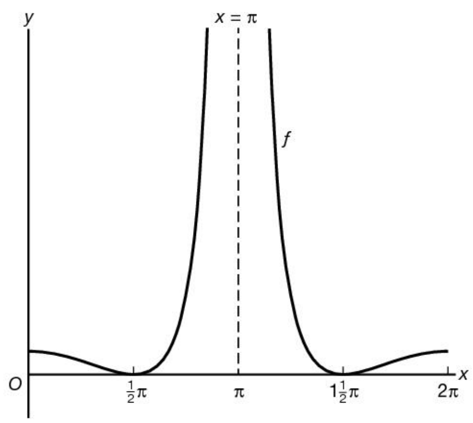
x op $[0, 2\pi]$ geeft $x=0 \vee x=\frac{1}{2}\pi \vee x=1\frac{1}{2}\pi \vee x=2\pi$

$\cos(x)+1=0$

$\cos(x)=-1$

$x=\pi+k\cdot 2\pi$

Dus de verticale asymptoot is de lijn $x=\pi$.



max. is $f(0)=\frac{1}{2}$

min. is $f\left(\frac{1}{2}\pi\right)=0$

min. is $f\left(1\frac{1}{2}\pi\right)=0$

max. is $f(2\pi)=\frac{1}{2}$

b $f(x) = \frac{1}{2}$ geeft

$\frac{\cos^2(x)}{\cos(x) + 1} = \frac{1}{2}$

$2\cos^2(x) = \cos(x) + 1$

$2\cos^2(x) - \cos(x) - 1 = 0$

Stel $\cos(x) = u$.

$2u^2 - u - 1 = 0$

$D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9$

$u = \frac{1+3}{4} = 1 \vee u = \frac{1-3}{4} = -\frac{1}{2}$

$\cos(x) = 1 \vee \cos(x) = -\frac{1}{2}$

$x = k \cdot 2\pi \vee x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$

x op $[0, 2\pi]$ geeft $x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = 2\pi$.

Dus $A\left(\frac{2}{3}\pi, \frac{1}{2}\right)$ en $B\left(1\frac{1}{3}\pi, \frac{1}{2}\right)$.

De afstand van A en B tot de lijn $x = \pi$ is $\frac{1}{3}\pi$.

29 **a** $x = \pi$ geeft $\sqrt{3} - 2\sin(a\pi) = 0 \wedge \cos(a\pi) \neq 0$

$2\sin(a\pi) = \sqrt{3} \wedge a\pi \neq \frac{1}{2}\pi + k \cdot \pi$

$\sin(a\pi) = \frac{1}{2}\sqrt{3} \wedge a \neq \frac{1}{2} + k \cdot 1$

$\left(a\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee a\pi = \frac{2}{3}\pi + k \cdot 2\pi\right) \wedge a \neq \frac{1}{2} + k \cdot 1$

$\left(a = \frac{1}{3} + k \cdot 2 \vee a = \frac{2}{3} + k \cdot 2\right) \wedge a \neq \frac{1}{2} + k \cdot 1$

$a = \frac{1}{3} + k \cdot 2 \vee a = \frac{2}{3} + k \cdot 2$

$0 < a < 1$ geeft $a = \frac{1}{3} \vee a = \frac{2}{3}$

b $f_{\frac{1}{2}}(x) = \frac{\cos\left(\frac{1}{2}x\right)}{\sqrt{3} - 2\sin\left(\frac{1}{2}x\right)}$ geeft

$f_{\frac{1}{2}}'(x) = \frac{\left(\sqrt{3} - 2\sin\left(\frac{1}{2}x\right)\right) \cdot -\frac{1}{2}\sin\left(\frac{1}{2}x\right) - \cos\left(\frac{1}{2}x\right) \cdot -\frac{1}{2} \cdot 2\cos\left(\frac{1}{2}x\right)}{\left(\sqrt{3} - 2\sin\left(\frac{1}{2}x\right)\right)^2}$

$= \frac{-\frac{1}{2}\sqrt{3} \cdot \sin\left(\frac{1}{2}x\right) + \sin^2\left(\frac{1}{2}x\right) + \cos^2\left(\frac{1}{2}x\right)}{\left(\sqrt{3} - 2\sin\left(\frac{1}{2}x\right)\right)^2} = \frac{-\frac{1}{2}\sqrt{3} \cdot \sin\left(\frac{1}{2}x\right) + 1}{\left(\sqrt{3} - 2\sin\left(\frac{1}{2}x\right)\right)^2}$

Stel k: $y = px + q$ met $p = f_{\frac{1}{2}}'(0) = \frac{-0 + 1}{\left(\sqrt{3} - 2 \cdot 0\right)^2} = \frac{1}{3}$ en $q = f_{\frac{1}{2}}(0) = \frac{1}{3}\sqrt{3}$.

Dus k: $y = \frac{1}{3}x + \frac{1}{3}\sqrt{3}$.

verticale asymptoten:

$\sqrt{3} - 2\sin\left(\frac{1}{2}x\right) = 0 \wedge \cos\left(\frac{1}{2}x\right) \neq 0$

$2\sin\left(\frac{1}{2}x\right) = \sqrt{3} \wedge \frac{1}{2}x \neq \frac{1}{2}\pi + k \cdot \pi$

$\sin\left(\frac{1}{2}x\right) = \frac{1}{2}\sqrt{3} \wedge x \neq \pi + k \cdot 2\pi$

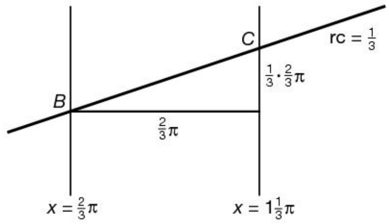
$\left(\frac{1}{2}x = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{2}{3}\pi + k \cdot 2\pi\right) \wedge x \neq \pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot 4\pi \vee x = 1\frac{1}{3}\pi + k \cdot 4\pi$

$0 \leq x \leq 2\pi$ geeft $x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi$

Dus de verticale asymptoten zijn de lijnen $x = \frac{2}{3}\pi$ en $x = 1\frac{1}{3}\pi$.

$y_C - y_B = \frac{1}{3}(x_C - x_B) = \frac{1}{3}\left(1\frac{1}{3}\pi - \frac{2}{3}\pi\right) = \frac{1}{3} \cdot \frac{2}{3}\pi = \frac{2}{9}\pi$

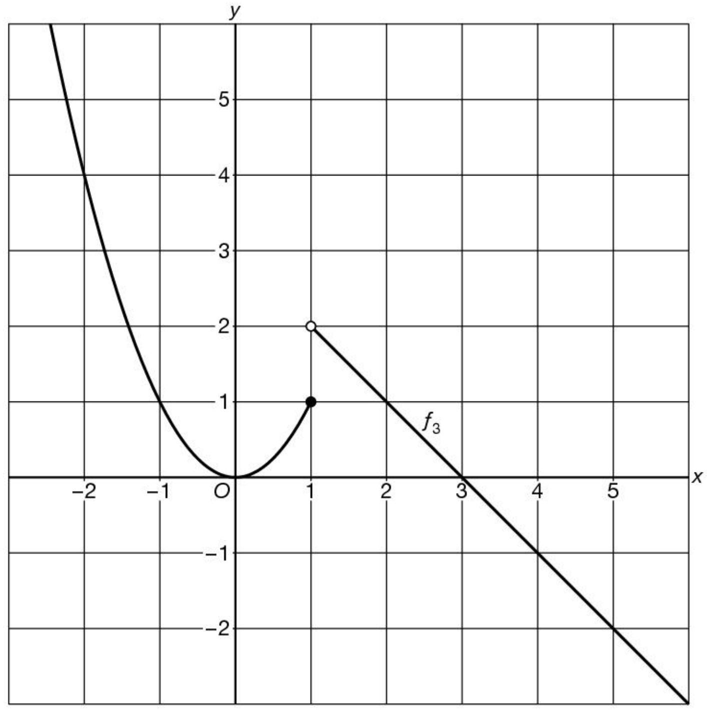


13.3 Limieten en perforaties

Bladzijde 28

30 **a** $f_3(x) = \begin{cases} x^2 \text{ voor } x \leq 1 \\ -x + 3 \text{ voor } x > 1 \end{cases}$

x	−3	−2	−1	0	1	2	3
$f(x)$	9	4	1	0	1	1	0



b Nee, de grafiek van f_3 is geen ononderbroken kromme.

c Voor $p = 2$ is de grafiek van f_p wel een ononderbroken kromme.

Bladzijde 29

31 **a** $\lim\limits_{x \uparrow 1} f_p(x) = \lim\limits_{x \uparrow 1} 2^{x-p} = 2^{1-p}$

$\lim\limits_{x \downarrow 1} f_p(x) = \lim\limits_{x \downarrow 1} (x^2 + 7) = 1 + 7 = 8$

$\lim\limits_{x \to 1} f_p(x)$ bestaat als $2^{1-p} = 8$

$1 - p = 3$

$p = -2$

b $\lim\limits_{x \uparrow 1} f_p(x) = \lim\limits_{x \uparrow 1} e^{x+p} = e^{1+p}$

$\lim\limits_{x \downarrow 1} f_p(x) = \lim\limits_{x \downarrow 1} (x + 3) = 1 + 3 = 4$

$\lim\limits_{x \to 1} f_p(x)$ bestaat als $e^{1+p} = 4$

$1 + p = \ln(4)$

$p = -1 + \ln(4)$

c $\lim\limits_{x \uparrow 1} f_p(x) = \lim\limits_{x \uparrow 1} \ln(x + p) = \ln(1 + p)$

$\lim\limits_{x \downarrow 1} f_p(x) = \lim\limits_{x \downarrow 1} (x^2 + 1) = 1 + 1 = 2$

$\lim\limits_{x \to 1} f_p(x)$ bestaat als $\ln(1 + p) = 2$

$1 + p = e^2$

$p = -1 + e^2$

d $\lim\limits_{x \uparrow 1} f_p(x) = \lim\limits_{x \uparrow 1} |px - 2| = |p - 2|$

$\lim\limits_{x \downarrow 1} f_p(x) = \lim\limits_{x \downarrow 1} (x^2 + 3x) = 1 + 3 = 4$

$\lim\limits_{x \to 1} f_p(x)$ bestaat als $|p - 2| = 4$

$p - 2 = -4 \vee p - 2 = 4$

$p = -2 \vee p = 6$

Bladzijde 30

32 $\lim\limits_{x\uparrow 2} f_{p,q}(x) = \lim\limits_{x\uparrow 2}(x^2 + p) = 4 + p$

$\lim\limits_{x\downarrow 2} f_{p,q}(x) = \lim\limits_{x\downarrow 2}(3x + q) = 6 + q$

$\lim\limits_{x\uparrow 4} f_{p,q}(x) = \lim\limits_{x\uparrow 4}(3x + q) = 12 + q$

$\lim\limits_{x\downarrow 4} f_{p,q}(x) = \lim\limits_{x\downarrow 4}(-x^2 - px + 11) = -16 - 4p + 11 = -4p - 5$

$\lim\limits_{x\to 2} f_{p,q}(x)$ bestaat als $4 + p = 6 + q$

$p - q = 2$

$\lim\limits_{x\to 4} f_{p,q}(x)$ bestaat als $12 + q = -4p - 5$

$4p + q = -17$

$$\begin{cases} p - q = 2 \\ 4p + q = -17 \end{cases} +$$
$$5p = -15$$

$$\left.\begin{matrix} p = -3 \\ p - q = 2 \end{matrix}\right\} \; -3 - q = 2$$
$$q = -5$$

Dus $p = -3$ en $q = -5$.

33 $\lim\limits_{x\uparrow \frac{1}{4}} f_{p,q}(x) = \lim\limits_{x\uparrow \frac{1}{4}} 4\sin(p\pi x) = 4\sin\left(\tfrac{1}{4}\pi p\right)$

$\lim\limits_{x\downarrow \frac{1}{4}} f_{p,q}(x) = \lim\limits_{x\downarrow \frac{1}{4}}(8x^2 + qx + 2) = \tfrac{1}{2} + \tfrac{1}{4}q + 2 = \tfrac{1}{4}q + 2\tfrac{1}{2}$

$\lim\limits_{x\uparrow 3} f_{p,q}(x) = \lim\limits_{x\uparrow 3}(8x^2 + qx + 2) = 72 + 3q + 2 = 3q + 74$

$\lim\limits_{x\downarrow 3} f_{p,q}(x) = (x^4 - 4x - 1) = 81 - 12 - 1 = 68$

$\lim\limits_{x\to 3} f_{p,q}(x)$ bestaat als $3q + 74 = 68$

$3q = -6$

$q = -2$

$\lim\limits_{x\to \frac{1}{4}} f_{p,q}(x)$ bestaat als $4\sin\left(\tfrac{1}{4}\pi p\right) = \tfrac{1}{4}q + 2\tfrac{1}{2}$

$q = -2$ geeft $4\sin\left(\tfrac{1}{4}\pi p\right) = -\tfrac{1}{2} + 2\tfrac{1}{2}$

$4\sin\left(\tfrac{1}{4}\pi p\right) = 2$

$\sin\left(\tfrac{1}{4}\pi p\right) = \tfrac{1}{2}$

$\tfrac{1}{4}\pi p = \tfrac{1}{6}\pi + k\cdot 2\pi \vee \tfrac{1}{4}\pi p = \tfrac{5}{6}\pi + k\cdot 2\pi$

$p = \tfrac{2}{3} + k\cdot 8 \vee p = 3\tfrac{1}{3} + k\cdot 8$

$0 < p < 5$ geeft $p = \tfrac{2}{3} \vee p = 3\tfrac{1}{3}$

Dus $\left(p = \tfrac{2}{3} \vee p = 3\tfrac{1}{3}\right) \wedge q = -2$.

34 $\lim\limits_{x\to 1} f_p(x)$ bestaat als $\lim\limits_{x\uparrow 1} f_p(x) = \lim\limits_{x\downarrow 1} f_p(x)$

$\lim\limits_{x\uparrow 1}(x^2 + 2p) = \lim\limits_{x\downarrow 1}|2x - p^2|$

$1 + 2p = |2 - p^2|$

$2 - p^2 \geq 0$ ofwel $-\sqrt{2} \leq p \leq \sqrt{2}$ geeft $1 + 2p = 2 - p^2$

$p^2 + 2p = 1$

$(p + 1)^2 - 1 = 1$

$(p + 1)^2 = 2$

$p + 1 = -\sqrt{2} \vee p + 1 = \sqrt{2}$

$p = -1 - \sqrt{2} \vee p = -1 + \sqrt{2}$

vold. niet vold.

$2 - p^2 < 0$ ofwel $p < -\sqrt{2} \vee p > \sqrt{2}$ geeft $1 + 2p = -2 + p^2$

$p^2 - 2p - 3 = 0$

$(p + 1)(p - 3) = 0$

$p = -1 \vee p = 3$

vold. niet vold.

Dus $p = -1 + \sqrt{2} \vee p = 3$.

35 $f(4)$ bestaat niet.

$$\lim_{x\uparrow 4} f(x) = \lim_{x\uparrow 4} \frac{2x^2-8x}{x-4} = \lim_{x\uparrow 4} \frac{2x(x-4)}{x-4} = \lim_{x\uparrow 4} 2x = 8 \text{ en}$$

$$\lim_{x\downarrow 4} f(x) = \lim_{x\downarrow 4} \frac{2x^2-8x}{x-4} = \lim_{x\downarrow 4} \frac{2x(x-4)}{x-4} = \lim_{x\downarrow 4} 2x = 8,$$ dus de perforatie is het punt (4, 8).

Bladzijde 32

36 Voor een perforatie mag de functiewaarde niet bestaan. Dat is alleen het geval als de noemer gelijk is aan nul.
Maar als de teller voor een nulpunt van de noemer ongelijk is aan nul, dan bestaat de limiet van $f(x)$ niet. De enige mogelijkheden voor een perforatie is er dus als de teller hetzelfde nulpunt heeft als de noemer.
Je moet nog wel controleren of de limiet van $f(x)$ bestaat.

37 **a** $f_a(x) = \dfrac{9x^2+6x-8}{3x+a} = \dfrac{(3x-2)(3x+4)}{3x+a}$

Dus er is een perforatie voor $a = -2$ en voor $a = 4$.

b $f_a(x) = \dfrac{4x^2-4x-15}{2x+a} = \dfrac{(2x+3)(2x-5)}{2x+a}$

Dus er is een perforatie voor $a = 3$ en voor $a = -5$.

c $f_a(x) = \dfrac{5x^2+3x-2}{x+a} = \dfrac{(5x-2)(x+1)}{x+a} = \dfrac{5\left(x-\frac{2}{5}\right)(x+1)}{x+a}$

Dus er is een perforatie voor $a = -\frac{2}{5}$ en voor $a = 1$.

d $f_a(x) = \dfrac{2x^2-11x+15}{2x+a} = \dfrac{(x-3)(2x-5)}{2x+a} = \dfrac{\frac{1}{2}(2x-6)(2x-5)}{2x+a}$

Dus er is een perforatie voor $a = -6$ en voor $a = -5$.

38 $f_a(x) = \dfrac{2x^2+x-10}{2x+a} = \dfrac{(2x+5)(x-2)}{2x+a} = \dfrac{\frac{1}{2}(2x+5)(2x-4)}{2x+a}$

$$\lim_{x\to -2\frac{1}{2}} f_5(x) = \lim_{x\to -2\frac{1}{2}} \frac{\frac{1}{2}(2x+5)(2x-4)}{2x+5} = \lim_{x\to -2\frac{1}{2}} \tfrac{1}{2}(2x-4) = \tfrac{1}{2}(-5-4) = -4\tfrac{1}{2}$$

Dus voor $a = 5$ is de perforatie $\left(-2\frac{1}{2}, -4\frac{1}{2}\right)$.

$$\lim_{x\to 2} f_{-4}(x) = \lim_{x\to 2} \frac{\frac{1}{2}(2x+5)(2x-4)}{2x-4} = \lim_{x\to 2} \tfrac{1}{2}(2x+5) = \tfrac{1}{2}(4+5) = 4\tfrac{1}{2}$$

Dus voor $a = -4$ is de perforatie $\left(2, 4\frac{1}{2}\right)$.

39 **a** $f_{-2}(x) = \dfrac{4x^2-9x-9}{4x-2} = \dfrac{x(4x-2)+2x-9x-9}{4x-2} = \dfrac{x(4x-2)-7x-9}{4x-2} = x + \dfrac{-7x-9}{4x-2}$

$= x + \dfrac{-\frac{7}{4}(4x-2)-3\frac{1}{2}-9}{4x-2} = x - 1\frac{3}{4} - \dfrac{12\frac{1}{2}}{4x-2}$

$\lim\limits_{x\to\infty} \dfrac{12\frac{1}{2}}{4x-2} = 0$ en $\lim\limits_{x\to-\infty} \dfrac{12\frac{1}{2}}{4x-2} = 0$, dus scheve asymptoot $y = x - 1\frac{3}{4}$.

b $f_4(x) = \dfrac{4x^2-9x-9}{4x+4}$ geeft

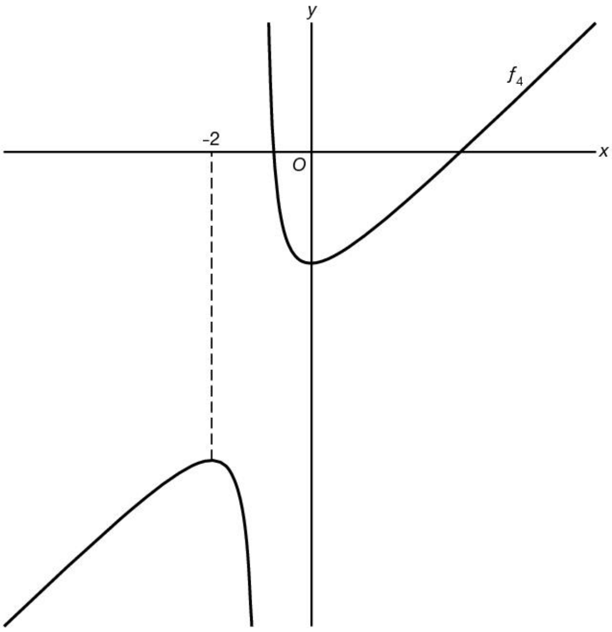
$f_4'(x) = \dfrac{(4x+4)\cdot(8x-9)-(4x^2-9x-9)\cdot 4}{(4x+4)^2} = \dfrac{32x^2-4x-36-16x^2+36x+36}{(4x+4)^2}$

$= \dfrac{16x^2+32x}{(4x+4)^2} = \dfrac{x^2+2x}{(x+1)^2}$

$f_4'(x) = 0$ geeft $x^2+2x = 0$

$x(x+2) = 0$

$x = 0 \vee x = -2$



max. is $f_4(-2) = -6\frac{1}{4}$ en min. is $f_4(0) = -2\frac{1}{4}$.

c $f_a(x) = \dfrac{4x^2 - 9x - 9}{4x + a} = \dfrac{(x-3)(4x+3)}{4x+a} = \dfrac{\frac{1}{4}(4x-12)(4x+3)}{4x+a}$

$\lim\limits_{x \to 3} f_{-12}(x) = \lim\limits_{x \to 3} \dfrac{\frac{1}{4}(4x-12)(4x+3)}{4x-12} = \lim\limits_{x \to 3} \frac{1}{4}(4x+3) = 3\frac{3}{4}$

Dus voor $a = -12$ is de perforatie $\left(3, 3\frac{3}{4}\right)$.

$\lim\limits_{x \to -\frac{3}{4}} f_3(x) = \lim\limits_{x \to -\frac{3}{4}} \dfrac{\frac{1}{4}(4x-12)(4x+3)}{4x+3} = \lim\limits_{x \to -\frac{3}{4}} \frac{1}{4}(4x-12) = \lim\limits_{x \to -\frac{3}{4}} (x-3) = -3\frac{3}{4}$

Dus voor $a = 3$ is de perforatie $\left(-\frac{3}{4}, -3\frac{3}{4}\right)$.

40 $f_a(x) = \dfrac{4x^2 - 9x - 9}{4x + a}$ geeft $f_a'(x) = \dfrac{(4x+a) \cdot (8x-9) - (4x^2 - 9x - 9) \cdot 4}{(4x+a)^2}$

$f_a'(1) = 0$ geeft $(4 + a) \cdot (8 - 9) - (4 - 9 - 9) \cdot 4 = 0$

$-4 - a + 56 = 0$

$a = 52$

$f_{52}'(x) = \dfrac{(4x+52) \cdot (8x-9) - (4x^2 - 9x - 9) \cdot 4}{(4x+52)^2}$

$f_{52}'(x) = 0$ geeft $(4x + 52) \cdot (8x - 9) - (4x^2 - 9x - 9) \cdot 4 = 0$

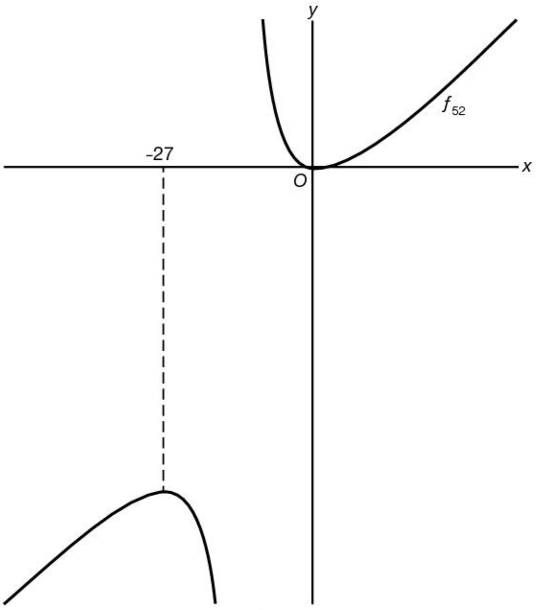
$32x^2 - 36x + 416x - 468 - 16x^2 + 36x + 36 = 0$

$16x^2 + 416x - 432 = 0$

$x^2 + 26x - 27 = 0$

$(x - 1)(x + 27) = 0$

$x = 1 \vee x = -27$



max. is $f_{52}(-27) = -56\frac{1}{4}$

Bladzijde 33

41 $f_a(x) = \dfrac{x^2 - 4ax + 3a^2}{x - a} = \dfrac{(x-a)(x-3a)}{x-a}$

$\lim\limits_{x \to a} f_a(x) = \lim\limits_{x \to a} \dfrac{(x-a)(x-3a)}{x-a} = \lim\limits_{x \to a} (x - 3a) = a - 3a = -2a$

Dus de perforatie is $(a, -2a)$.

$(a, -2a)$ op de lijn $y = x - 3$ geeft $-2a = a - 3$

$-3a = -3$

$a = 1$

42 **a** $\dfrac{3x^2 + 11x}{3x - 1} = \dfrac{x(3x-1) + x + 11x}{3x - 1} = x + \dfrac{12x}{3x-1} = x + \dfrac{4(3x-1)+4}{3x-1} = x + 4 + \dfrac{4}{3x-1}$

$\lim\limits_{x \to -\infty} \dfrac{4}{3x-1} = 0$ en $\lim\limits_{x \to \infty} \dfrac{4}{3x-1} = 0$

Dus de scheve asymptoot is de lijn k: $y = x + 4$.

$|f_0(x) - (x+4)| < 0{,}01$

$\left|x + 4 + \dfrac{4}{3x-1} - x - 4\right| < 0{,}01$

$\left|\dfrac{4}{3x-1}\right| < 0{,}01$

$-0{,}01 < \dfrac{4}{3x-1} < 0{,}01$

$\dfrac{4}{3x-1} = -0{,}01$ geeft $-0{,}03x + 0{,}01 = 4$

$-3x + 1 = 400$

$-3x = 399$

$x = -133$

$\dfrac{4}{3x-1} = 0{,}01$ geeft $0{,}03x - 0{,}01 = 4$

$3x - 1 = 400$

$3x = 401$

$x = 133\frac{2}{3}$

$|f_0(x) - (x+4)| < 0{,}01$ geeft $x < -133 \vee x > 133\frac{2}{3}$

b Wil de grafiek van $f_a(x) = \dfrac{3x^2 + 11x + a}{3x - 1}$ een perforatie hebben, dan moet het nulpunt van de noemer $\left(\frac{1}{3}\right)$ ook nulpunt van de teller zijn.

Dit geeft $3 \cdot \left(\frac{1}{3}\right)^2 + 11 \cdot \frac{1}{3} + a = 0$

$\frac{1}{3} + 3\frac{2}{3} + a = 0$

$a = -4$

$$\lim_{x \to \frac{1}{3}} \frac{3x^2 + 11x - 4}{3x - 1} = \lim_{x \to \frac{1}{3}} \frac{(3x - 1)(x + 4)}{3x - 1} = \lim_{x \to \frac{1}{3}} (x + 4) = 4\tfrac{1}{3}$$

Dus de perforatie is het punt $\left(\frac{1}{3}, 4\frac{1}{3}\right)$.

43 $f(x) = 0$ geeft $x^2 - 9 = 0 \wedge x^2 - 3x \neq 0$

$x^2 = 9 \wedge x(x - 3) \neq 0$

$(x = -3 \vee x = 3) \wedge x \neq 0 \wedge x \neq 3$

$x = -3$

Dus $A(-3, 0)$.

verticale asymptoten:

$x^2 - 3x = 0 \wedge x^2 - 9 \neq 0$

$x(x - 3) = 0 \wedge x^2 \neq 9$

$(x = 0 \vee x = 3) \wedge x \neq -3 \wedge x \neq 3$

$x = 0$

Dus de verticale asymptoot is de lijn $x = 0$.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2 - 9}{x^2 - 3x} = \frac{1 - \dfrac{9}{x^2}}{1 - \dfrac{3}{x}} = \frac{1 - 0}{1 - 0} = 1$$

Dus de horizontale asymptoot is de lijn $y = 1$.

Dus $B(0, 1)$.

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2 - 9}{x^2 - 3x} = \lim_{x \to 3} \frac{(x - 3)(x + 3)}{x(x - 3)} = \lim_{x \to 3} \frac{x + 3}{x} = \frac{3 + 3}{3} = 2$$

Dus de perforatie is $(3, 2)$.

Stel k: $y = ax + 1$ met $a = \text{rc}_k = \dfrac{1 - 0}{0 - -3} = \frac{1}{3}$.

Dus k: $y = \frac{1}{3}x + 1$.

$\left.\begin{array}{l} k\text{: } y = \frac{1}{3}x + 1 \\ \text{door } (3, 2) \end{array}\right\} \frac{1}{3} \cdot 3 + 1 = 2$ klopt.

Dus de lijn k gaat door de perforatie van de grafiek van f.

Bladzijde 34

44 $f(x) = \dfrac{x^3 - 6x^2 + 8x}{x^2 + x - 6} = \dfrac{x(x^2 - 6x + 8)}{(x - 2)(x + 3)} = \dfrac{x(x - 2)(x - 4)}{(x - 2)(x + 3)}$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x(x - 2)(x - 4)}{(x - 2)(x + 3)} = \lim_{x \to 2} \frac{x(x - 4)}{x + 3} = \frac{2(2 - 4)}{2 + 3} = -\tfrac{4}{5}$$

Dus de perforatie is $A\left(2, -\frac{4}{5}\right)$.

verticale asymptoot:

$(x - 2)(x + 3) = 0 \wedge x(x - 2)(x - 4) \neq 0$

$(x = 2 \vee x = -3) \wedge x \neq 0 \wedge x \neq 2 \wedge x \neq 4$

$x = -3$

Dus de verticale asymptoot is de lijn $x = -3$.

scheve asymptoot:

$$f(x) = \frac{x^3 - 6x^2 + 8x}{x^2 + x - 6} = \frac{x(x^2 - 6x + 8)}{(x - 2)(x + 3)} = \frac{x(x - 2)(x - 4)}{(x - 2)(x + 3)} = \frac{x(x - 4)}{x + 3} = \frac{x^2 - 4x}{x + 3}$$

$$= \frac{x(x + 3) - 3x - 4x}{x + 3} = x - \frac{7x}{x + 3} = x - \frac{7(x + 3) - 21}{x + 3} = x - 7 + \frac{21}{x + 3}$$

$\lim_{x \to \infty} \dfrac{21}{x + 3} = 0$ en $\lim_{x \to -\infty} \dfrac{21}{x + 3} = 0$

Dus de scheve asymptoot is de lijn $y = x - 7$.

$\left.\begin{array}{l} y = x - 7 \\ x = -3 \end{array}\right\} y = -3 - 7 = -10$

Dus $B(-3, -10)$.

Stel k: $y = ax + b$ met $a = \text{rc}_k = \dfrac{-10 - -\frac{4}{5}}{-3 - 2} = \frac{46}{25}$

$$\left.\begin{array}{l} y = \frac{46}{25}x + b \\ \text{door } B(-3, -10) \end{array}\right\} \frac{46}{25} \cdot -3 + b = -10$$

$$-\frac{138}{25} + b = -10$$

$$b = -4\frac{12}{25}$$

Dus k: $y = 1\frac{21}{25}x - 4\frac{12}{25}$ en het snijpunt met de y-as is $C\left(0, -4\frac{12}{25}\right)$.

45 **a** $f_{10}(x) = \dfrac{16x^2 + 12x - 10}{4x + 10}$

$f_{10}(0) = \dfrac{0 + 0 - 10}{0 + 10} = -1$, dus $A(0, -1)$.

$f_{10}(x) = \dfrac{16x^2 + 12x - 10}{4x + 10}$ geeft

$$f_{10}'(x) = \frac{(4x + 10) \cdot (32x + 12) - (16x^2 + 12x - 10) \cdot 4}{(4x + 10)^2}$$

$$= \frac{128x^2 + 48x + 320x + 120 - 64x^2 - 48x + 40}{(4x + 10)^2} = \frac{64x^2 + 320x + 160}{(4x + 10)^2}$$

Stel k: $y = ax - 1$ met $a = f_{10}'(0) = \dfrac{160}{100} = 1\frac{3}{5}$

Dus k: $y = 1\frac{3}{5}x - 1$.

b $f_{14}(x) = \dfrac{16x^2 + 12x - 10}{4x + 14}$ geeft

$$f_{14}'(x) = \frac{(4x + 14) \cdot (32x + 12) - (16x^2 + 12x - 10) \cdot 4}{(4x + 14)^2}$$

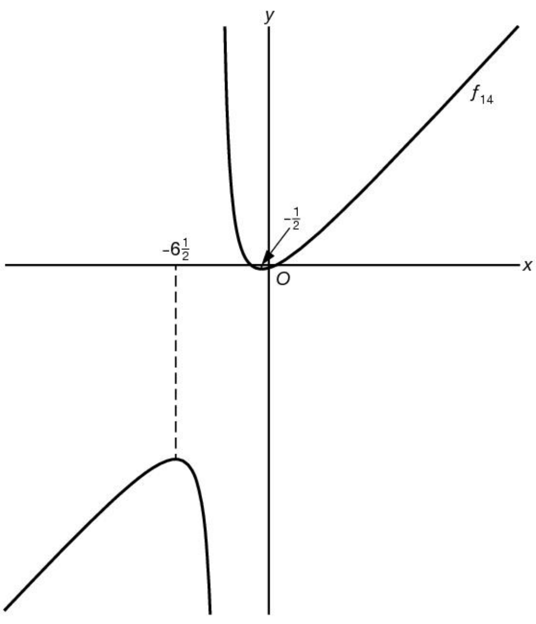
$$= \frac{128x^2 + 48x + 448x + 168 - 64x^2 - 48x + 40}{(4x + 14)^2} = \frac{64x^2 + 448x + 208}{(4x + 14)^2}$$

$f_{14}'(x) = 0$ geeft $64x^2 + 448x + 208 = 0$

$$x^2 + 7x + \tfrac{13}{4} = 0$$

$$\left(x + \tfrac{1}{2}\right)\left(x + \tfrac{13}{2}\right) = 0$$

$$x = -\tfrac{1}{2} \vee x = -6\tfrac{1}{2}$$



max. is $f_{14}\left(-6\frac{1}{2}\right) = -49$ en min. is $f_{14}\left(-\frac{1}{2}\right) = -1$.

c $f_a(x) = \dfrac{16x^2 + 12x - 10}{4x + a} = \dfrac{(4x-2)(4x+5)}{4x+a}$

$\lim\limits_{x \to \frac{1}{2}} f_{-2}(x) = \lim\limits_{x \to \frac{1}{2}} \dfrac{(4x-2)(4x+5)}{4x-2} = \lim\limits_{x \to \frac{1}{2}} (4x+5) = 7$

Dus voor $a = -2$ is de perforatie $\left(\frac{1}{2}, 7\right)$.

$\lim\limits_{x \to -1\frac{1}{4}} f_5(x) = \lim\limits_{x \to -1\frac{1}{4}} \dfrac{(4x-2)(4x+5)}{4x+5} = \lim\limits_{x \to -1\frac{1}{4}} (4x-2) = -7$

Dus voor $a = 5$ is de perforatie $\left(-1\frac{1}{4}, -7\right)$.

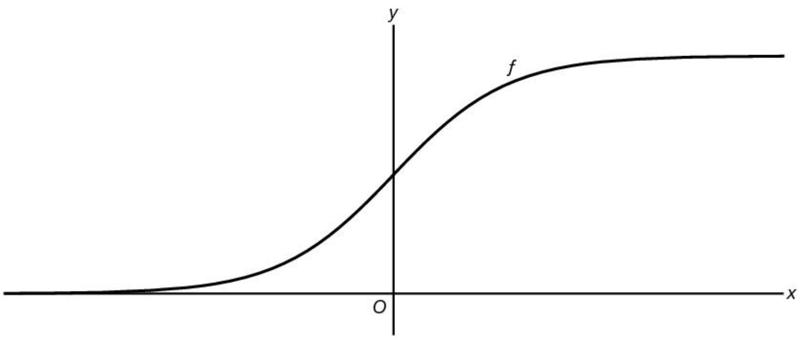
13.4 Limieten bij exponentiële en logaritmische functies

Bladzijde 36

46 a

x	−6	−3	0	3	6
$f(x)$	0,01	0,28	3	5,72	5,99

b



c De lijnen $y = 0$ en $y = 6$.

Bladzijde 38

47 a $\lim\limits_{x \to -\infty} e^{\frac{1}{x}} = e^0 = 1$

$\lim\limits_{x \to \infty} e^{\frac{1}{x}} = e^0 = 1$

Dus de horizontale asymptoot is de lijn $y = 1$.

b $f'(x) = e^{\frac{1}{x}}$ geeft $f'(x) = e^{\frac{1}{x}} \cdot \left[\dfrac{1}{x}\right]' = e^{\frac{1}{x}} \cdot \left[x^{-1}\right]' = e^{\frac{1}{x}} \cdot -x^{-2} = -\dfrac{1}{x^2} \cdot e^{\frac{1}{x}}$

c $f'(-0{,}1) \approx -4{,}5 \times 10^{-3}$ en $f'(-0{,}05) \approx -8{,}2 \times 10^{-7}$

$\lim\limits_{x \uparrow 0} f'(x) = 0$

48 a verticale asymptoot:

$e^x - 2 = 0$

$e^x = 2$

$x = \ln(2)$

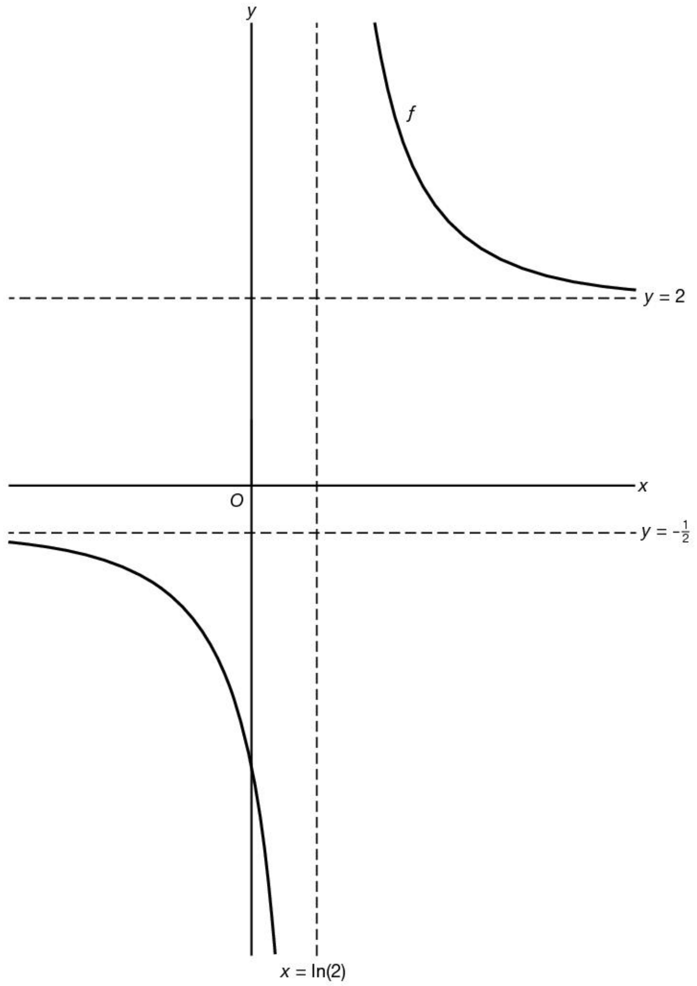
Dus de verticale asymptoot is de lijn $x = \ln(2)$.

horizontale asymptoten:

$\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{2e^x + 1}{e^x - 2} = \lim\limits_{x \to \infty} \dfrac{2 + \dfrac{1}{e^x}}{1 - \dfrac{2}{e^x}} = \dfrac{2+0}{1-0} = 2$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \dfrac{2e^x + 1}{e^x - 2} = \dfrac{0+1}{0-2} = -\frac{1}{2}$

Dus de horizontale asymptoten zijn de lijnen $y = 2$ en $y = -\frac{1}{2}$.



b $f(0) = \dfrac{2e^0 + 1}{e^0 - 2} = \dfrac{3}{-1} = -3$, dus $A(0, -3)$.

$f(x) = \dfrac{2e^x + 1}{e^x - 2}$ geeft $f'(x) = \dfrac{(e^x - 2) \cdot 2e^x - (2e^x + 1) \cdot e^x}{(e^x - 2)^2} = \dfrac{2e^{2x} - 4e^x - 2e^{2x} - e^x}{(e^x - 2)^2} = \dfrac{-5e^x}{(e^x - 2)^2}$

Stel k: $y = ax - 3$ met $a = f'(0) = \dfrac{-5e^0}{(e^0 - 2)^2} = \dfrac{-5}{1} = -5$

Dus k: $y = -5x - 3$.

$y = -\frac{1}{2}$ geeft $-5x - 3 = -\frac{1}{2}$

$-5x = 2\frac{1}{2}$

$x = -\frac{1}{2}$

Dus het snijpunt is $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ en dat ligt op de lijn $y = x$.

49 **a** $\lim\limits_{x \to \infty} \left(10 - 4 \cdot \left(\frac{1}{2}\right)^x\right) = 10 - 4 \cdot 0 = 10$

b $\lim\limits_{x \to \infty} (2 + 4 \cdot e^{-x}) = 2 + 4 \cdot 0 = 2$

c $\lim\limits_{x \to -\infty} \dfrac{e^x - 3}{e^x + 2} = \dfrac{0 - 3}{0 + 2} = -1\frac{1}{2}$

d $\lim\limits_{x \to -\infty} \dfrac{2 \cdot 3^x + 1}{3 \cdot 2^x + 1} = \dfrac{2 \cdot 0 + 1}{3 \cdot 0 + 1} = 1$

50 **a** verticale asymptoot:

$e^x - e = 0 \wedge 5e^x \neq 0$

$e^x = e^1$

$x = 1$

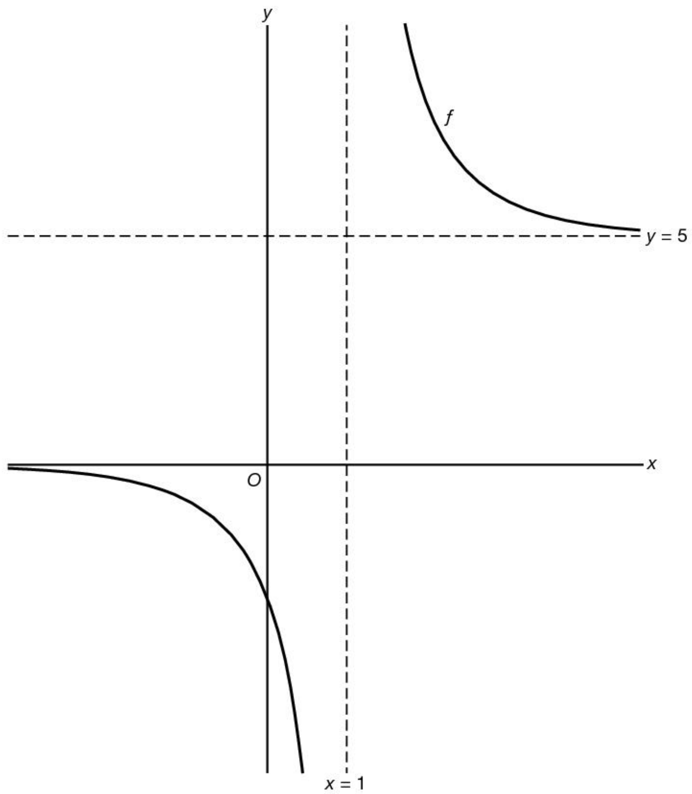
Dus de verticale asymptoot is de lijn $x = 1$.

horizontale asymptoten:

$\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{5e^x}{e^x - e} = \lim\limits_{x \to \infty} \dfrac{5}{1 - e^{1-x}} = \dfrac{5}{1 - 0} = 5$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \dfrac{5e^x}{e^x - e} = \dfrac{0}{0 - e} = 0$

Dus de horizontale asymptoten zijn de lijnen $y = 5$ en $y = 0$.



b $B_f = \langle \leftarrow, 0 \rangle$ en $\langle 5, \rightarrow \rangle$.

c $f(x) = \dfrac{5e^x}{e^x - e}$ geeft $f'(x) = \dfrac{(e^x - e) \cdot 5e^x - 5e^x \cdot e^x}{(e^x - e)^2} = \dfrac{5e^{2x} - 5e^{x+1} - 5e^{2x}}{(e^x - e)^2} = \dfrac{-5e^{x+1}}{(e^x - e)^2}$

$rc_k = f'(0) = \dfrac{-5e^1}{(e^0 - e)^2} = \dfrac{-5e}{(1 - e)^2}$

51 **a** Geen verticale asymptoot, dus het stelsel vergelijkingen $e^x + q = 0 \wedge -e^x - 1 \neq 0$ heeft geen oplossing.
Dat is het geval als $q \geq 0$.

b $\lim\limits_{x \to -\infty} f_{p,q}(x) = -3$

$\lim\limits_{x \to -\infty} \dfrac{pe^x - 1}{e^x + q} = -3$

$\dfrac{p \cdot 0 - 1}{0 + q} = -3$

$\dfrac{-1}{q} = -3$

$q = \frac{1}{3}$

$\lim\limits_{x \to \infty} f_{p,q}(x) = 4$

$\lim\limits_{x \to \infty} \dfrac{pe^x - 1}{e^x + q} = 4$

$\lim\limits_{x \to \infty} \dfrac{p - \dfrac{1}{e^x}}{1 + \dfrac{q}{e^x}} = 4$

$\dfrac{p - 0}{1 + 0} = 4$

$p = 4$

Dus $p = 4$ en $q = \frac{1}{3}$.

c $e^2 + q = 0$

$q = -e^2$

$\lim\limits_{x \to \infty} f_{p,q}(x) = 2$

$\lim\limits_{x \to \infty} \dfrac{pe^x - 1}{e^x + q} = 2$

$$\lim_{x\to\infty} \frac{p - \frac{1}{e^x}}{1 + \frac{q}{e^x}} = 2$$

$$\frac{p-0}{1+0} = 2$$

$p = 2$

Dus $p = 2$ en $q = -e^2$.

Dit geeft $f_{2,-e^2}(x) = \dfrac{2e^x - 1}{e^x - e^2}$.

$$\lim_{x\to -\infty} f_{2,-e^2}(x) = \lim_{x\to -\infty} \frac{2e^x - 1}{e^x - e^2} = \frac{2\cdot 0 - 1}{0 - e^2} = \frac{1}{e^2}$$

Dus voor $x \to -\infty$ is de horizontale asymptoot de lijn $y = \dfrac{1}{e^2}$.

d Voor een perforatie voor $x = -\ln(2)$ moet $-\ln(2)$ nulpunt zijn van de teller en van de noemer van $\dfrac{pe^x - 1}{e^x + q}$, dus $pe^{-\ln(2)} - 1 = 0 \wedge e^{-\ln(2)} + q = 0$

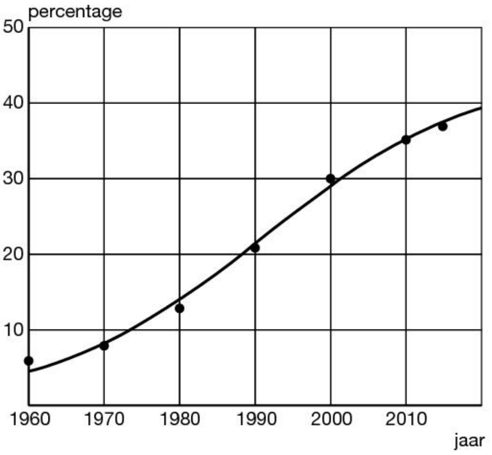
$pe^{\ln(\frac{1}{2})} - 1 = 0 \wedge e^{\ln(\frac{1}{2})} + q = 0$

$p \cdot \frac{1}{2} - 1 = 0 \wedge \frac{1}{2} + q = 0$

$p = 2 \wedge q = -\frac{1}{2}$

Bladzijde 39

52 **a**



b De toename gaat steeds langzamer. Het ziet eruit dat de grafiek van P nog net boven 40 uitkomt.

$$\lim_{x\to\infty} \frac{be^{ct+d}}{e^{ct+d}+1} = \lim_{x\to\infty} \frac{b}{1 + \frac{1}{e^{ct+d}}} = \frac{b}{1+0} = b$$

Dus $b = 42$.

c $\left.\begin{array}{l} P = \dfrac{42e^{ct+d}}{e^{ct+d}+1} \\ t = 0 \text{ en } P = 6 \end{array}\right\}$ $\dfrac{42e^d}{e^d+1} = 6$

$42e^d = 6e^d + 6$

$36e^d = 6$

$e^d = \frac{1}{6}$

$d = \ln\left(\frac{1}{6}\right) = -1{,}791...$

$\left.\begin{array}{l} p = \dfrac{42e^{ct-1{,}791...}}{e^{ct-1{,}791...}+1} \\ t = 55 \text{ en } P = 37 \end{array}\right\}$ $\dfrac{42e^{55c-1{,}791...}}{e^{55c-1{,}791...}+1} = 37$

$42e^{55c-1{,}791...} = 37e^{55c-1{,}791...} + 37$

$5e^{55c-1{,}791...} = 37$

$e^{55c-1{,}791...} = 7{,}4$

$55c - 1{,}791... = \ln(7{,}4)$

$55c - 1{,}791... = 2{,}001...$

$55c = 3{,}793...$

$c = 0{,}0689...$

Dus $c \approx 0{,}069$ en $d \approx -1{,}8$.

d Voer in $y_1 = \dfrac{42e^{0,069x-1,8}}{e^{0,069x-1,8}+1}$.

jaar	1960	1970	1980	1990	2000	2010	2015
percentage	6	8	13	21	30	35	37
model	5,95	10,41	16,65	23,81	30,37	35,23	36,97
verschil	0,05	2,41	3,65	2,81	0,37	0,23	0,03

Dus in 1980 is het verschil het grootst.

53 **a** $\lim\limits_{x \uparrow 0} f(x) = \lim\limits_{\frac{1}{x} \to -\infty} \dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1} = \dfrac{0}{0+1} = 0$

$\lim\limits_{x \downarrow 0} f(x) = \lim\limits_{\frac{1}{x} \to \infty} \dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1} = \lim\limits_{e^{\frac{1}{x}} \to \infty} \dfrac{2}{1+\frac{1}{e^{\frac{1}{x}}}} = \dfrac{2}{1+0} = 2$

b $\lim\limits_{x \to \infty} f(x) = \lim\limits_{\frac{1}{x} \to 0} \dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1} = \dfrac{2e^0}{e^0+1} = \dfrac{2}{1+1} = 1$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{\frac{1}{x} \to 0} \dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1} = \dfrac{2e^0}{e^0+1} = \dfrac{2}{1+1} = 1$

Er is dus één horizontale asymptoot, de formule ervan is $y = 1$.

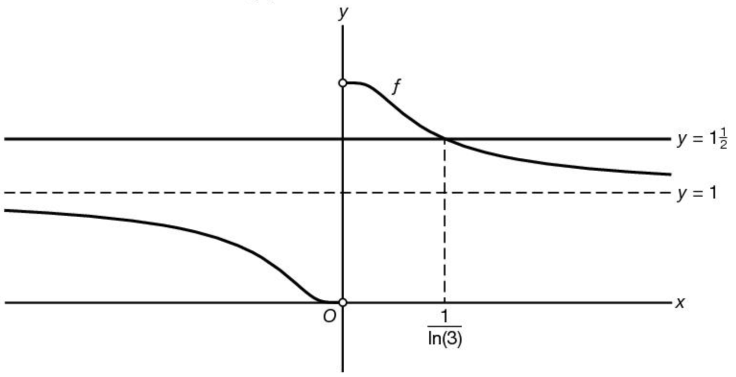
c $f(x) = 1\frac{1}{2}$ geeft $\dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1} = \frac{3}{2}$

$4e^{\frac{1}{x}} = 3e^{\frac{1}{x}} + 3$

$e^{\frac{1}{x}} = 3$

$\dfrac{1}{x} = \ln(3)$

$x = \dfrac{1}{\ln(3)}$



$f(x) \leq 1\frac{1}{2}$ geeft $x < 0 \vee x \geq \dfrac{1}{\ln(3)}$

d $f(x) = \dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$ geeft

$$f'(x) = \frac{\left(e^{\frac{1}{x}}+1\right) \cdot 2 \cdot e^{\frac{1}{x}} \cdot -\frac{1}{x^2} - 2e^{\frac{1}{x}} \cdot e^{\frac{1}{x}} \cdot -\frac{1}{x^2}}{\left(e^{\frac{1}{x}}+1\right)^2} = \frac{2e^{\frac{2}{x}} + 2e^{\frac{1}{x}} - 2e^{\frac{2}{x}}}{\left(e^{\frac{1}{x}}+1\right)^2} \cdot -\frac{1}{x^2} = \frac{-2e^{\frac{1}{x}}}{x^2\left(e^{\frac{1}{x}}+1\right)^2}$$

$\text{rc}_k = f'(1) = \dfrac{-2e^1}{1 \cdot (e^1+1)^2} = \dfrac{-2e}{(e+1)^2}$

Bladzijde 40

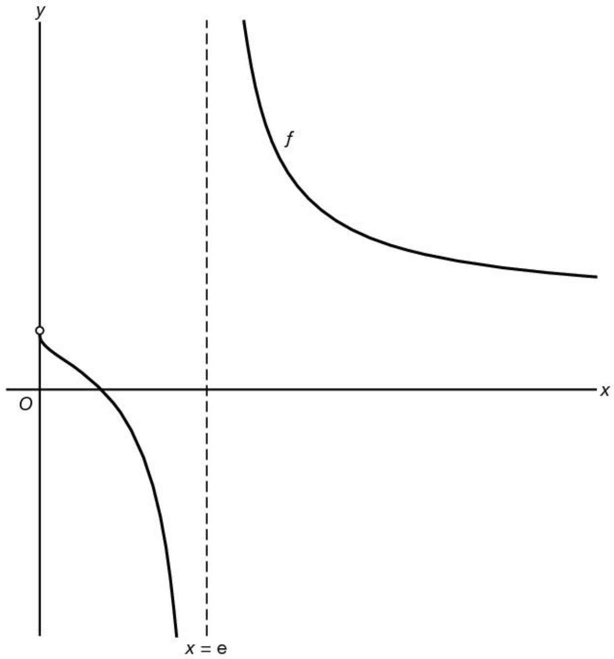
54 **a** verticale asymptoot:
$\ln(x) - 1 = 0 \wedge \ln(x) \neq 0$
$\ln(x) = 1$
$x = \mathrm{e}$
Dus de verticale asymptoot is de lijn $x = \mathrm{e}$.

b $f(x) = 0$ geeft $\ln(x) = 0$
$x = 1$
Dus het nulpunt van f is 1.

c

x	0,00001	0,0001	10	100
$f(x)$	0,92	0,90	1,76	1,28

d



e $\lim\limits_{x \downarrow 0} f(x) = 1$

f ja

Bladzijde 41

55 **a** $\lim\limits_{x \to \infty} \dfrac{4\ln(x)}{1 + \ln(x)} = \lim\limits_{\ln(x) \to \infty} \dfrac{4}{\dfrac{1}{\ln(x)} + 1} = \dfrac{4}{0 + 1} = 4$

b $\lim\limits_{x \to 0} \dfrac{\ln(x^2)}{1 + \ln(x^2)} = \lim\limits_{\ln(x^2) \to 0} \dfrac{1}{\dfrac{1}{\ln(x^2)} + 1} = \dfrac{1}{0 + 1} = 1$

c $\lim\limits_{x \to -\infty} \dfrac{\ln|x|}{4 + \ln(x^2)} = \lim\limits_{x \to -\infty} \dfrac{\ln|x|}{4 + 2\ln|x|} = \lim\limits_{x \to -\infty} \dfrac{\ln(-x)}{4 + 2\ln(-x)} = \lim\limits_{\ln(-x) \to \infty} \dfrac{1}{\dfrac{4}{\ln(-x)} + 2} = \dfrac{1}{0 + 2} = \frac{1}{2}$

d $\lim\limits_{x \to \infty} \dfrac{\ln^2(x)}{2 - \ln^2(x)} = \lim\limits_{\ln(x) \to \infty} \dfrac{1}{\dfrac{2}{\ln^2(x)} - 1} = \dfrac{1}{0 - 1} = -1$

Bladzijde 42

56 **a** verticale asymptoot:

$2\ln(x) - 3 = 0 \wedge \ln(x) - 1 \neq 0$

$2\ln(x) = 3 \wedge \ln(x) \neq 1$

$\ln(x) = 1\frac{1}{2} \wedge x \neq \text{e}$

$x = \text{e}\sqrt{\text{e}}$

Dus de verticale asymptoot is de lijn $x = \text{e}\sqrt{\text{e}}$.

horizontale asymptoot:

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\ln(x) - 1}{2\ln(x) - 3} = \lim_{\ln(x) \to \infty} \frac{1 - \frac{1}{\ln(x)}}{2 - \frac{3}{\ln(x)}} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$$

Dus de horizontale asymptoot is de lijn $y = \frac{1}{2}$.

b $$\lim_{x \downarrow 0} f(x) = \lim_{x \downarrow 0} \frac{\ln(x) - 1}{2\ln(x) - 3} = \lim_{\ln(x) \to -\infty} \frac{1 - \frac{1}{\ln(x)}}{2 - \frac{3}{\ln(x)}} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$$

c $f(x) = 0$ geeft $\ln(x) - 1 = 0 \wedge 2\ln(x) - 3 \neq 0$

$\ln(x) = 1 \wedge 2\ln(x) \neq 3$

$x = \text{e} \wedge \ln(x) \neq 1\frac{1}{2}$

$x = \text{e}$

Dus $A(\text{e}, 0)$.

$f(x) = \dfrac{\ln(x) - 1}{2\ln(x) - 3}$ geeft

$$f'(x) = \frac{(2\ln(x) - 3) \cdot \frac{1}{x} - (\ln(x) - 1) \cdot \frac{2}{x}}{(2\ln(x) - 3)^2} = \frac{2\ln(x) - 3 - 2\ln(x) + 2}{x(2\ln(x) - 3)^2} = \frac{-1}{x(2\ln(x) - 3)^2}$$

Stel k: $y = ax + b$ met $a = f'(\text{e}) = \dfrac{-1}{\text{e}(2\ln(\text{e}) - 3)^2} = \dfrac{-1}{\text{e}(2 - 3)^2} = -\dfrac{1}{\text{e}}$

$$\left.\begin{array}{l} y = -\frac{1}{\text{e}}x + b \\ A(\text{e}, 0) \end{array}\right\} \begin{array}{l} -\frac{1}{\text{e}} \cdot \text{e} + b = 0 \\ -1 + b = 0 \\ b = 1 \end{array}$$

Dus k: $y = -\dfrac{1}{\text{e}}x + 1$.

d Voor f geldt $y = \dfrac{\ln(x) - 1}{2\ln(x) - 3}$, dus voor f^{inv} geldt $x = \dfrac{\ln(y) - 1}{2\ln(y) - 3}$

$2x\ln(y) - 3x = \ln(y) - 1$

$2x\ln(y) - \ln(y) = 3x - 1$

$(2x - 1)\ln(y) = 3x - 1$

$\ln(y) = \dfrac{3x - 1}{2x - 1}$

$y = \text{e}^{\frac{3x - 1}{2x - 1}}$

Dus $f^{\text{inv}} = \text{e}^{\frac{3x - 1}{2x - 1}}$.

e De horizontale asymptoot van de grafiek van f is de lijn $y = \frac{1}{2}$.

Dus de verticale asymptoot van de grafiek van de inverse is de lijn $x = \frac{1}{2}$.

De verticale asymptoot van de grafiek van f is de lijn $x = \text{e}\sqrt{\text{e}}$.

Dus de horizontale asymptoot van de grafiek van de inverse is de lijn $y = \text{e}\sqrt{\text{e}}$.

57 **a** $\lim\limits_{x \uparrow 0} f(x) = \lim\limits_{x \uparrow 0} \dfrac{\ln(x^2) - 4}{\ln(x^2) - 1} = \lim\limits_{\ln(x^2) \to -\infty} \dfrac{1 - \dfrac{4}{\ln(x^2)}}{1 - \dfrac{1}{\ln(x^2)}} = \dfrac{1-0}{1-0} = 1$

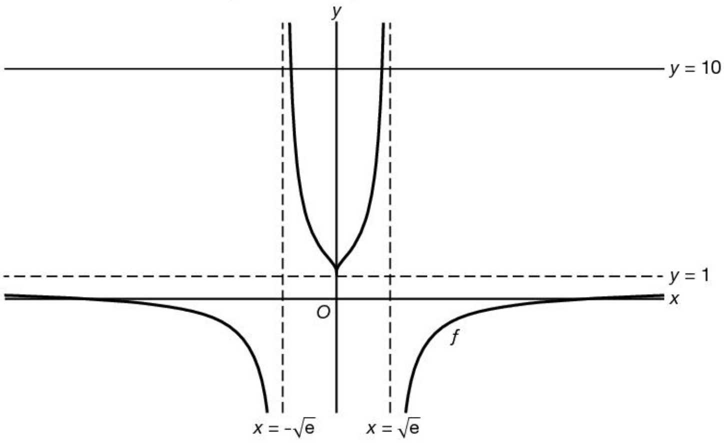
$\lim\limits_{x \downarrow 0} f(x) = \lim\limits_{x \downarrow 0} \dfrac{\ln(x^2) - 4}{\ln(x^2) - 1} = \lim\limits_{\ln(x^2) \to -\infty} \dfrac{1 - \dfrac{4}{\ln(x^2)}}{1 - \dfrac{1}{\ln(x^2)}} = \dfrac{1-0}{1-0} = 1$

b verticale asymptoot:
$\ln(x^2) - 1 = 0 \wedge \ln(x^2) - 4 \neq 0$
$\ln(x^2) = 1 \wedge \ln(x^2) \neq 4$
$x^2 = \mathrm{e}$
$x = \sqrt{\mathrm{e}} \vee x = -\sqrt{\mathrm{e}}$
Dus verticale asymptoten zijn de lijnen $x = \sqrt{\mathrm{e}}$ en $x = -\sqrt{\mathrm{e}}$.
horizontale asymptoot:

$\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to \infty} \dfrac{\ln(x^2) - 4}{\ln(x^2) - 1} = \lim\limits_{\ln(x^2) \to \infty} \dfrac{1 - \dfrac{4}{\ln(x^2)}}{1 - \dfrac{1}{\ln(x^2)}} = \dfrac{1-0}{1-0} = 1$

Dus horizontale asymptoot is de lijn $y = 1$.

c $f(x) = 10$ geeft $\dfrac{\ln(x^2) - 4}{\ln(x^2) - 1} = 10$
$10\ln(x^2) - 10 = \ln(x^2) - 4$
$9\ln(x^2) = 6$
$\ln(x^2) = \frac{2}{3}$
$x^2 = \mathrm{e}^{\frac{2}{3}}$
$x = \mathrm{e}^{\frac{1}{3}} \vee x = -\mathrm{e}^{\frac{1}{3}}$
$x = \sqrt[3]{\mathrm{e}} \vee x = -\sqrt[3]{\mathrm{e}}$



$f(x) \leq 10$ geeft $x < -\sqrt{\mathrm{e}} \vee -\sqrt[3]{\mathrm{e}} \leq x < 0 \vee 0 < x \leq \sqrt[3]{\mathrm{e}} \vee x > \sqrt{\mathrm{e}}$

Diagnostische toets

Bladzijde 44

1 $\left.\begin{array}{l} y = a \cdot \dfrac{2x+3}{5-x} \\ x = 11 \text{ en } y = -5 \end{array}\right\} a \cdot \dfrac{22+3}{5-11} = -5$
$-\frac{25}{6}a = -5$
$a = \frac{30}{25} = 1\frac{1}{5}$
Dus $y = 1\frac{1}{5} \cdot \dfrac{2x+3}{5-x}$.

$\lim\limits_{x \to \infty} 1\frac{1}{5} \cdot \dfrac{2x+3}{5-x} = 1\frac{1}{5} \cdot \dfrac{2 + \dfrac{3}{x}}{\dfrac{5}{x} - 1} = 1\frac{1}{5} \cdot \dfrac{2+0}{0-1} = 1\frac{1}{5} \cdot -2 = -2\frac{2}{5}$

2 $\left.\begin{array}{l}\dfrac{|x|}{2x+5}=\dfrac{a}{y}\\ x=-6 \text{ en } y=2\end{array}\right\}$ $\dfrac{a}{2}=\dfrac{6}{-12+5}$

$\dfrac{a}{2}=-\frac{6}{7}$

$a=-\frac{12}{7}$

$-\dfrac{12}{7y}=\dfrac{|x|}{2x+5}$ geeft $7y|x|=-12(2x+5)$

Dus $y=\dfrac{-24x-60}{7|x|}$

$$\lim_{x\to-\infty}\frac{-24x-60}{7|x|}=\lim_{x\to-\infty}\frac{-24x-60}{-7x}=\lim_{x\to-\infty}\frac{-24-\frac{60}{x}}{-7}=\frac{-24}{-7}=3\tfrac{3}{7}$$

3 **a** Voor f geldt $y=\dfrac{2x-3}{x+2}$, dus voor f^{inv} geldt $x=\dfrac{2y-3}{y+2}$

$xy+2x=2y-3$

$xy-2y=-2x-3$

$y(x-2)=-2x-3$

$y=\dfrac{-2x-3}{x-2}$

Dus $f^{\text{inv}}(x)=\dfrac{-2x-3}{x-2}$.

b Voor g geldt $y=2+\dfrac{3}{5x+2}$, dus voor g^{inv} geldt $x=2+\dfrac{3}{5y+2}$

$x-2=\dfrac{3}{5y+2}$

$5xy+2x-10y-4=3$

$(5x-10)y=-2x+7$

$y=\dfrac{-2x+7}{5x-10}$

Dus $g^{\text{inv}}(x)=\dfrac{-2x+7}{5x-10}$.

c Voor h geldt $y=\dfrac{8x-5}{x}$, dus voor h^{inv} geldt $x=\dfrac{8y-5}{y}$

$xy=8y-5$

$xy-8y=-5$

$y(x-8)=-5$

$y=\dfrac{-5}{x-8}$

Dus $h^{\text{inv}}(x)=\dfrac{-5}{x-8}$.

4 **a** $$\lim_{x\to-\infty}\frac{x^4+5x}{(x^2+2)(x^2-2)}=\lim_{x\to-\infty}\frac{x^4+5x}{x^4-4}=\lim_{x\to-\infty}\frac{1+\frac{5}{x^3}}{1-\frac{4}{x^4}}=\frac{1+0}{1-0}=1$$

b $$\lim_{x\to\infty}\frac{x^2+2x-2}{x^3+x^2+3}=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^2}-\frac{2}{x^3}}{1+\frac{1}{x}+\frac{3}{x^3}}=\frac{0+0-0}{1+0+0}=\frac{0}{1}=0$$

c $$\lim_{x\to-\infty}\frac{|x^3+x+3|}{(x^2+2)(x-2)}=\lim_{x\to-\infty}\frac{-x^3-x-3}{x^3-2x^2+2x-4}=\lim_{x\to-\infty}\frac{-1-\frac{1}{x^2}-\frac{3}{x^3}}{1-\frac{2}{x}+\frac{2}{x^2}-\frac{4}{x^3}}=\frac{-1-0-0}{1-0+0-0}=-1$$

d $$\lim_{x\to\infty}\frac{|3-x^2|+5x^2+3}{2x^2-5x}=\lim_{x\to\infty}\frac{-3+x^2+5x^2+3}{2x^2-5x}=\lim_{x\to\infty}\frac{6x^2}{2x^2-5x}=\lim_{x\to\infty}\frac{6}{2-\frac{5}{x}}=\frac{6}{2-0}=3$$

5 **a** verticale asymptoot: $x - 1 = 0 \wedge x^2 - 7x + 10 \neq 0$

$x = 1 \wedge x^2 - 7x + 10 \neq 0$

$x = 1$

Dus de verticale asymptoot is de lijn $x = 1$.

$$f(x) = \frac{x^2 - 7x + 10}{x - 1} = \frac{x(x-1) + x - 7x + 10}{x - 1} = x + \frac{-6x + 10}{x - 1} = x + \frac{-6(x-1) - 6 + 10}{x - 1} = x - 6 + \frac{4}{x - 1}$$

$\lim\limits_{x \to \infty} \frac{4}{x - 1} = 0$ en $\lim\limits_{x \to -\infty} \frac{4}{x - 1} = 0$, dus de scheve asymptoot is de lijn $y = x - 6$.

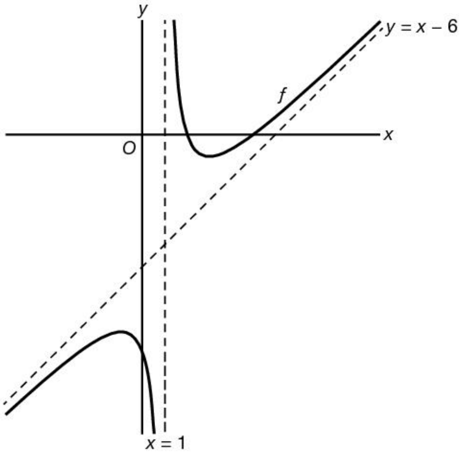
b $f(x) = \frac{x^2 - 7x + 10}{x - 1}$ geeft

$$f'(x) = \frac{(x-1)(2x-7) - (x^2 - 7x + 10)}{(x-1)^2} = \frac{2x^2 - 7x - 2x + 7 - x^2 + 7x - 10}{(x-1)^2} = \frac{x^2 - 2x - 3}{(x-1)^2}$$

$f'(x) = 0$ geeft $x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1 \vee x = 3$



max. is $f(-1) = -9$ en min. is $f(3) = -1$.

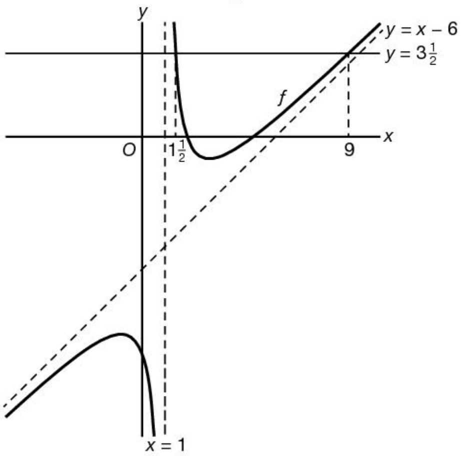
c $f(x) = 3\frac{1}{2}$ geeft $\frac{x^2 - 7x + 10}{x - 1} = 3\frac{1}{2}$

$x^2 - 7x + 10 = 3\frac{1}{2}x - 3\frac{1}{2}$

$x^2 - 10\frac{1}{2}x + 13\frac{1}{2} = 0$

$\left(x - 1\frac{1}{2}\right)(x - 9) = 0$

$x = 1\frac{1}{2} \vee x = 9$



$f(x) \geq 3\frac{1}{2}$ geeft $1 < x \leq 1\frac{1}{2} \vee x \geq 9$

6 $f(x) = \dfrac{\sin(x) + 2}{2\cos(x) - 1}$ geeft

$$f'(x) = \frac{(2\cos(x) - 1) \cdot \cos(x) - (\sin(x) + 2) \cdot -2\sin(x)}{(2\cos(x) - 1)^2} = \frac{2\cos^2(x) - \cos(x) + 2\sin^2(x) + 4\sin(x)}{(2\cos(x) - 1)^2}$$

$$= \frac{2 - \cos(x) + 4\sin(x)}{(2\cos(x) - 1)^2}$$

Stel k: $y = ax + b$ met $a = f'(\pi) = \dfrac{2 - \cos(\pi) + 4\sin(\pi)}{(2\cos(\pi) - 1)^2} = \dfrac{2 - -1 + 0}{(-2 - 1)^2} = \frac{3}{9} = \frac{1}{3}$.

verticale asymptoten: $2\cos(x) - 1 = 0 \wedge \sin(x) + 2 \neq 0$
$\cos(x) = \frac{1}{2}$
$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$
x op $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 1\frac{2}{3}\pi$.
$x_C - x_B = 1\frac{2}{3}\pi - \frac{1}{3}\pi = 1\frac{1}{3}\pi$
$rc_k = \frac{1}{3}$ geeft $y_C - y_B = \frac{1}{3} \cdot 1\frac{1}{3}\pi = \frac{4}{9}\pi$
De stelling van Pythagoras geeft

$$d(B, C) = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{\left(\tfrac{4}{3}\pi\right)^2 + \left(\tfrac{4}{9}\pi\right)^2} = \sqrt{\tfrac{16}{9}\pi^2 + \tfrac{16}{81}\pi^2} = \sqrt{\tfrac{160}{81}\pi^2} = \tfrac{4}{9}\pi\sqrt{10}$$

Bladzijde 45

7 $\lim\limits_{x\uparrow 1} f_{p,q}(x) = \lim\limits_{x\downarrow 1} f_{p,q}(x)$ en $\lim\limits_{x\uparrow 2} f_{p,q}(x) = \lim\limits_{x\downarrow 2} f_{p,q}(x)$

$\lim\limits_{x\uparrow 1} 2\cos(\pi x) = \lim\limits_{x\downarrow 1}(px^2 + q)$ en $\lim\limits_{x\uparrow 2}(px^2 + q) = \lim\limits_{x\downarrow 2}(2x + 5)$

$-2 = p + q$ en $4p + q = 9$

$$\begin{cases} p + q = -2 \\ 4p + q = 9 \end{cases} -$$
$-3p = -11$
$p = \frac{11}{3} = 3\frac{2}{3}$, $p + q = 2$ } $3\frac{2}{3} + q = 2$
$q = -1\frac{2}{3}$
Dus $p = 2\frac{1}{3}$ en $q = -1\frac{2}{3}$.

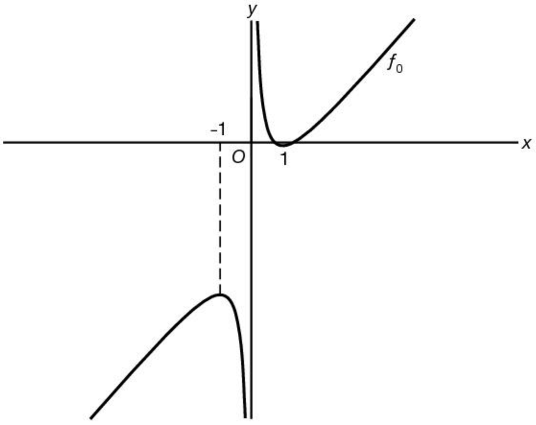
8 **a** $f_1(x) = \dfrac{12x^2 - 25x + 12}{x + 1} = \dfrac{12x(x + 1) - 12x - 25x + 12}{x + 1} = 12x + \dfrac{-37x + 12}{x + 1}$

$= 12x + \dfrac{-37(x + 1) + 37 + 12}{x + 1} = 12x - 37 + \dfrac{49}{x + 1}$

$\lim\limits_{x \to \infty} \dfrac{49}{x + 1} = 0$ en $\lim\limits_{x \to -\infty} \dfrac{49}{x + 1} = 0$, dus de scheve asymptoot is de lijn $y = 12x - 37$.

b $f_0(x) = \dfrac{12x^2 - 25x + 12}{x} = 12x - 25 + 12x^{-1}$ geeft $f_0'(x) = 12 - 12x^{-2} = 12 - \dfrac{12}{x^2}$

$f_0'(x) = 0$ geeft $12 - \dfrac{12}{x^2} = 0$
$x^2 = 1$
$x = -1 \vee x = 1$



max. is $f_0(-1) = -49$ en min. is $f_0(1) = -1$.

c Voor een perforatie is het nulpunt $-a$ van de noemer ook een nulpunt van de teller.
Dus $12(-a)^2 - 25 \cdot -a + 12 = 0$

$12a^2 + 25a + 12 = 0$

$D = 25^2 - 4 \cdot 12 \cdot 12 = 49$

$a = \dfrac{-25-7}{24} = -\frac{4}{3} \vee a = \dfrac{-25+7}{24} = -\frac{3}{4}$

$f_{-\frac{3}{4}}(x) = \dfrac{12\left(x - \frac{3}{4}\right)\left(x - \frac{4}{3}\right)}{x - \frac{3}{4}} = 12\left(x - \frac{4}{3}\right)$ mits $x \neq \frac{3}{4}$

$\lim\limits_{x \to \frac{3}{4}} 12\left(x - \frac{4}{3}\right) = -7$

Dus $\left(\frac{3}{4}, -7\right)$ is een perforatie van de grafiek van $f_{-\frac{3}{4}}$.

$f_{-\frac{4}{3}}(x) = \dfrac{12\left(x - \frac{3}{4}\right)\left(x - \frac{4}{3}\right)}{x - \frac{4}{3}} = 12\left(x - \frac{3}{4}\right)$ mits $x \neq \frac{4}{3}$

$\lim\limits_{x \to \frac{4}{3}} 12\left(x - \frac{3}{4}\right) = 7$

Dus $\left(\frac{4}{3}, 7\right)$ is een perforatie van de grafiek van $f_{-\frac{4}{3}}$.

9 **a** $\lim\limits_{x \to -\infty} \dfrac{2e^x + 5}{3e^x + 2} = \dfrac{0+5}{0+2} = 2\frac{1}{2}$

b $\lim\limits_{x \to \infty} \left(4 - \frac{1}{2}e^{-2x+1}\right) = 4 - 0 = 4$

10 **a** verticale asymptoot:

$e^x - 2 = 0 \wedge e^x + 2 \neq 0$

$e^x = 2$

$x = \ln(2)$

Dus de verticale asymptoot is de lijn $x = \ln(2)$.

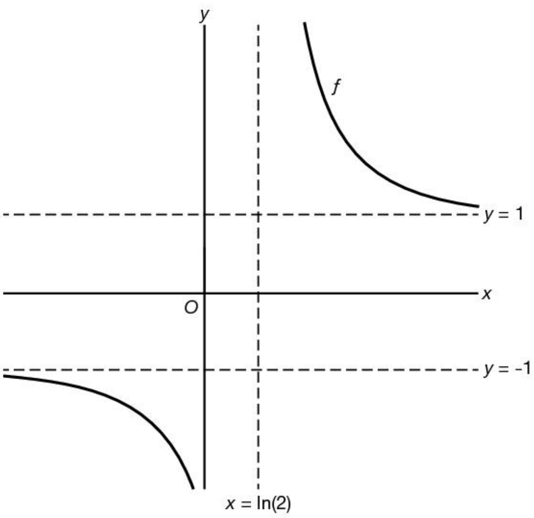
horizontale asymptoten:

$\lim\limits_{x \to -\infty} \dfrac{e^x + 2}{e^x - 2} = \dfrac{0+2}{0-2} = -1$

Dus een horizontale asymptoot is de lijn $y = -1$.

$\lim\limits_{x \to \infty} \dfrac{e^x + 2}{e^x - 2} = \lim\limits_{x \to \infty} \dfrac{1 + \dfrac{2}{e^x}}{1 - \dfrac{2}{e^x}} = \dfrac{1+0}{1-0} = 1$

Dus een horizontale asymptoot is de lijn $y = 1$.



$B_f = \langle \leftarrow, -1 \rangle$ en $\langle 1, \rightarrow \rangle$.

b $f(0) = \dfrac{e^0 + 2}{e^0 - 2} = \dfrac{1 + 2}{1 - 2} = -3$

Dus $A(0, -3)$.

$f(x) = \dfrac{e^x + 2}{e^x - 2}$ geeft $f'(x) = \dfrac{(e^x - 2) \cdot e^x - (e^x + 2) \cdot e^x}{(e^x - 2)^2} = \dfrac{e^{2x} - 2e^x - e^{2x} - 2e^x}{(e^x - 2)^2} = \dfrac{-4e^x}{(e^x - 2)^2}$

Stel k: $y = ax - 3$ met $a = f'(0) = \dfrac{-4e^0}{(e^0 - 2)^2} = \dfrac{-4}{(1 - 2)^2} = -4$

Dus k: $y = -4x - 3$.

$y = 0$ geeft $-4x - 3 = 0$

$-4x = 3$

$x = -\frac{3}{4}$

Dus $S\left(-\frac{3}{4}, 0\right)$.

11 **a** $\lim\limits_{x \downarrow 0} \dfrac{\ln(x) + 1}{1 - \ln(x^2)} = \lim\limits_{x \downarrow 0} \dfrac{\ln(x) + 1}{1 - 2\ln(x)} = \lim\limits_{\ln(x) \to -\infty} \dfrac{1 + \dfrac{1}{\ln(x)}}{\dfrac{1}{\ln(x)} - 2} = \dfrac{1 + 0}{0 - 2} = -\frac{1}{2}$

b $\lim\limits_{x \to \infty} \dfrac{\ln(x) - 6}{\ln(x^2) + 2} = \lim\limits_{x \to \infty} \dfrac{\ln(x) - 6}{2\ln(x) + 2} = \lim\limits_{\ln(x) \to \infty} \dfrac{1 - \dfrac{6}{\ln(x)}}{2 + \dfrac{2}{\ln(x)}} = \dfrac{1 - 0}{2 + 0} = \frac{1}{2}$

12 **a** De nulpunten van de noemer van $\dfrac{\ln(x^2)}{\ln(x^2) + 2}$ zijn geen nulpunten van de teller.

De teller en noemer bestaan beide niet voor $x = 0$. De enige x-coördinaat die in aanmerking komt voor een perforatie is 0.

$\lim\limits_{x \to 0} \dfrac{\ln(x^2)}{\ln(x^2) + 2} = \lim\limits_{\ln(x^2) \to -\infty} \dfrac{\ln(x^2)}{\ln(x^2) + 2} = \lim\limits_{\ln(x^2) \to -\infty} \dfrac{1}{1 + \dfrac{2}{\ln(x^2)}} = \dfrac{1}{1 + 0} = 1$

Dus de perforatie is het punt $(0, 1)$.

b Verticale asymptoten:

$\ln(x^2) + 2 = 0 \wedge \ln(x^2) \neq 0$

$\ln(x^2) = -2$

$x^2 = e^{-2}$

$x = e^{-1} \vee x = -e^{-1}$

Dus de verticale asymptoten zijn de lijnen $x = \dfrac{1}{e}$ en $x = -\dfrac{1}{e}$.

Horizontale asymptoot:

$\lim\limits_{x \to -\infty} \dfrac{\ln(x^2)}{\ln(x^2) + 2} = \lim\limits_{\ln(x^2) \to \infty} \dfrac{1}{1 + \dfrac{2}{\ln(x^2)}} = \dfrac{1}{1 + 0} = 1$

Dus de horizontale asymptoot is de lijn $y = 1$.

14 Meetkunde toepassen

Voorkennis Goniometrie, stellingen en lijnen

Bladzijde 50

1 De cosinusregel in $\triangle BCN$ geeft $CN^2 = BN^2 + BC^2 - 2 \cdot BN \cdot BC \cdot \cos(\angle B)$

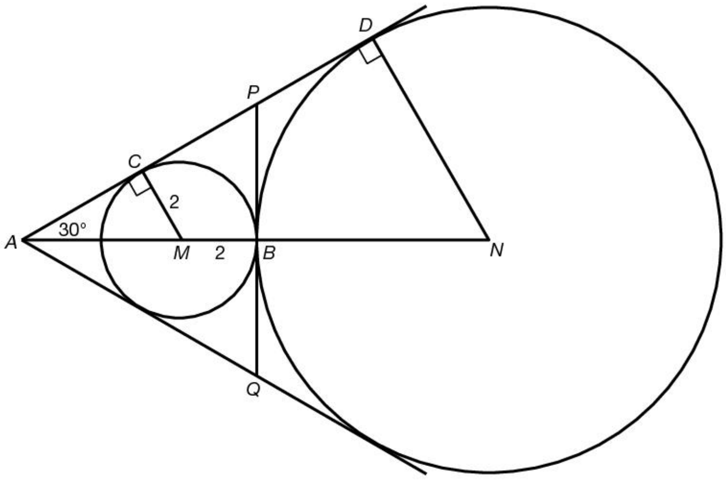
$$CN^2 = \left(2\tfrac{1}{2}\sqrt{3}\right)^2 + \left(2\sqrt{3}\right)^2 - 2 \cdot 2\tfrac{1}{2}\sqrt{3} \cdot 2\sqrt{3} \cdot \cos(60°)$$

$$CN^2 = \tfrac{75}{4} + 12 - 30 \cdot \tfrac{1}{2} = 15\tfrac{3}{4}$$

$$CN = \sqrt{\tfrac{63}{4}} = \tfrac{3}{2}\sqrt{7} = 1\tfrac{1}{2}\sqrt{7}$$

Bladzijde 51

2 **a** Zie de figuur met de punten B, C en D.



$AC = \sqrt{3} \cdot MC = 2\sqrt{3}$ en $AM = 2 \cdot MC = 2 \cdot 2 = 4$

Stel de straal van de cirkel met middelpunt N is r.

Uit $\triangle AMC \backsim \triangle AND$ volgt $\dfrac{MC}{ND} = \dfrac{AM}{AN}$

$$\frac{2}{r} = \frac{4}{4+2+r}$$

$$4r = 12 + 2r$$

$$2r = 12$$

$$r = 6$$

Dus de straal van de cirkel met middelpunt N is 6.

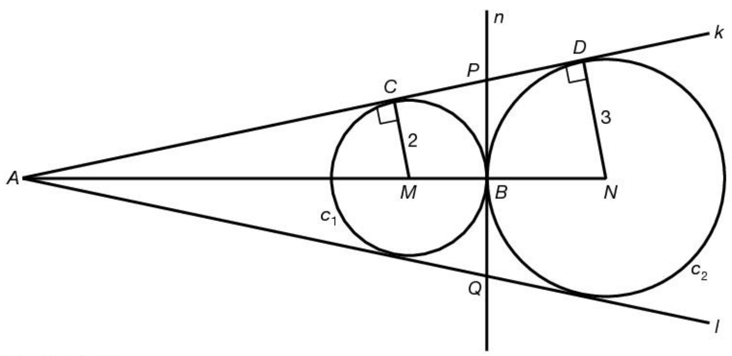
b Uit $\triangle AMC \backsim \triangle APB$ volgt $\dfrac{AC}{AB} = \dfrac{MC}{PB}$

$$\frac{2\sqrt{3}}{4+2} = \frac{2}{PB}$$

$$PB = \frac{12}{2\sqrt{3}} = 2\sqrt{3}$$

$PQ = 2 \cdot PB = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$

3 Zie de figuur met de punten C en D.



Stel $AM = x$.
Uit $\triangle AMC \backsim \triangle AND$ volgt $\frac{AM}{AN} = \frac{MC}{ND}$, dus $\frac{x}{x+5} = \frac{2}{3}$

$3x = 2x + 10$

$x = 10$

Dus $AM = 10$.
$AC = \sqrt{AM^2 - CM^2} = \sqrt{10^2 - 2^2} = \sqrt{96} = 4\sqrt{6}$
Uit $\triangle AMC \backsim \triangle APB$ volgt $\frac{AC}{AB} = \frac{MC}{PB}$

$\frac{4\sqrt{6}}{12} = \frac{2}{PB}$

$PB = \frac{12 \cdot 2}{4\sqrt{6}} = \sqrt{6}$

Dus $PQ = 2 \times PB = 2\sqrt{6}$.

Bladzijde 52

4 Stel l: $y = ax + b$ met $a = \frac{7-1}{5-2} = 2$.

$\left.\begin{array}{l} y = 2x + b \\ A(2, 1) \end{array}\right\} \begin{array}{l} 2 \cdot 2 + b = 1 \\ b = -3 \end{array}$

Dus de formule is l: $y = 2x - 3$.
$2x - 3 = 0$ geeft $x = 1\frac{1}{2}$, dus $\left(1\frac{1}{2}, 0\right)$ en $(0, -3)$ geeft de assenvergelijking l: $\frac{x}{1\frac{1}{2}} + \frac{y}{-3} = 1$.
l: $\frac{x}{1\frac{1}{2}} + \frac{y}{-3} = 1$ geeft de lineaire vergelijking l: $2x - y = 3$.

Stel $x = t$, dan is $y = 2t - 3$, dus een parametervoorstelling is l: $x(t) = t \wedge y(t) = 2t - 3$.

$x = t \wedge y = 2t - 3$ geeft l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, dus een vectorvoorstelling is l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

5 **a** k: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ geeft $\vec{n}_k = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\left.\begin{array}{l} 2x + y = c \\ \text{door } (3, 4) \end{array}\right\} c = 2 \cdot 3 + 4 = 10$

Dus k: $2x + y = 10$.

b k: $2x + y = 7$ geeft k: $\frac{2x}{7} + \frac{y}{7} = 1$ ofwel k: $\frac{x}{3\frac{1}{2}} + \frac{y}{7} = 1$.

c k: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ geeft k: $x = 3 - \lambda \wedge y = 4 + 2\lambda$ ofwel k: $x(t) = 3 - t \wedge y(t) = 4 + 2t$.

6 $(0, -4)$ op $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ b \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ geeft $2 + 2\lambda = 0 \wedge b + 5\lambda = -4$

$2\lambda = -2 \wedge b = -4 - 5\lambda$

$\lambda = -1 \wedge b = -4 - 5 \cdot -1$

$\lambda = -1 \wedge b = 1$

$(2, 1)$ op $y = ax - 4$ geeft $2a - 4 = 1$

$2a = 5$

$a = 2\frac{1}{2}$

Dus $a = 2\frac{1}{2}$ en $b = 1$.

7 $t = 0$ geeft het punt $(-3, 2)$.
$(-3, 2)$ op m: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ p \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ geeft $\begin{cases} 2 - \lambda = -3 \\ p + 3\lambda = 2 \end{cases}$

$\begin{cases} \lambda = 5 \\ p + 15 = 2 \end{cases}$

Dus $p = -13$.
$(2, -13)$ op $x(t) = 2t - 3 \wedge y(t) = qt + 2$ geeft $2t - 3 = 2 \wedge qt + 2 = -13$

$2t = 5 \wedge qt = -15$

$t = 2\frac{1}{2} \wedge 2\frac{1}{2}q = -15$

$t = 2\frac{1}{2} \wedge q = -6$

Dus $p = -13$ en $q = -6$.

Bladzijde 53

8 **a** $\cos(\angle(k, l)) = |\cos(\angle(\vec{n}_k, \vec{n}_l))| = \dfrac{\left|\begin{pmatrix} 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 3 \\ -4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right|} = \dfrac{|3-8|}{\sqrt{9+16} \cdot \sqrt{1+4}} = \dfrac{5}{5\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

$\angle(k, l) \approx 63{,}4°$

b $\cos(\angle(m, n)) = |\cos(\angle(\vec{n}_m, \vec{n}_n))| = \dfrac{\left|\begin{pmatrix} 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix}\right|}{\left|\begin{pmatrix} 4 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ -1 \end{pmatrix}\right|} = \dfrac{|12-1|}{\sqrt{16+1} \cdot \sqrt{9+1}} = \dfrac{11}{\sqrt{17} \cdot \sqrt{10}} = \dfrac{11}{\sqrt{170}}$

$\angle(m, n) \approx 32{,}5°$

c $\cos(\angle(p, q)) = |\cos(\angle(\vec{r}_p, \vec{r}_q))| = \dfrac{\left|\begin{pmatrix} 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 1 \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ 4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -5 \\ 1 \end{pmatrix}\right|} = \dfrac{|-5+4|}{\sqrt{1+16} \cdot \sqrt{25+1}} = \dfrac{1}{\sqrt{17} \cdot \sqrt{26}} = \dfrac{1}{\sqrt{442}}$

$\angle(p, q) \approx 87{,}3°$

9 **a** $\vec{r}_k = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ geeft $\vec{n}_k = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$\cos(\angle(k, l)) = |\cos(\angle(\vec{n}_k, \vec{n}_l))| = \dfrac{\left|\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \end{pmatrix}\right|}{\left|\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 4 \\ 5 \end{pmatrix}\right|} = \dfrac{|16-5|}{\sqrt{16+1} \cdot \sqrt{16+25}} = \dfrac{11}{\sqrt{17} \cdot \sqrt{41}} = \dfrac{11}{\sqrt{697}}$

$\angle(k, l) \approx 65{,}4°$

b $\vec{r}_m = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ geeft $\text{rc}_m = 7$
$\tan(\alpha) = 7$ geeft $\alpha = 81{,}86\ldots°$
$\tan(\beta) = 3$ geeft $\beta = 71{,}56\ldots°$
$\alpha - \beta = 10{,}30\ldots°$
Dus $\angle(m, n) \approx 10{,}3°$.

c $\text{rc}_p = \dfrac{3-0}{0-2} = -1{,}5$ en $\text{rc}_q = \dfrac{-2-4}{5-4} = -6$
$\tan(\alpha) = -1{,}5$ geeft $\alpha = -56{,}30\ldots°$
$\tan(\beta) = -6$ geeft $\beta = -80{,}53\ldots°$
$\alpha - \beta = 24{,}22\ldots°$
Dus $\angle(p, q) \approx 24{,}2°$.

14.1 Vergelijkingen in de meetkunde

Bladzijde 54

1 **a** $AB = x$ geeft $BC = \dfrac{x}{\sqrt{3}} = \frac{1}{3}x\sqrt{3}$ en $AC = \frac{2}{3}x\sqrt{3}$

$AC = \frac{2}{3}x\sqrt{3}$ geeft $CD = \dfrac{\frac{2}{3}x\sqrt{3}}{\sqrt{2}} = \dfrac{\frac{2}{3}x\sqrt{3}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\frac{2}{3}x\sqrt{6}}{2} = \frac{1}{3}x\sqrt{6}$ en $AD = \frac{1}{3}x\sqrt{6}$

Dus de omtrek van $ABCD$ is $AB + BC + CD + AD = x + \frac{1}{3}x\sqrt{3} + \frac{1}{3}x\sqrt{6} + \frac{1}{3}x\sqrt{6} = x + \frac{1}{3}x\sqrt{3} + \frac{2}{3}x\sqrt{6}$.

b omtrek = 10 geeft $x + \frac{1}{3}x\sqrt{3} + \frac{2}{3}x\sqrt{6} = 10$

$x\left(1 + \frac{1}{3}\sqrt{3} + \frac{2}{3}\sqrt{6}\right) = 10$

$x = \dfrac{10}{1 + \frac{1}{3}\sqrt{3} + \frac{2}{3}\sqrt{6}} \approx 3{,}115$

Dus $AB \approx 3{,}115$.

Bladzijde 55

2 **a** $\dfrac{14}{3-\sqrt{2}} = \dfrac{14}{3-\sqrt{2}} \cdot \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{42+14\sqrt{2}}{9-2} = \dfrac{42+14\sqrt{2}}{7} = 6 + 2\sqrt{2}$

b $\frac{4}{\sqrt{3}-1} = \frac{4}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4\sqrt{3}+4}{3-1} = \frac{4\sqrt{3}+4}{2} = 2\sqrt{3}+2 = 2+2\sqrt{3}$

c $\frac{3\sqrt{2}}{\sqrt{2}+\sqrt{5}} = \frac{3\sqrt{2}}{\sqrt{2}+\sqrt{5}} \cdot \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}} = \frac{6-3\sqrt{10}}{2-5} = \frac{6-3\sqrt{10}}{-3} = -2+\sqrt{10}$

d $\frac{12\sqrt{2}}{\sqrt{10}-\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{10}-\sqrt{2}} \cdot \frac{\sqrt{10}+\sqrt{2}}{\sqrt{10}+\sqrt{2}} = \frac{12\sqrt{20}+24}{10-2} = \frac{24\sqrt{5}+24}{8} = 3\sqrt{5}+3 = 3+3\sqrt{5}$

e $\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} \cdot \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \frac{6+2\sqrt{12}+2}{6-2} = \frac{8+2\sqrt{2}}{4} = 2+\frac{1}{2}\sqrt{2}$

f $\frac{6-5\sqrt{2}}{1-\sqrt{2}} = \frac{6-5\sqrt{2}}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{6+\sqrt{2}-10}{1-2} = \frac{-4+\sqrt{2}}{-1} = 4-\sqrt{2}$

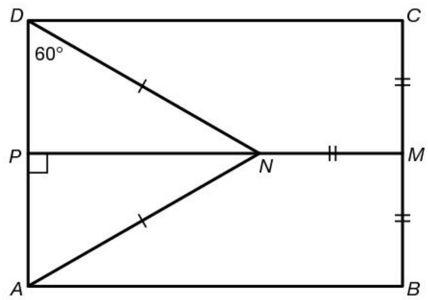
3 Stel $PQ = x$.

$\left.\begin{array}{l} PQ = x \text{ geeft } PM = \frac{x}{\sqrt{2}} = \frac{1}{2}x\sqrt{2} \\ AB = 4 \text{ geeft } AM = \frac{4}{\sqrt{2}} = 2\sqrt{2} \end{array}\right\} AP = AM - PM = 2\sqrt{2} - \frac{1}{2}x\sqrt{2}$

omtrek = 9
$AB + BQ + PQ + AP = 9$
$4 + 2\sqrt{2} - \frac{1}{2}x\sqrt{2} + x + 2\sqrt{2} - \frac{1}{2}x\sqrt{2} = 9$
$x - x\sqrt{2} = 5 - 4\sqrt{2}$
$x(1-\sqrt{2}) = 5 - 4\sqrt{2}$
$x = \frac{5-4\sqrt{2}}{1-\sqrt{2}} = \frac{5-4\sqrt{2}}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{5+\sqrt{2}-8}{1-2} = \frac{-3+\sqrt{2}}{-1} = 3-\sqrt{2}$
Dus $PQ = 3 - \sqrt{2}$.
Dus $a = 3$, $b = -1$ en $c = 2$.

4 Verleng MN tot het punt P op AD.
Stel $AD = x$ geeft $AP = \frac{1}{2}x$, $NM = BM = \frac{1}{2}x$ en $NP = \frac{1}{2}x\sqrt{3}$,
dus $CD = MP = \frac{1}{2}x + \frac{1}{2}x\sqrt{3}$.
omtrek = 20 geeft
$2 \cdot AD + 2 \cdot CD = 20$
$2x + 2\left(\frac{1}{2}x + \frac{1}{2}x\sqrt{3}\right) = 20$
$2x + x + x\sqrt{3} = 20$
$3x + x\sqrt{3} = 20$
$x(3+\sqrt{3}) = 20$
$x = \frac{20}{3+\sqrt{3}} = \frac{20}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{60-20\sqrt{3}}{9-3} = 10 - 3\frac{1}{3}\sqrt{3}$

Dus $AD = 10 - 3\frac{1}{3}\sqrt{3}$.



Bladzijde 56

5 Teken PQ loodrecht op AB.
Stel $AP = x$.
$AP = x$ geeft $AQ = PQ = \frac{x}{\sqrt{2}}$
$PQ = \frac{x}{\sqrt{2}}$ geeft $BQ = \frac{PQ}{\sqrt{3}} = \frac{x}{\sqrt{6}}$

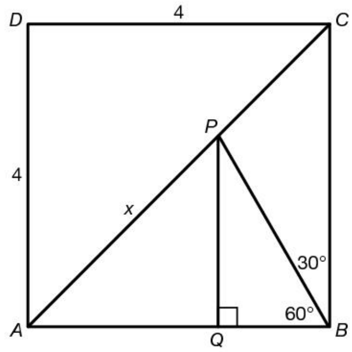
$\left.\begin{array}{l} AB = 4 \\ AB = AQ + BQ \end{array}\right\} \frac{x}{\sqrt{2}} + \frac{x}{\sqrt{6}} = 4$

$x\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\right) = 4$

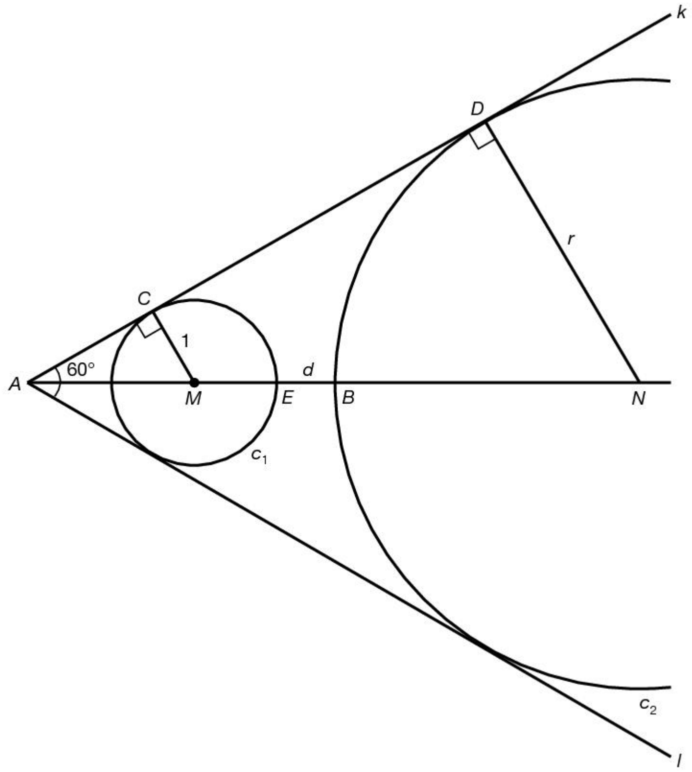
$x = \frac{4}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}} = \frac{4\sqrt{12}}{\frac{\sqrt{12}}{\sqrt{2}} + \frac{\sqrt{12}}{\sqrt{6}}} = \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}} = \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$= \frac{24\sqrt{2}-8\sqrt{6}}{6-2} = 6\sqrt{2} - 2\sqrt{6}$

Dus $AP = 6\sqrt{2} - 2\sqrt{6}$.

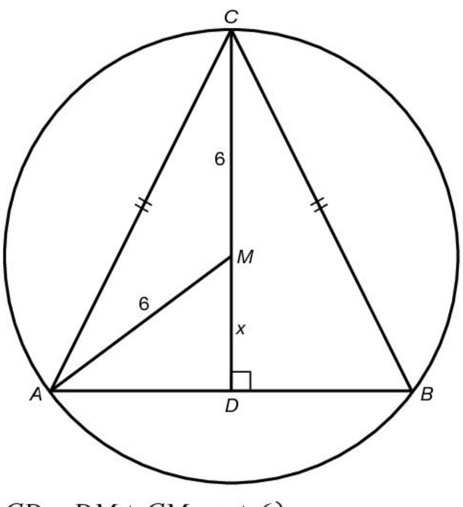


6 Zie de figuur met de letters B, C, D en E.



$MC = 1$ geeft $AM = 2$
$ND = r$ geeft $AN = 2r$
$d = AN - AM - ME - BN = 2r - 2 - 1 - r = r - 3$
$d = r - 3$ geeft $d' = 1$
Dus de snelheid waarmee d verandert is onafhankelijk van r.

7 **a**



b $\left.\begin{array}{l} CD = DM + CM = x + 6 \\ AB = CD \end{array}\right\} \begin{array}{l} AB = x + 6 \\ AD = \frac{1}{2}(x + 6) = \frac{1}{2}x + 3 \end{array}$

c $AD^2 + DM^2 = AM^2$
$\left(\frac{1}{2}x + 3\right)^2 + x^2 = 6^2$
$\frac{1}{4}x^2 + 3x + 9 + x^2 = 36$
$1\frac{1}{4}x^2 + 3x - 27 = 0$

d $1\frac{1}{4}x^2 + 3x - 27 = 0$
$D = 3^2 - 4 \cdot 1\frac{1}{4} \cdot -27 = 144$
$x = \dfrac{-3 - 12}{2\frac{1}{2}} \vee x = \dfrac{-3 + 12}{2\frac{1}{2}}$
$x = -6 \quad \vee x = 3{,}6$
vold. niet   vold.
$\left.\begin{array}{l} AB = x + 6 \\ x = 3{,}6 \end{array}\right\} AB = 9{,}6$

Bladzijde 57

8 Stel $AD = x$.

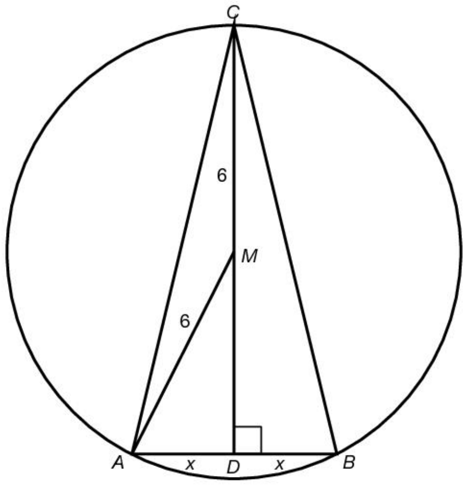
$$\left.\begin{array}{l} AD = x \text{ geeft } AB = 2x \\ CD = 2AB \end{array}\right\} CD = 4x$$

$$\left.\begin{array}{l} CD = 4x \\ CM = 6 \end{array}\right\} MD = 4x - 6$$

De stelling van Pythagoras in $\triangle ADM$ geeft $AD^2 + DM^2 = AM^2$

$$\begin{array}{l} x^2 + (4x-6)^2 = 6^2 \\ x^2 + 16x^2 - 48x + 36 = 36 \\ 17x^2 - 48x = 0 \\ x(17x - 48) = 0 \\ x = 0 \quad \vee \quad x = 2\tfrac{14}{17} \\ \text{vold. niet} \quad \text{vold.} \end{array}$$

$x = 2\frac{14}{17}$ geeft $AB = 2x = 5\frac{11}{17}$



9 Teken de lijn door M en N. Deze snijdt AD in het punt P.
P is het midden van AD, dus $DP = 1$.

$$\left.\begin{array}{l} PN = AD = 2 \\ MN = r \end{array}\right\} MP = 2 - r$$

De stelling van Pythagoras in $\triangle APM$ geeft

$AM^2 = AP^2 + MP^2$

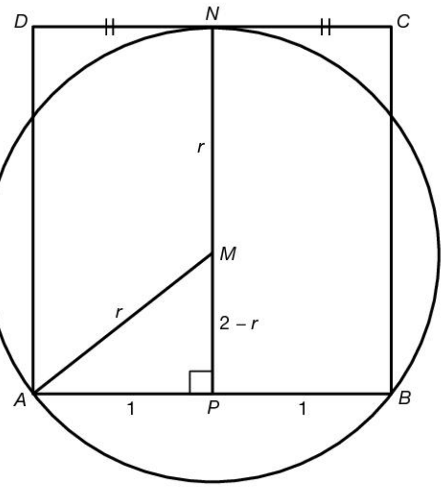
$r^2 = 1^2 + (2-r)^2$

$r^2 = 1 + 4 - 4r + r^2$

$4r = 5$

$r = 1\frac{1}{4}$

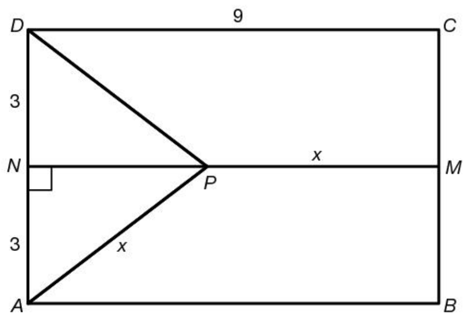
Dus de straal van de cirkel is $1\frac{1}{4}$.



10 Verleng MP.
Stel $AP = x$. Dan is $MP = x$ en $NP = NM - MP = 9 - x$.
De stelling van Pythagoras in $\triangle ANP$ geeft $AP^2 = AN^2 + NP^2$

$$\begin{array}{l} x^2 = 3^2 + (9-x)^2 \\ x^2 = 9 + 81 - 18x + x^2 \\ 18x = 90 \\ x = 5 \end{array}$$

Dus $AP = 5$.



Bladzijde 58

11 Verleng CM.
Stel $AD = x$. Dan is $AC = 1\frac{1}{2}AB = 1\frac{1}{2} \cdot 2x = 3x$.
De stelling van Pythagoras in $\triangle ACD$ geeft $AD^2 + CD^2 = AC^2$

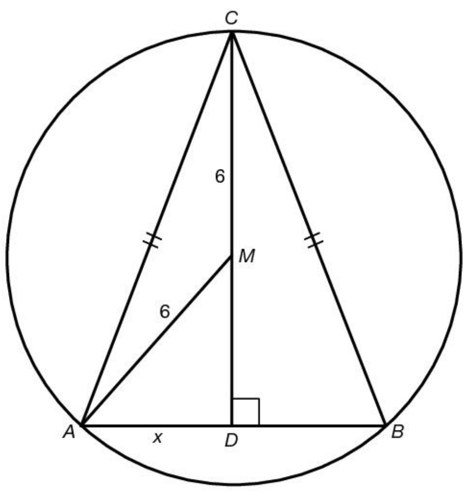
$$\begin{array}{l} x^2 + CD^2 = (3x)^2 \\ CD^2 = 9x^2 - x^2 \\ CD^2 = 8x^2 \\ CD = \sqrt{8x^2} = 2x\sqrt{2} \end{array}$$

$DM = CD - CM = 2x\sqrt{2} - 6$

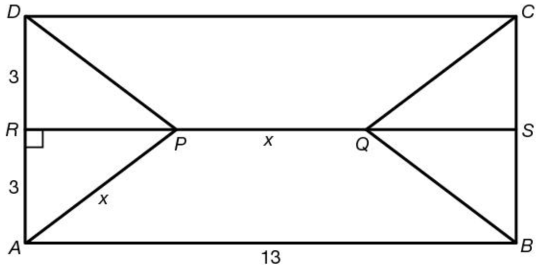
De stelling van Pythagoras in $\triangle ADM$ geeft $AD^2 + DM^2 = AM^2$

$$\begin{array}{l} x^2 + \left(2x\sqrt{2} - 6\right)^2 = 6^2 \\ x^2 + 8x^2 - 24x\sqrt{2} + 36 = 36 \\ 9x^2 - 24x\sqrt{2} = 0 \\ x\left(9x - 24\sqrt{2}\right) = 0 \\ x = 0 \quad \vee \quad x = \tfrac{24}{9}\sqrt{2} \\ \text{vold. niet} \quad \text{vold.} \end{array}$$

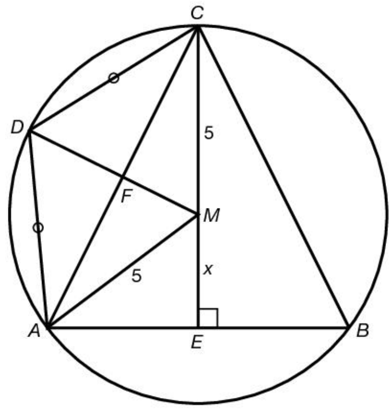
Dus $AB = 2x = 2 \cdot 2\frac{2}{3}\sqrt{2} = 5\frac{1}{3}\sqrt{2}$.



12 Teken RS.
Stel $AP = x$.
$PR = \frac{1}{2}(13 - x) = 6\frac{1}{2} - \frac{1}{2}x$
De stelling van Pythagoras in $\triangle APR$ geeft
$PR^2 + AR^2 = AP^2$
$\left(6\frac{1}{2} - \frac{1}{2}x\right)^2 + 3^2 = x^2$
$42\frac{1}{4} - 6\frac{1}{2}x + \frac{1}{4}x^2 + 9 = x^2$
$169 - 26x + x^2 + 36 = 4x^4$
$3x^2 + 26x - 205 = 0$
$D = 26^2 - 4 \cdot 3 \cdot -205 = 3136$
$x = \dfrac{-26 - 56}{6} = -13\frac{2}{3} \vee x = \dfrac{-26 + 56}{6} = 5$
vold. niet vold.
Dus $AP = 5$.



13 Stel $ME = x$, dan is $CE = x + 5$.
$AB = CE$ geeft $AB = x + 5$ en $AE = \frac{1}{2}AB = \frac{1}{2}x + 2\frac{1}{2}$
De stelling van Pythagoras in $\triangle AEM$ geeft $AE^2 + EM^2 = AM^2$
$\left(\frac{1}{2}x + 2\frac{1}{2}\right)^2 + x^2 = 5^2$
$\frac{1}{4}x^2 + 2\frac{1}{2}x + 6\frac{1}{4} + x^2 = 25$
$1\frac{1}{4}x^2 + 2\frac{1}{2}x - 18\frac{3}{4} = 0$
$5x^2 + 10x - 75 = 0$
$x^2 + 2x - 15 = 0$
$(x - 3)(x + 5) = 0$
$x = 3 \vee x = -5$
vold. vold. niet
Dus $ME = 3$, $AB = CE = 8$, $AE = 4$ en $AC = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}$.
DM staat loodrecht op AC en $\triangle ACM$ is gelijkbenig, dus $AF = \frac{1}{2}AC = \frac{1}{2} \cdot 4\sqrt{5} = 2\sqrt{5}$.
$FM = \sqrt{AM^2 - AF^2} = \sqrt{5^2 - \left(2\sqrt{5}\right)^2} = \sqrt{25 - 20} = \sqrt{5}$
$DF = 5 - \sqrt{5}$
$AD = \sqrt{AF^2 + DF^2} = \sqrt{\left(2\sqrt{5}\right)^2 + \left(5 - \sqrt{5}\right)^2} = \sqrt{20 + 25 - 10\sqrt{5} + 5} = \sqrt{50 - 10\sqrt{5}}$



14 **a** De cosinusregel in $\triangle ABC$ geeft $BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(\alpha)$
$\left(\sqrt{10}\right)^2 = 3^3 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos(\alpha)$
$24\cos(\alpha) = 9 + 16 - 10$
$\cos(\alpha) = \dfrac{15}{24} = \frac{5}{8}$

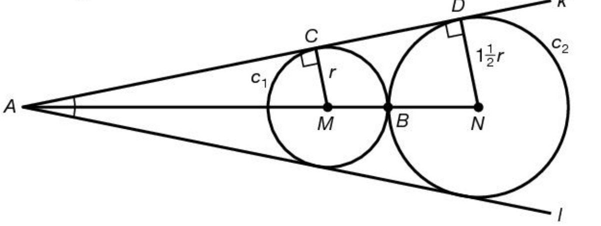
b $\left.\begin{array}{l}\sin^2(\alpha) + \cos^2(\alpha) = 1 \\ \cos(\alpha) = \frac{5}{8}\end{array}\right\}$ $\sin^2(\alpha) + \frac{25}{64} = 1$
$\sin^2(\alpha) = \frac{39}{64}$
$\sin(\alpha) = \sqrt{\frac{39}{64}} = \frac{1}{8}\sqrt{39} \vee \sin(\alpha) = -\frac{1}{8}\sqrt{39}$
vold. vold. niet

Dus $\sin(\alpha) = \frac{1}{8}\sqrt{39}$.

c $O(\triangle ABC) = \frac{1}{2} \cdot AB \cdot AC \cdot \sin(\alpha) = \frac{1}{2} \cdot 3 \cdot 4 \cdot \frac{1}{8}\sqrt{39} = \frac{3}{4}\sqrt{39}$

Bladzijde 60

15



Stel $AM = x$.

Uit $\triangle AMC \backsim \triangle AND$ volgt $\frac{AM}{AN} = \frac{MC}{ND}$, dus $\frac{x}{x + 2\frac{1}{2}r} = \frac{r}{1\frac{1}{2}r}$

$$\frac{x}{x + 2\frac{1}{2}r} = \frac{2}{3}$$
$$3x = 2x + 5r$$
$$x = 5r$$

$$\left.\begin{array}{l}\sin^2(\angle MAC) + \cos^2(\angle MAC) = 1 \\ \sin(\angle MAC) = \frac{MC}{AM} = \frac{r}{5r} = \frac{1}{5}\end{array}\right\} \begin{array}{l}\frac{1}{25} + \cos^2(\angle MAC) = 1 \\ \cos^2(\angle MAC) = \frac{24}{25}\end{array}$$

$\cos(\angle MAC) = \sqrt{\frac{24}{25}} = \frac{2}{5}\sqrt{6} \vee \cos(\angle MAC) = -\frac{2}{5}\sqrt{6}$

vold. vold. niet

$\sin(\angle(k, l)) = \sin(2 \cdot \angle MAC) = 2\sin(\angle MAC)\cos(\angle MAC) = 2 \cdot \frac{1}{5} \cdot \frac{2}{5}\sqrt{6} = \frac{4}{25}\sqrt{6}$

16 **a** De stelling van Pythagoras in $\triangle ABD$ geeft $AD = \sqrt{5^2 + 2^2} = \sqrt{29}$.

$\sin(\angle BAD) = \frac{2}{\sqrt{29}}$ en $\cos(\angle BAD) = \frac{5}{\sqrt{29}}$ geeft

$\cos(\angle BAC) = \cos(2 \cdot \angle BAD) = \cos^2(\angle BAD) - \sin^2(\angle BAD) = \frac{25}{29} - \frac{4}{29} = \frac{21}{29}$

b $$\left.\begin{array}{l}\cos(\angle BAC) = \frac{21}{29} \\ \cos(\angle BAC) = \frac{AB}{AC}\end{array}\right\} \begin{array}{l}\frac{5}{AC} = \frac{21}{29} \\ AC = \frac{5 \cdot 29}{21} = 6\frac{19}{21}\end{array}$$

De stelling van Pythagoras in $\triangle ABC$ geeft $BC = \sqrt{AC^2 - AB^2} = \sqrt{\left(6\frac{19}{21}\right)^2 - 5^2} = 4\frac{16}{21}$.

Dus $CD = BC - BD = 4\frac{16}{21} - 2 = 2\frac{16}{21}$.

17 Stel $BD = x$.

$\tan(\alpha) = \frac{x}{5}$, dus $x = 5\tan(\alpha)$.

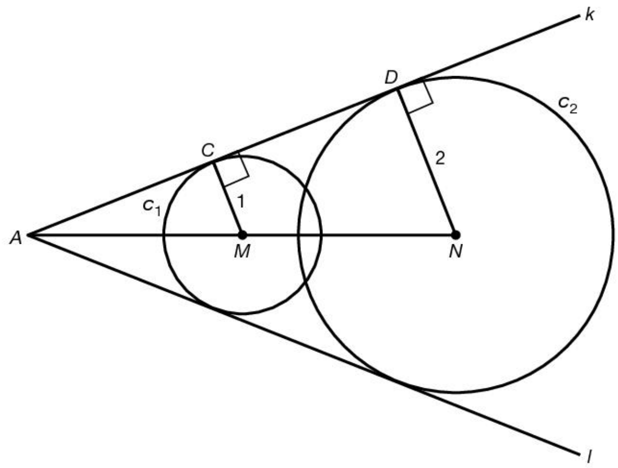
$\tan(2\alpha) = \frac{x + 2}{5}$, dus $x + 2 = 5\tan(2\alpha)$ ofwel $x = 5\tan(2\alpha) - 2$.

Voer in $y_1 = 5\tan(x)$ en $y_2 = 5\tan(2x) - 2$.

Intersect geeft $x = 17{,}95...$ en $y = 1{,}619...$

Dus $BD \approx 1{,}62$.

18



Stel $\angle(k, l) = 2\alpha$, dan is $\angle MAC = \alpha$.

$$\left.\begin{array}{l}\cos(2\alpha) = 2\cos^2(\alpha) - 1 \\ \cos(2\alpha) = \frac{119}{169}\end{array}\right\} 2\cos^2(\alpha) - 1 = \frac{119}{169}$$

$$2\cos^2(\alpha) = \frac{288}{169}$$

$$\cos^2(\alpha) = \frac{144}{169}$$

$\cos(\alpha) = \frac{12}{13} \vee \cos(\alpha) = -\frac{12}{13}$

vold. vold. niet

$$\left.\begin{array}{l}\sin^2(\alpha)+\cos^2(\alpha)=1\\ \cos(\alpha)=\frac{12}{13}\end{array}\right\}\ \sin^2(\alpha)+\frac{144}{169}=1$$

$$\sin^2(\alpha)=\frac{25}{169}$$

$$\sin(\alpha)=\frac{5}{13}\ \vee\ \sin(\alpha)=-\frac{5}{13}$$

vold. vold. niet

$$\left.\begin{array}{l}\sin(\alpha)=\frac{5}{13}\\ \sin(\alpha)=\dfrac{1}{AM}\end{array}\right\}\ \dfrac{1}{AM}=\frac{5}{13}$$

$$AM=2\tfrac{3}{5}$$

$$\left.\begin{array}{l}\sin(\alpha)=\frac{5}{13}\\ \sin(\alpha)=\dfrac{2}{AN}\end{array}\right\}\ \dfrac{2}{AN}=\frac{5}{13}$$

$$AN=5\tfrac{1}{5}$$

$d(M, N)=AN-AM=5\frac{1}{5}-2\frac{3}{5}=2\frac{3}{5}$

14.2 Lijnen en cirkels

Bladzijde 62

19 **a** $\vec{n}_m=\overrightarrow{AB}=\begin{pmatrix}5-1\\0-3\end{pmatrix}=\begin{pmatrix}4\\-3\end{pmatrix}$

m door $M\left(\dfrac{1+5}{2}, \dfrac{3+0}{2}\right)=M(3, 1\tfrac{1}{2})$

$$\left.\begin{array}{l}m\text{: } 4x-3y=c\\ \text{door } M(3, 1\tfrac{1}{2})\end{array}\right\}\ c=4\cdot 3-3\cdot 1\tfrac{1}{2}=7\tfrac{1}{2}$$

Dus m: $4x-3y=7\frac{1}{2}$.

b b is de bissectrice, dus $\angle PAQ=\angle PAR$

$$\sin(\angle PAQ)=\sin(\angle PAR)$$

$$\frac{PQ}{PA}=\frac{PR}{PA}$$

$$PQ=PR$$

c

$$\left.\begin{array}{l}|\overrightarrow{AB}|=\left|\begin{pmatrix}5-2\\2-1\end{pmatrix}\right|=\left|\begin{pmatrix}3\\1\end{pmatrix}\right|=\sqrt{9+1}=\sqrt{10}\\ |\overrightarrow{AC}|=\left|\begin{pmatrix}3-2\\4-1\end{pmatrix}\right|=\left|\begin{pmatrix}1\\3\end{pmatrix}\right|=\sqrt{1+9}=\sqrt{10}\end{array}\right\}\ |\overrightarrow{AB}|=|\overrightarrow{AC}|$$

Dus de somvector $\overrightarrow{AB}+\overrightarrow{AC}$ is de diagonaal van een ruit en valt dus op de bissectrice van $\angle BAC$.

$\vec{r}_k=\overrightarrow{AB}+\overrightarrow{AC}=\begin{pmatrix}3\\1\end{pmatrix}+\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}4\\4\end{pmatrix}\triangleq\begin{pmatrix}1\\1\end{pmatrix}$

Dus k: $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\end{pmatrix}$.

d $\overrightarrow{AD}=\begin{pmatrix}8-2\\3-1\end{pmatrix}=\begin{pmatrix}6\\2\end{pmatrix}=2\cdot\begin{pmatrix}3\\1\end{pmatrix}=2\cdot\overrightarrow{AB}$

Dus D ligt op het verlengde van AB en dus is k ook bissectrice van $\angle DAC$.

$\vec{r}_k=\overrightarrow{AB}+\overrightarrow{AC}=\begin{pmatrix}3\\1\end{pmatrix}+\begin{pmatrix}1\\3\end{pmatrix}=\frac{1}{2}\cdot\begin{pmatrix}6\\2\end{pmatrix}+\begin{pmatrix}1\\3\end{pmatrix}=\frac{1}{2}\cdot\overrightarrow{AD}+\overrightarrow{AC}$

Bladzijde 64

20 **a** $P(x, y)$ op een bissectrice geeft

$d(P, k)=d(P, l)$

$$\frac{|3x-4y-12|}{5}=\frac{|5x+12y-48|}{13}$$

$39x - 52y - 156 = 25x + 60y - 240 \vee 39x - 52y - 156 = -(25x + 60y - 240)$
$14x - 112y = -84 \vee 64x + 8y = 396$
$x - 8y = -6 \vee 8x + y = 49\frac{1}{2}$
Dus m: $x - 8y = -6$ en n: $8x + y = 49\frac{1}{2}$.

b De snijpunten met de y-as zijn $A\left(0, \frac{3}{4}\right)$ en $B\left(0, 49\frac{1}{2}\right)$.
$d(A, B) = y_B - y_A = 49\frac{1}{2} - \frac{3}{4} = 48\frac{3}{4}$

Bladzijde 65

21 **a** BC: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 15 \end{pmatrix} + \lambda\begin{pmatrix} -4 \\ -3 \end{pmatrix}$ en BA: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 15 \end{pmatrix} + \mu\begin{pmatrix} 8 \\ -15 \end{pmatrix}$ geeft

k: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 15 \end{pmatrix} + v\left(17\begin{pmatrix} -4 \\ -3 \end{pmatrix} + 5\begin{pmatrix} 8 \\ -15 \end{pmatrix}\right)$

k: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 15 \end{pmatrix} + v\begin{pmatrix} -28 \\ -126 \end{pmatrix}$

k: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 15 \end{pmatrix} + p\begin{pmatrix} 2 \\ 9 \end{pmatrix}$

b k snijden met AC: $x + y = 12$ geeft $4 + 2p + 15 + 9p = 12$
$11p = -7$
$p = -\frac{7}{11}$

$p = -\frac{7}{11}$ geeft $D\left(4 - \frac{7}{11} \cdot 2, 15 - \frac{7}{11} \cdot 9\right)$ ofwel $D\left(2\frac{8}{11}, 9\frac{3}{11}\right)$

22 **a** $O(0, 0)$ en $A(8, 0)$ geeft middelloodlijn van OA de lijn $x = 4$
$O(0, 0)$ en $B(2, 6)$ geeft $\vec{r}_{OB} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, dus mll_{OB}: $x + 3y = c$.

$\left.\begin{array}{l} x + 3y = c \\ \text{door } (1, 3)\text{, het midden van } OB \end{array}\right\} c = 1 + 3 \cdot 3 = 10$

Dus de middelloodlijn van OB is de lijn $x + 3y = 10$.
Snijden van de lijnen $x + 3y = 10$ en $x = 4$ geeft $4 + 3y = 10$
$3y = 6$
$y = 2$

Dus $M(4, 2)$.

b $r = d(M, O) = \sqrt{4^2 + 2^2} = \sqrt{20}$
Dus de omgeschreven cirkel is $(x - 4)^2 + (y - 2)^2 = 20$.

23 k is de middelloodlijn van het lijnstuk AB en l is de middelloodlijn van het lijnstuk BC.
$A(3, 0)$ en $B(7, 4)$ geeft het midden M van AB is $(5, 2)$ en $\vec{r}_{AB} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$\left.\begin{array}{l} k\text{: } x + y = d \\ \text{door } M(5, 2) \end{array}\right\} d = 5 + 2 = 7$

Dus k: $x + y = 7$.

$B(7, 4)$ en $C(5, 6)$ geeft het midden N van BC is $(6, 5)$ en $\vec{r}_{BC} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

$\left.\begin{array}{l} l\text{: } -x + y = p \\ \text{door } N(6, 5) \end{array}\right\} p = -6 + 5 = -1$

Dus l: $-x + y = -1$.

k en l snijden geeft $\begin{cases} -x + y = -1 \\ x + y = 7 \end{cases}$ +

$2y = 6$

$\left.\begin{array}{l} y = 3 \\ x + y = 7 \end{array}\right\} \begin{array}{l} x + 3 = 7 \\ x = 4 \end{array}$

Dus het middelpunt van de cirkel c door A, B en C is $M(4, 3)$.

$\left.\begin{array}{l} c\text{: } (x - 4)^2 + (y - 3)^2 = r^2 \\ \text{door } (3, 0) \end{array}\right\} r^2 = (3 - 4)^2 + (0 - 3)^2 = 1 + 9 = 10$

$\left.\begin{array}{l} c\text{: } (x - 4)^2 + (y - 3)^2 = 10 \\ D(1, 4) \end{array}\right\} \begin{array}{l} (1 - 4)^2 + (4 - 3)^2 = 10 \\ 9 + 1 = 10 \end{array}$

Dit klopt, dus D ligt ook op c.
Dus de punten A, B, C en D liggen op één cirkel.

24 **a** $\lambda = 4$ geeft (10, 13), dus (10, 13) op k.

(10, 13) op l_p geeft $\begin{pmatrix}-3\\3\end{pmatrix} + \mu\begin{pmatrix}p\\1\end{pmatrix} = \begin{pmatrix}10\\13\end{pmatrix}$

$-3 + \mu p = 10 \wedge 3 + \mu = 13$

$\mu p = 13 \wedge \mu = 10$

$10p = 13$

$p = 1\frac{3}{10}$

Dus voor $p = 1\frac{3}{10}$.

b $k \perp l_p$ geeft $\begin{pmatrix}2\\3\end{pmatrix} \cdot \begin{pmatrix}p\\1\end{pmatrix} = 0$

$2p + 3 = 0$

$2p = -3$

$p = -1\frac{1}{2}$

c $\cos(\angle(k, l_2)) = \dfrac{\left|\begin{pmatrix}2\\3\end{pmatrix} \cdot \begin{pmatrix}2\\1\end{pmatrix}\right|}{\left|\begin{pmatrix}2\\3\end{pmatrix}\right| \cdot \left|\begin{pmatrix}2\\1\end{pmatrix}\right|} = \dfrac{|4 + 3|}{\sqrt{13} \cdot \sqrt{5}} = \dfrac{7}{\sqrt{65}}$

Dit geeft $\angle(k, l_2) \approx 30°$.

d $\cos(45°) = \dfrac{\left|\begin{pmatrix}2\\3\end{pmatrix} \cdot \begin{pmatrix}p\\1\end{pmatrix}\right|}{\left|\begin{pmatrix}2\\3\end{pmatrix}\right| \cdot \left|\begin{pmatrix}p\\1\end{pmatrix}\right|}$

$\frac{1}{2}\sqrt{2} = \dfrac{|2p + 3|}{\sqrt{13} \cdot \sqrt{p^2 + 1}}$

$\frac{1}{2}\sqrt{26(p^2 + 1)} = |2p + 3|$

$\sqrt{26(p^2 + 1)} = |4p + 6|$

$26p^2 + 26 = 16p^2 + 48p + 36$

$10p^2 - 48p - 10 = 0$

$5p^2 - 24p - 5 = 0$

$(5p + 1)(p - 5) = 0$

$p = -\frac{1}{5} \vee p = 5$

Bladzijde 66

25 **a** k: $x + y = 6$ geeft $\vec{n}_k = \begin{pmatrix}1\\1\end{pmatrix}$ en $\vec{r}_k = \begin{pmatrix}1\\-1\end{pmatrix}$

$\angle(k, l) = 60°$ geeft $\cos(60°) = \dfrac{|\vec{r}_k \cdot \vec{r}_l|}{|\vec{r}_k| \cdot |\vec{r}_l|}$

$\frac{1}{2} = \dfrac{\left|\begin{pmatrix}1\\-1\end{pmatrix} \cdot \begin{pmatrix}1\\p\end{pmatrix}\right|}{\left|\begin{pmatrix}1\\-1\end{pmatrix}\right| \cdot \left|\begin{pmatrix}1\\p\end{pmatrix}\right|}$

$\frac{1}{2} = \dfrac{|1 - p|}{\sqrt{2} \cdot \sqrt{1 + p^2}}$

$2|1 - p| = \sqrt{2 + 2p^2}$

$4(1 - p)^2 = 2 + 2p^2$

$4(1 - 2p + p^2) = 2 + 2p^2$

$4 - 8p + 4p^2 = 2 + 2p^2$

$2p^2 - 8p + 2 = 0$

$p^2 - 4p + 1 = 0$

$(p - 2)^2 - 4 + 1 = 0$

$(p - 2)^2 = 3$

$p - 2 = \sqrt{3} \vee p - 2 = -\sqrt{3}$

$p = 2 + \sqrt{3} \vee p = 2 - \sqrt{3}$

b $p = 2 + \sqrt{3}$ geeft l_1: $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2 + \sqrt{3}\end{pmatrix}$ en

$p = 2 - \sqrt{3}$ geeft l_2: $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2 - \sqrt{3}\end{pmatrix}$.

26 **a** $\angle(k, l) = 60°$ geeft $\cos(60°) = \dfrac{|\vec{r}_k \cdot \vec{r}_l|}{|\vec{r}_k| \cdot |\vec{r}_l|}$

$$\tfrac{1}{2} = \frac{\left|\begin{pmatrix}\sqrt{3}\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\p\end{pmatrix}\right|}{\left|\begin{pmatrix}\sqrt{3}\\-1\end{pmatrix}\right|\cdot\left|\begin{pmatrix}1\\p\end{pmatrix}\right|}$$

$$\tfrac{1}{2} = \frac{|\sqrt{3} - p|}{2 \cdot \sqrt{1 + p^2}}$$

$$|\sqrt{3} - p| = \sqrt{1 + p^2}$$

$$(\sqrt{3} - p)^2 = 1 + p^2$$

$$3 - 2p\sqrt{3} + p^2 = 1 + p^2$$

$$-2p\sqrt{3} = -2$$

$$p = \frac{-2}{-2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tfrac{1}{3}\sqrt{3}$$

b $\angle(k, l) = 60°$ geeft $\cos(60°) = \dfrac{|\vec{r}_k \cdot \vec{r}_l|}{|\vec{r}_k| \cdot |\vec{r}_l|}$

$$\tfrac{1}{2} = \frac{\left|\begin{pmatrix}\sqrt{3}\\-1\end{pmatrix}\cdot\begin{pmatrix}q\\1\end{pmatrix}\right|}{\left|\begin{pmatrix}\sqrt{3}\\-1\end{pmatrix}\right|\cdot\left|\begin{pmatrix}q\\1\end{pmatrix}\right|}$$

$$\tfrac{1}{2} = \frac{|q\sqrt{3} - 1|}{2 \cdot \sqrt{q^2 + 1}}$$

$$|q\sqrt{3} - 1| = \sqrt{q^2 + 1}$$

$$(q\sqrt{3} - 1)^2 = q^2 + 1$$

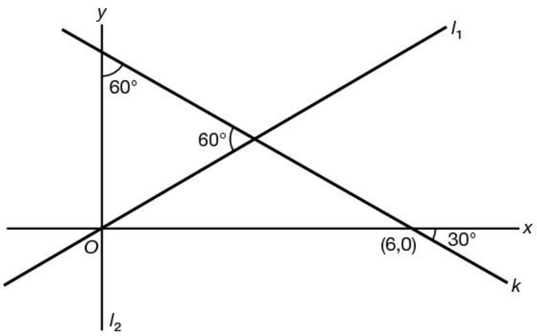
$$3q^2 - 2q\sqrt{3} + 1 = q^2 + 1$$

$$2q^2 - 2q\sqrt{3} = 0$$

$$2q(q - \sqrt{3}) = 0$$

$$q = 0 \vee q = \sqrt{3}$$

c



l_2 is verticaal, dus is de richtingsvector niet voor te stellen met $\begin{pmatrix}1\\p\end{pmatrix}$.

27 $\left.\begin{array}{l}\overrightarrow{OA} = \begin{pmatrix}7\\0\end{pmatrix} \triangleq \begin{pmatrix}5\\0\end{pmatrix}\\ \overrightarrow{OB} = \begin{pmatrix}3\\4\end{pmatrix}\end{array}\right\} \vec{r}_k = \begin{pmatrix}5\\0\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}8\\4\end{pmatrix} \triangleq \begin{pmatrix}2\\1\end{pmatrix}$

Dus k: $y = \tfrac{1}{2}x$.

$\left.\begin{array}{l}\overrightarrow{OB} = \begin{pmatrix}3\\4\end{pmatrix} \text{ geeft } l\text{: } 3x + 4y = c\\ \qquad\qquad \text{door het midden } (1\tfrac{1}{2}, 2) \text{ van } OB\end{array}\right\} c = 3 \cdot 1\tfrac{1}{2} + 4 \cdot 2 = 12\tfrac{1}{2}$

$\left.\begin{array}{l}k\text{: } y = \tfrac{1}{2}x\\ l\text{: } 3x + 4y = 12\tfrac{1}{2}\end{array}\right\} \begin{array}{l}3x + 4 \cdot \tfrac{1}{2}x = 12\tfrac{1}{2}\\ 5x = 12\tfrac{1}{2}\end{array}$

$\left.\begin{array}{l}x = 2\tfrac{1}{2}\\ y = \tfrac{1}{2}x\end{array}\right\} y = \tfrac{1}{2} \cdot 2\tfrac{1}{2} = 1\tfrac{1}{4}$

Dus $S\left(2\frac{1}{2}, 1\frac{1}{4}\right)$.

Stel m: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2\frac{1}{2} \\ 1\frac{1}{4} \end{pmatrix} + \lambda \begin{pmatrix} a \\ 1 \end{pmatrix}$.

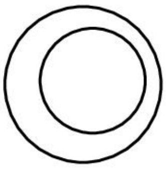
$\vec{r}_{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\angle(m, AB) = 30°$ geeft $\cos(30°) = \dfrac{\left|\begin{pmatrix} a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right|}{\left|\begin{pmatrix} a \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right|}$

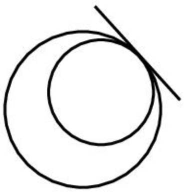
$$\tfrac{1}{2}\sqrt{3} = \frac{|a-1|}{\sqrt{a^2+1} \cdot \sqrt{2}}$$
$$\tfrac{1}{2}\sqrt{6(a^2+1)} = |a-1|$$
$$\sqrt{6(a^2+1)} = 2|a-1|$$
$$6a^2 + 6 = 4(a^2 - 2a + 1)$$
$$6a^2 + 6 = 4a^2 - 8a + 4$$
$$2a^2 + 8a + 2 = 0$$
$$a^2 + 4a + 1 = 0$$
$$(a+2)^2 - 4 + 1 = 0$$
$$(a+2)^2 = 3$$
$$a + 2 = \sqrt{3} \vee a + 2 = -\sqrt{3}$$
$$a = -2 + \sqrt{3} \vee a = -2 - \sqrt{3}$$

Dus m_1: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2\frac{1}{2} \\ 1\frac{1}{4} \end{pmatrix} + \lambda \begin{pmatrix} -2+\sqrt{3} \\ 1 \end{pmatrix}$ en m_2: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2\frac{1}{2} \\ 1\frac{1}{4} \end{pmatrix} + \lambda \begin{pmatrix} -2-\sqrt{3} \\ 1 \end{pmatrix}$.

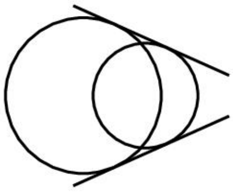
28



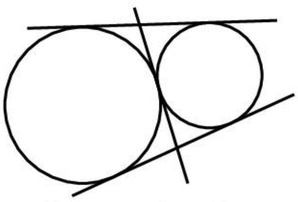
geen gemeenschappelijke raaklijn



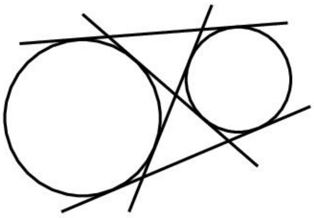
één gemeenschappelijke raaklijn



twee gemeenschappelijke raaklijnen



drie gemeenschappelijke raaklijnen



vier gemeenschappelijke raaklijnen

Bladzijde 69

29 **a** $b = -4a + \frac{2}{3}$ invullen in $|3a - 1 + b| = \sqrt{5a^2+5}$ geeft $|3a - 1 - 4a + \frac{2}{3}| = \sqrt{5a^2+5}$
$$|-a - \tfrac{1}{3}| = \sqrt{5a^2+5}$$
$$a^2 + \tfrac{2}{3}a + \tfrac{1}{9} = 5a^2 + 5$$
$$4a^2 - \tfrac{2}{3}a + 4\tfrac{8}{9} = 0$$
$$D = \left(-\tfrac{2}{3}\right)^2 - 4 \cdot 4 \cdot 4\tfrac{8}{9} = -77\tfrac{7}{9}$$

$D < 0$, dus $b = -4a + \frac{2}{3}$ voldoet niet.

b Elke lijn k: $y = ax + b$ heeft een richtingscoëfficiënt, dus de verticale richting zit er niet bij.

30 c_1 heeft middelpunt $M(3, 0)$ en $r = 2$.
c_2 heeft middelpunt $N(8, 0)$ en $r = 3$.
Stel k: $y = ax + b$ ofwel $ax - y + b = 0$ of l: $x = c$.

$d(M, k) = 2$ geeft $\dfrac{|3a - 0 + b|}{\sqrt{a^2 + 1}} = 2$

$|3a + b| = 2\sqrt{a^2 + 1}$

$d(N, k) = 3$ geeft $\dfrac{|8a - 0 + b|}{\sqrt{a^2 + 1}} = 3$

$|8a + b| = 3\sqrt{a^2 + 1}$

Hieruit volgt $3|3a + b| = 2|8a + b|$

$9a + 3b = 16a + 2b \vee 9a + 3b = -16a - 2b$

$-7a + b = 0 \vee 25a + 5b = 0$

$b = 7a \vee b = -5a$

$b = 7a$ invullen in $|3a + b| = 2\sqrt{a^2 + 1}$ geeft $|3a + 7a| = 2\sqrt{a^2 + 1}$

$|10a| = 2\sqrt{a^2 + 1}$

$|5a| = \sqrt{a^2 + 1}$

$25a^2 = a^2 + 1$

$24a^2 = 1$

$a^2 = \frac{1}{24}$

$a = \frac{1}{12}\sqrt{6} \vee a = -\frac{1}{12}\sqrt{6}$

$\left.\begin{array}{l} a = \frac{1}{12}\sqrt{6} \\ b = 7a \end{array}\right\} b = \frac{7}{12}\sqrt{6}$

Dus k_1: $y = \frac{1}{12}x\sqrt{6} + \frac{7}{12}\sqrt{6}$.

$\left.\begin{array}{l} a = -\frac{1}{12}\sqrt{6} \\ b = 7a \end{array}\right\} b = -\frac{7}{12}\sqrt{6}$

Dus k_2: $y = -\frac{1}{12}x\sqrt{6} - \frac{7}{12}\sqrt{6}$.

$b = -5a$ invullen in $|3a + b| = 2\sqrt{a^2 + 1}$ geeft $|3a - 5a| = 2\sqrt{a^2 + 1}$

$|-2a| = 2\sqrt{a^2 + 1}$

$|-a| = \sqrt{a^2 + 1}$

$a^2 = a^2 + 1$

geen opl.

$d(M, l) = 2$ geeft $\dfrac{|3 - c|}{1} = 2$ en $d(N, l) = 3$ geeft $\dfrac{|8 - c|}{1} = 3$

$|3 - c| = 2$ | $|8 - c| = 3$

$3 - c = 2 \vee 3 - c = -2$ | $8 - c = 3 \vee 8 - c = -3$

$c = 1 \vee c = 5$ | $c = 5 \vee c = 11$

Dus $c = 5$. Dit geeft de gemeenschappelijke raaklijn k_3: $x = 5$.

31 **a** $PM = r + 2$

de stelling van Pythagoras in $\triangle OMP$ geeft $p^2 + 9 = (r + 2)^2$.

$PN = r + 3$

de stelling van Pythagoras in $\triangle ONP$ geeft $p^2 + 64 = (r + 3)^2$.

b Uit vraag a volgt $(r + 2)^2 - 9 = (r + 3)^2 - 64$

$r^2 + 4r + 4 - 9 = r^2 + 6r + 9 - 64$

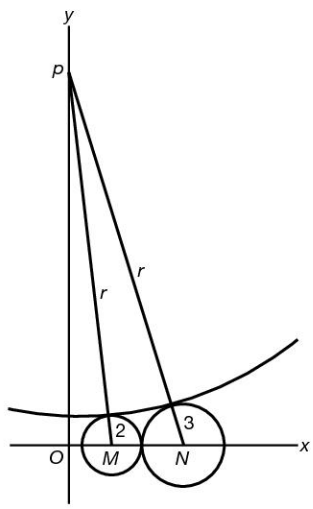
$-2r = -50$

$r = 25$

$\left.\begin{array}{l} r = 25 \\ p^2 + 9 = (r + 2)^2 \end{array}\right\} \begin{array}{l} p^2 + 9 = 27^2 \\ p^2 = 720 \end{array}$

$p = 12\sqrt{5} \vee p = -12\sqrt{5}$

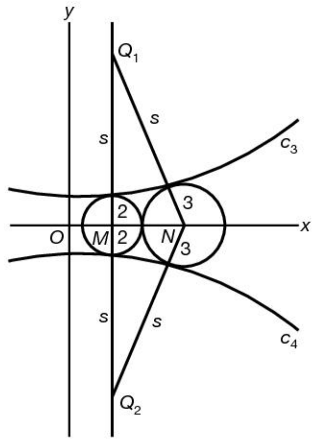
P op de positieve y-as, dus $p = 12\sqrt{5}$.



c Stel de straal van de cirkel is s en het middelpunt is $Q(3, q)$.
$QM = s + 2$ en $QN = s + 3$
De stelling van Pythagoras in $\triangle MNQ$ geeft $(s + 3)^2 = (s + 2)^2 + 5^2$

$$s^2 + 6s + 9 = s^2 + 4s + 4 + 25$$
$$2s = 20$$
$$s = 10$$

$s = 10$ geeft $Q_1(3, 12)$ en $Q_2(3, -12)$
Dus c_3: $(x - 3)^2 + (y - 12)^2 = 100$ en c_4: $(x - 3)^2 + (y + 12)^2 = 100$.



32 **a** k: $y = -2x + 5$ ofwel k: $2x + y - 5 = 0$

$$d(M, k) = \frac{|2 \cdot 4 + 2 - 5|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

Dus c: $(x - 4)^2 + (y - 2)^2 = 5$.

b $\vec{n}_l = \overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\left.\begin{array}{l} l: x + 2y = c \\ \text{door } A(5, 4) \end{array}\right\} c = 5 + 2 \cdot 4 = 13$

Dus l: $x + 2y = 13$.

c Stel m: $y = 2x + c$ ofwel $2x - y + c = 0$.

$d(M, m) = r$ geeft $\dfrac{|2 \cdot 4 - 2 + c|}{\sqrt{5}} = \sqrt{5}$

$$|6 + c| = 5$$
$$6 + c = 5 \vee 6 + c = -5$$
$$c = -1 \vee c = -11$$

Dus m_1: $y = 2x - 1$ en m_2: $y = 2x - 11$.

d Stel n: $y = ax + 2\frac{1}{2}$ ofwel $ax - y + 2\frac{1}{2} = 0$.

$d(M, n) = r$ geeft $\dfrac{|4a - 2 + 2\frac{1}{2}|}{\sqrt{a^2 + 1}} = \sqrt{5}$

$$|4a + \tfrac{1}{2}| = \sqrt{5a^2 + 5}$$
$$16a^2 + 4a + \tfrac{1}{4} = 5a^2 + 5$$
$$11a^2 + 4a - 4\tfrac{3}{4} = 0$$
$$D = 4^2 - 4 \cdot 11 \cdot -4\tfrac{3}{4} = 225$$
$$a = \frac{-4 + 15}{22} = \tfrac{1}{2} \vee a = \frac{-4 - 15}{22} = -\tfrac{19}{22}$$

Dus n_1: $y = \frac{1}{2}x + 2\frac{1}{2}$ en n_2: $y = -\frac{19}{22}x + 2\frac{1}{2}$.

Bladzijde 70

33 **a** $y_P = p$ op m: $x - 2y = 0$ geeft $x_P = 2p$, dus $M(2p, p)$ ligt op m.

b $d(M, k) = d(M, l)$ geeft $\dfrac{|2 \cdot 2p - p - 1|}{\sqrt{5}} = \dfrac{|2p - 2p - 5|}{\sqrt{5}}$

$$\frac{|4p - p - 1|}{\sqrt{5}} = \frac{|-5|}{\sqrt{5}}$$
$$|3p - 1| = |-5|$$
$$3p - 1 = 5 \vee 3p - 1 = -5$$
$$3p = 6 \vee 3p = -4$$
$$p = 2 \vee p = -1\tfrac{1}{3}$$

c Zie vraag b, $d(M, k) = d(M, l) = \sqrt{5}$.
$p = 2$ geeft $M(4, 2)$, dus c_1: $(x - 4)^2 + (y - 2)^2 = 5$.
$p = -1\frac{1}{3}$ geeft $M\left(-2\frac{2}{3}, -1\frac{1}{3}\right)$, dus c_2: $\left(x + 2\frac{2}{3}\right)^2 + \left(y + 1\frac{1}{3}\right)^2 = 5$.

34 Voor het middelpunt M van de cirkels geldt $d(M, k) = d(M, l)$.
Dus M op de bissectrices van de lijnen k en l.
Omdat M ook op m ligt, bereken je dus de coördinaten van de snijpunten van de bissectrices van k en l met m.

35 $d(M, k) = d(M, l)$ geeft $\dfrac{|3x - y|}{\sqrt{10}} = \dfrac{|x - 3y + 8|}{\sqrt{10}}$

$3x - y = x - 3y + 8 \vee 3x - y = -x + 3y - 8$

$2x + 2y = 8 \vee 4x - 4y = -8$

$x + y = 4 \vee x - y = -2$

Dus de bissectrices van k en l zijn de lijnen b_1: $x + y = 4$ en b_2: $x - y = -2$.

b_1 snijden met m geeft $\begin{cases} x + y = 4 \\ x - 2y = -4 \end{cases}$ −

$3y = 8$

$\left.\begin{array}{l} y = 2\frac{2}{3} \\ x + y = 4 \end{array}\right\} x + 2\frac{2}{3} = 4$

$x = 1\frac{1}{3}$

$d\left(\left(1\frac{1}{3}, 2\frac{2}{3}\right), k\right) = \dfrac{|4 - 2\frac{2}{3}|}{\sqrt{10}} = \dfrac{1\frac{1}{3}}{\sqrt{10}} = \frac{2}{15}\sqrt{10}$, dus $r^2 = \frac{4}{225} \cdot 10 = \frac{8}{45}$.

Dus c_1: $\left(x - 1\frac{1}{3}\right)^2 + \left(y - 2\frac{2}{3}\right)^2 = \frac{8}{45}$.

b_2 snijden met m geeft $\begin{cases} x - y = -2 \\ x - 2y = -4 \end{cases}$ −

$\left.\begin{array}{l} y = 2 \\ x - y = -2 \end{array}\right\} x - 2 = -2$

$x = 0$

$d((0, 2), k) = \dfrac{|0 - 2|}{\sqrt{10}} = \dfrac{2}{\sqrt{10}} = \frac{1}{5}\sqrt{10}$, dus $r^2 = \frac{1}{25} \cdot 10 = \frac{2}{5}$.

Dus c_2: $x^2 + (y - 2)^2 = \frac{2}{5}$.

36 **a** $\begin{cases} x^2 + y^2 - 2x - 4y = 0 \\ x^2 + y^2 - 14x + 24 = 0 \end{cases}$ −

$12x - 4y - 24 = 0$

$3x - y - 6 = 0$

$\left.\begin{array}{l} y = 3x - 6 \\ x^2 + y^2 - 2x - 4y = 0 \end{array}\right\} x^2 + (3x - 6)^2 - 2x - 4(3x - 6) = 0$

$x^2 + 9x^2 - 36x + 36 - 2x - 12x + 24 = 0$

$10x^2 - 50x + 60 = 0$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2 \vee x = 3$

$x = 2$ geeft $y = 3 \cdot 2 - 6 = 0$ en $x = 3$ geeft $y = 3 \cdot 3 - 6 = 3$.

Dus $A(3, 3)$ en $B(2, 0)$.

b $\left.\begin{array}{l} y = ax + b \\ \text{door } A(3, 3) \end{array}\right\} 3a + b = 3$

$b = 3 - 3a$

Dus k: $y = ax + 3 - 3a$.

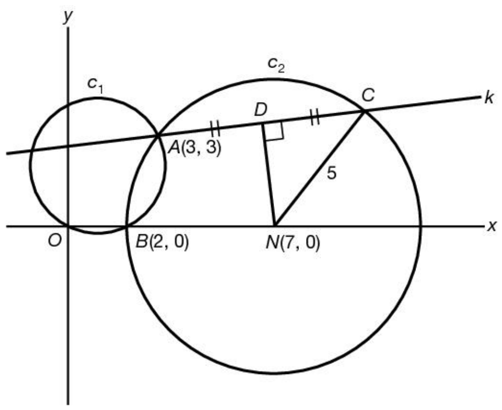
c c_2: $x^2 + y^2 - 14x + 24 = 0$

$x^2 - 14x + y^2 + 24 = 0$

$(x - 7)^2 - 49 + y^2 + 24 = 0$

$(x - 7)^2 + y^2 = 25$

Dus middelpunt $N(7, 0)$ en straal 5.



De loodlijn ND op AC deelt AC middendoor, dus $CD = 2\frac{1}{2}\sqrt{2}$.

De stelling van Pythagoras in $\triangle NCD$ geeft $ND = 2\frac{1}{2}\sqrt{2}$, dus $d(N, k) = 2\frac{1}{2}\sqrt{2}$.

d $d(N, k) = 2\frac{1}{2}\sqrt{2}$ geeft

$$\frac{|7a - 0 + 3 - 3a|}{\sqrt{a^2 + 1}} = 2\tfrac{1}{2}\sqrt{2}$$

$|4a + 3| = 2\frac{1}{2}\sqrt{2a^2 + 2}$
$16a^2 + 24a + 9 = 6\frac{1}{4}(2a^2 + 2)$
$16a^2 + 24a + 9 = 12\frac{1}{2}a^2 + 12\frac{1}{2}$
$3\frac{1}{2}a^2 + 24a - 3\frac{1}{2} = 0$

$D = 24^2 - 4 \cdot 3\frac{1}{2} \cdot -3\frac{1}{2} = 625$

$$a = \frac{-24 + 25}{7} = \tfrac{1}{7} \vee a = \frac{-24 - 25}{7} = -7$$

e $a = \frac{1}{7}$ geeft k_1: $y = \frac{1}{7}x + 3 - \frac{3}{7}$
$a = -7$ geeft k_2: $y = -7x + 3 - 21$
Dus k_1: $y = \frac{1}{7}x + 2\frac{4}{7}$ en k_2: $y = -7x - 18$.

37 $AP = 5\sqrt{2}$ geeft dat P op de cirkel met middelpunt A en straal $5\sqrt{2}$ ligt.
Dus c_3: $(x - 3)^2 + (y - 3)^2 = 50$.

Bladzijde 71

38 **a** $d(M, k) = d(M, l)$ geeft $\dfrac{|3x - y - 9|}{\sqrt{10}} = \dfrac{|x - 3y - 3|}{\sqrt{10}}$

$3x - y - 9 = x - 3y - 3 \vee 3x - y - 9 = -x + 3y + 3$
$2x + 2y = 6 \vee 4x - 4y = 12$
$x + y = 3 \vee x - y = 3$

M op $x + y = 3$ en $x_M = p$ geeft $M(p, 3 - p)$.

$d(M, k) = \sqrt{10}$ geeft $\dfrac{|3p - (3 - p) - 9|}{\sqrt{10}} = \sqrt{10}$

$|3p - 3 + p - 9| = 10$
$|4p - 12| = 10$
$4p - 12 = 10 \vee 4p - 12 = -10$
$4p = 22 \vee 4p = 2$
$p = 5\frac{1}{2} \vee p = \frac{1}{2}$

$p = 5\frac{1}{2}$ geeft $M\left(5\frac{1}{2}, -2\frac{1}{2}\right)$ vold. niet.

$p = \frac{1}{2}$ geeft $M\left(\frac{1}{2}, 2\frac{1}{2}\right)$

Dus c_1: $\left(x - \frac{1}{2}\right)^2 + \left(y - 2\frac{1}{2}\right)^2 = 10$.
M op $x - y = 3$ en $y_M = q$ geeft $M(q + 3, q)$.

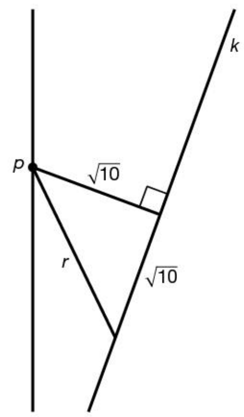
$d(M, k) = \sqrt{10}$ geeft $\dfrac{|3(q + 3) - q - 9|}{\sqrt{10}} = \sqrt{10}$

$|3q + 9 - q - 9| = 10$
$|2q| = 10$
$2q = 10 \vee 2q = -10$
$q = 5 \vee q = -5$

$q = 5$ geeft $M(8, 5)$
$q = -5$ geeft $M(-2, -5)$ vold. niet.
Dus c_2: $(x - 8)^2 + (y - 5)^2 = 10$.

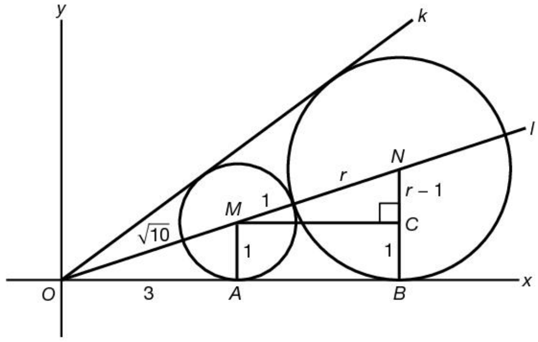
b $d(P, k) = \dfrac{|0 - 1 - 9|}{\sqrt{10}} = \sqrt{10}$

$r = \sqrt{10} \cdot \sqrt{2} = 2\sqrt{5}$.
Dus c_3: $x^2 + (y - 1)^2 = 20$.



39 $\vec{r}_k = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ en $\vec{r}_{x\text{-as}} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ en l de bissectrice van $\angle(k, x\text{-as})$
geeft $\vec{r}_l = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.
Dus voor een punt P op bissectrice l geldt $P(3p, p)$.
De straal van c_1 is 1, dus $M(3, 1)$ en $OM = \sqrt{3^2 + 1^2} = \sqrt{10}$.
$\triangle OAM \backsim \triangle MCN$ geeft $\dfrac{OM}{MN} = \dfrac{AM}{CN}$

$$\frac{\sqrt{10}}{r+1} = \frac{1}{r-1}$$
$$r\sqrt{10} - \sqrt{10} = r + 1$$
$$r\sqrt{10} - r = \sqrt{10} + 1$$
$$r(\sqrt{10} - 1) = \sqrt{10} + 1$$
$$r = \frac{\sqrt{10}+1}{\sqrt{10}-1} = \frac{\sqrt{10}+1}{\sqrt{10}-1} \cdot \frac{\sqrt{10}+1}{\sqrt{10}+1} = \frac{10 + 2\sqrt{10} + 1}{10 - 1} = 1\tfrac{2}{9} + \tfrac{2}{9}\sqrt{10}$$



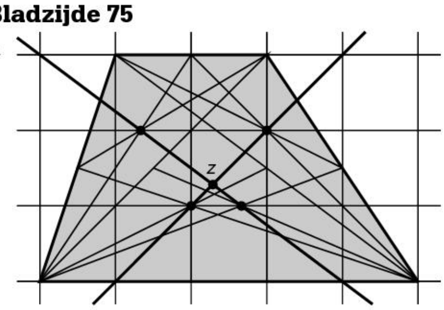
14.3 Zwaartepunten

Bladzijde 73

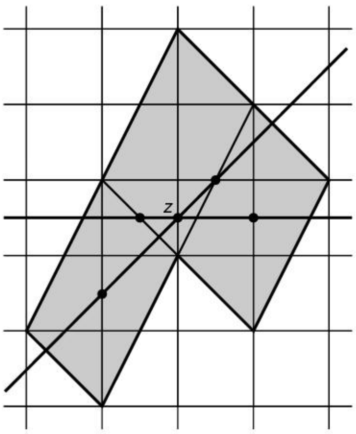
40 Ja.

Bladzijde 75

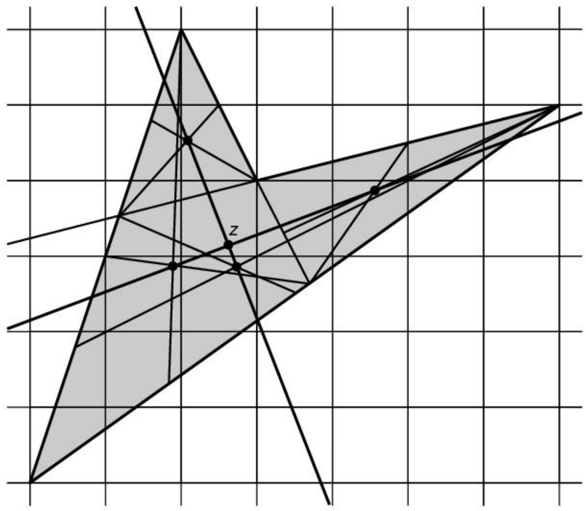
41 **a**



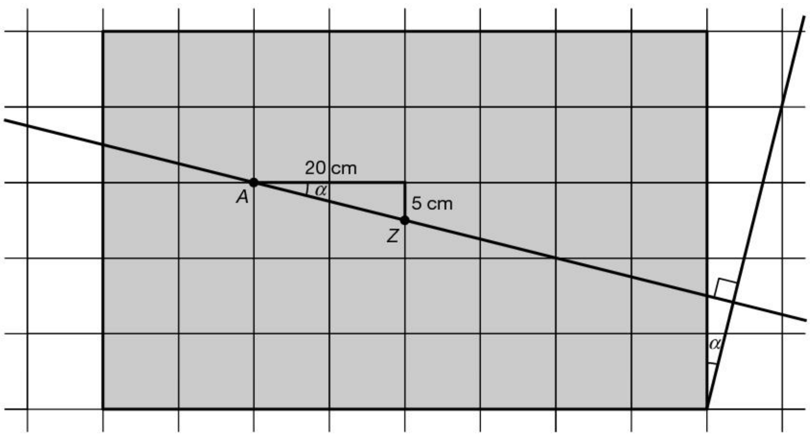
b



c



42



De lijn door het gaatje A en het zwaartepunt Z van de rechthoek komt verticaal te hangen.
$\tan(\alpha) = \dfrac{5}{20} = \tfrac{1}{4}$ geeft $\alpha = 14{,}03...°$
Dus $\alpha \approx 14°$.

43 Dichter bij *B*.

Bladzijde 77

44 $m_1 = 4$ en $m_2 = 4$

$$z = \frac{1}{4+4}\left(4 \cdot -1\tfrac{1}{2} + 4 \cdot \tfrac{1}{2}\right) = \tfrac{1}{8} \cdot -4 = -\tfrac{1}{2}$$

Het zwaartepunt ligt $\frac{1}{2}$ onder 0.

alternatieve uitwerking

$m_1 = 2$, $m_2 = 4$ en $m_3 = 2$

$$z = \frac{1}{2+4+2}(2 \cdot 1 + 4 \cdot 0 + 2 \cdot -3) = \tfrac{1}{8} \cdot -4 = -\tfrac{1}{2}$$

Het zwaartepunt ligt $\frac{1}{2}$ onder 0.

Bladzijde 78

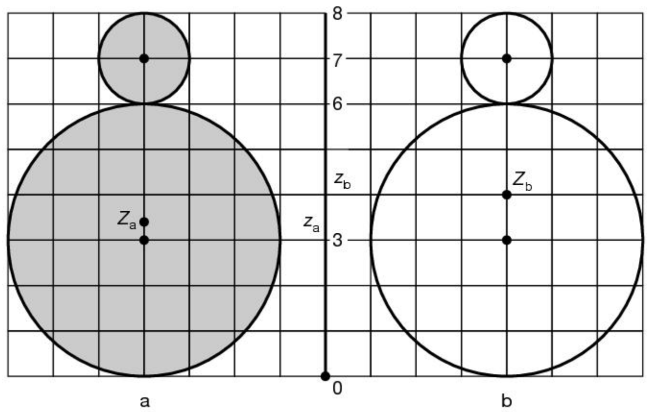
45 **a** De hefboomwet geeft $m_1 \cdot (z - a_1) = m_2 \cdot (a_2 - z)$

$$m_1 z - m_1 a_1 = m_2 a_2 - m_2 z$$
$$m_1 z + m_2 z = m_1 a_1 + m_2 a_2$$
$$(m_1 + m_2)z = m_1 a_1 + m_2 a_2$$
$$z = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{1}{M}(m_1 \cdot a_1 + m_2 \cdot a_2)$$

b m_1 en m_2 met Z_{12} en massa $(m_1 + m_2)$ en m_3 met Z_3 geeft

$$z = \frac{1}{(m_1+m_2)+m_3}((m_1+m_2) \cdot z_{12} + m_3 \cdot a_3) = \frac{1}{m_1+m_2+m_3}\left((m_1+m_2)\frac{m_1 a_1 + m_2 a_2}{m_1+m_2} + m_3 a_3\right)$$
$$= \frac{1}{M}(m_1 a_1 + m_2 a_2 + m_3 a_3)$$

46



Het zwaartepunt Z_a ligt op de symmetrieas van figuur a.

$$z_a = \frac{1}{\pi \cdot 3^2 + \pi \cdot 1^2}(\pi \cdot 3^2 \cdot 3 + \pi \cdot 1^2 \cdot 7) = \frac{27\pi + 7\pi}{9\pi + \pi} = \frac{34\pi}{10\pi} = 3\tfrac{2}{5}$$

Dus Z_a ligt $3\frac{2}{5}$ boven 0.

Het zwaartepunt Z_b ligt op de symmetrieas van figuur b.

$$z_b = \frac{1}{2\pi \cdot 3 + 2\pi \cdot 1}(2\pi \cdot 3 \cdot 3 + 2\pi \cdot 1 \cdot 7) = \frac{18\pi + 14\pi}{6\pi + 2\pi} = \frac{32\pi}{8\pi} = 4$$

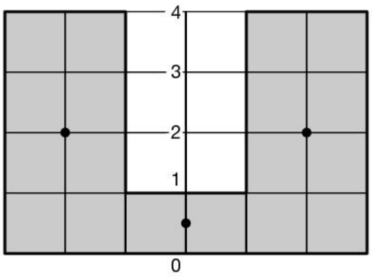
Dus Z_b ligt 4 boven 0.

47 Neem 0 in het middelpunt van de Aarde.

$$z = \frac{1}{M} \cdot (m_1 a_1 + m_2 a_2) = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{5{,}975 \times 10^{24} \cdot 0 + 7{,}343 \times 10^{22} \cdot 384\,400}{5{,}975 \times 10^{24} + 7{,}343 \times 10^{22}} = \frac{2{,}82... \times 10^{28}}{6{,}04... \times 10^{24}} = 4666{,}74...$$

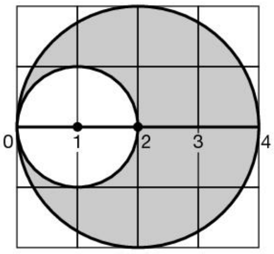
Het gemeenschappelijk zwaartepunt bevindt zich op $6371 - 4666{,}74... \approx 1704$ km onder het aardoppervlak.

Bladzijde 79

48 

Het zwaartepunt van figuur a ligt op de symmetrieas.
$M = 8 + 2 + 8 = 18$
$z = \frac{1}{18}\left(8 \cdot 2 + 2 \cdot \frac{1}{2} + 8 \cdot 2\right) = \frac{1}{18} \cdot 33 = 1\frac{5}{6}$
Dus het zwaartepunt van figuur a ligt $1\frac{5}{6}$ boven het midden van de onderkant.

Reken eerst met de hele cirkel en trek daarna het gat ervan af.
Het zwaartepunt van figuur b ligt op de symmetrieas.



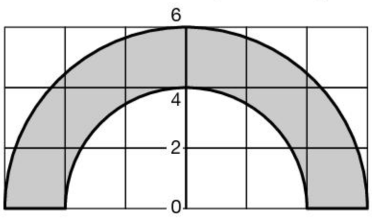
$M = \pi \cdot 2^2 - \pi \cdot 1^2 = 3\pi$

$z = \dfrac{1}{3\pi}(\pi \cdot 2^2 \cdot 2 - \pi \cdot 1^2 \cdot 1) = \dfrac{1}{3\pi} \cdot 7\pi = 2\frac{1}{3}$
Dus het zwaartepunt van figuur b ligt $\frac{1}{3}$ rechts van het middelpunt van de grote cirkel op de lijn door de middelpunten.

49 $M = 4 + 6 + 2 + 3 = 15$
$x_Z = \frac{1}{15}(4 \cdot 0 + 6 \cdot 3 + 2 \cdot 3 + 3 \cdot 7) = 3$

$y_Z = \frac{1}{15}(4 \cdot 3 + 6 \cdot 5 + 2 \cdot 1 + 3 \cdot 3) = \frac{53}{15} = 3\frac{8}{15}$
Dus $Z\left(3,\ 3\frac{8}{15}\right)$.

50 Het zwaartepunt ligt op de symmetrieas.



$M = \frac{1}{2} \cdot \pi \cdot 6^2 - \frac{1}{2} \cdot \pi \cdot 4^2 = 10\pi$

$$z = \frac{1}{10\pi}\left(\frac{1}{2} \cdot \pi \cdot 6^2 \cdot \frac{4}{3\pi} \cdot 6 - \frac{1}{2} \cdot \pi \cdot 4^2 \cdot \frac{4}{3\pi} \cdot 4\right) = \frac{1}{10\pi}\left(144 - 42\tfrac{2}{3}\right) \approx 3{,}23$$

Dus het zwaartepunt ligt 3,23 m boven het middelpunt van de cirkelbogen.

Bladzijde 80

51 **a** Neem een as langs AB met de oorsprong in Z.

De hefboomwet geeft $0 = \dfrac{AZ \cdot 4 - BZ \cdot 3}{4 + 3}$

$4AZ = 3BZ$

$\dfrac{AZ}{BZ} = \dfrac{3}{4}$

Dus $AZ : BZ = 3 : 4$.

b $\vec{z} = \overrightarrow{OA} + \frac{3}{7} \cdot \overrightarrow{AB} = \vec{a} + \frac{3}{7}\left(\vec{b} - \vec{a}\right) = \vec{a} + \frac{3}{7}\vec{b} - \frac{3}{7}\vec{a} = \frac{4}{7}\vec{a} + \frac{3}{7}\vec{b} = \frac{1}{7}\left(4\vec{a} + 3\vec{b}\right)$

Bladzijde 81

52 Voor het zwaartepunt $\vec{z}_1$ van $\vec{a}$ en $\vec{b}$ geldt $\vec{z}_1 = \dfrac{1}{m_1 + m_2}\left(m_1 \cdot \vec{a} + m_2 \cdot \vec{b}\right)$.

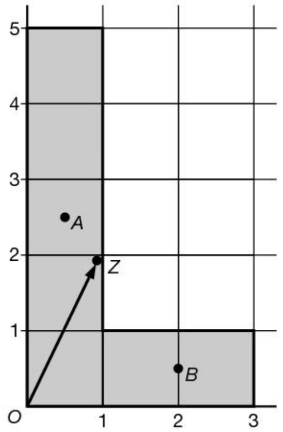
Voor het zwaartepunt $\vec{z}$ van $\vec{z}_1$ en $\vec{c}$ geldt

$$\vec{z} = \frac{m_1 + m_2}{m_1 + m_2 + m_3} \cdot \frac{m_1 \cdot \vec{a} + m_2 \cdot \vec{b}}{m_1 + m_2} + \frac{m_3}{m_1 + m_2 + m_3} \cdot \vec{c} = \frac{m_1 \cdot \vec{a} + m_2 \cdot \vec{b}}{m_1 + m_2 + m_3} + \frac{m_3 \cdot \vec{c}}{m_1 + m_2 + m_3}$$

$$= \frac{m_1 \cdot \vec{a} + m_2 \cdot \vec{b} + m_3 \cdot \vec{c}}{m_1 + m_2 + m_3} = \frac{1}{M}\left(m_1 \cdot \vec{a} + m_2 \cdot \vec{b} + m_3 \cdot \vec{c}\right)$$

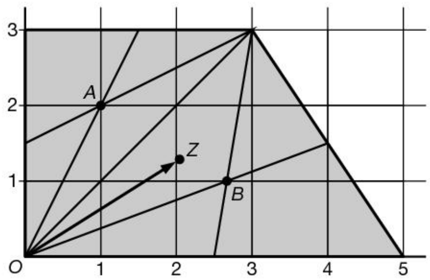
Bladzijde 82

53 *A* heeft massa 5, *B* heeft massa 2.



$$\vec{z} = \tfrac{1}{7}\left(5 \cdot \begin{pmatrix} \frac{1}{2} \\ 2\frac{1}{2} \end{pmatrix} + 2 \cdot \begin{pmatrix} 2 \\ \frac{1}{2} \end{pmatrix}\right) = \tfrac{1}{7}\left(\begin{pmatrix} 2\frac{1}{2} \\ 12\frac{1}{2} \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix}\right) = \tfrac{1}{7} \cdot \begin{pmatrix} 6\frac{1}{2} \\ 13\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{13}{14} \\ 1\frac{13}{14} \end{pmatrix}$$

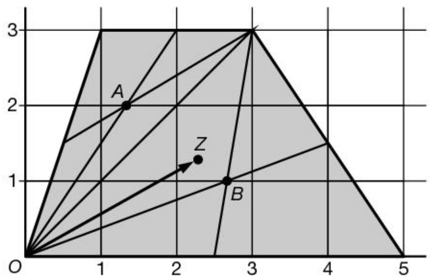
A heeft massa $4\frac{1}{2}$, *B* heeft massa $7\frac{1}{2}$.



$$\vec{z} = \tfrac{1}{12}\left(4\tfrac{1}{2} \cdot \tfrac{1}{3} \cdot \begin{pmatrix} 0+3+0 \\ 0+3+3 \end{pmatrix} + 7\tfrac{1}{2} \cdot \tfrac{1}{3} \cdot \begin{pmatrix} 0+5+3 \\ 0+0+3 \end{pmatrix}\right) = \tfrac{1}{12}\left(4\tfrac{1}{2} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 7\tfrac{1}{2} \cdot \begin{pmatrix} 2\frac{2}{3} \\ 1 \end{pmatrix}\right) = \tfrac{1}{12}\left(\begin{pmatrix} 4\frac{1}{2} \\ 9 \end{pmatrix} + \begin{pmatrix} 20 \\ 7\frac{1}{2} \end{pmatrix}\right)$$

$$= \tfrac{1}{12} \cdot \begin{pmatrix} 24\frac{1}{2} \\ 16\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2\frac{1}{24} \\ 1\frac{3}{8} \end{pmatrix}$$

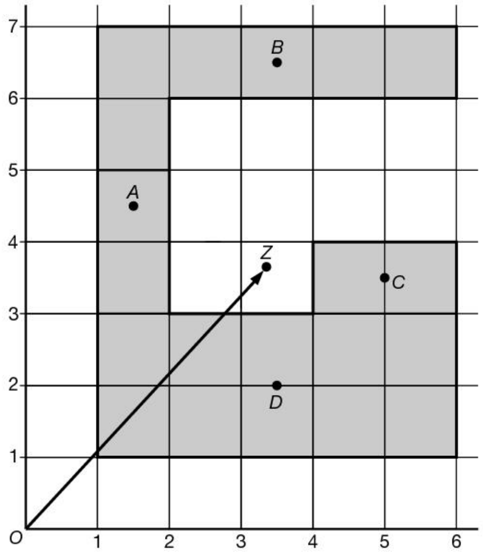
A heeft massa 3, *B* heeft massa $7\frac{1}{2}$.



$$\vec{z} = \frac{1}{10\frac{1}{2}}\left(3 \cdot \tfrac{1}{3} \cdot \begin{pmatrix} 0+3+1 \\ 0+3+3 \end{pmatrix} + 7\tfrac{1}{2} \cdot \tfrac{1}{3} \cdot \begin{pmatrix} 0+5+3 \\ 0+0+3 \end{pmatrix}\right) = \tfrac{2}{21}\left(3 \cdot \begin{pmatrix} 1\frac{1}{3} \\ 2 \end{pmatrix} + 7\tfrac{1}{2} \cdot \begin{pmatrix} 2\frac{2}{3} \\ 1 \end{pmatrix}\right) = \tfrac{2}{21}\left(\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 20 \\ 7\frac{1}{2} \end{pmatrix}\right)$$

$$= \tfrac{2}{21} \cdot \begin{pmatrix} 24 \\ 13\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2\frac{2}{7} \\ 1\frac{2}{7} \end{pmatrix}$$

54

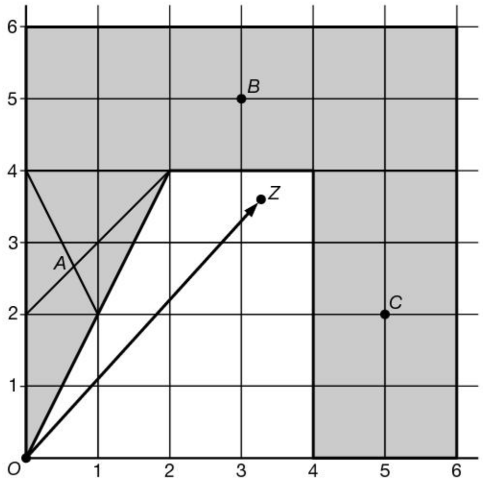


De totale oppervlakte is 3 + 5 + 2 + 10 = 20.

$$\vec{z} = \tfrac{1}{20}\left(3\begin{pmatrix}1\frac{1}{2}\\4\frac{1}{2}\end{pmatrix} + 5\begin{pmatrix}3\frac{1}{2}\\6\frac{1}{2}\end{pmatrix} + 2\begin{pmatrix}5\\3\frac{1}{2}\end{pmatrix} + 10\begin{pmatrix}3\frac{1}{2}\\2\end{pmatrix}\right) = \begin{pmatrix}3\frac{7}{20}\\3\frac{13}{20}\end{pmatrix}$$

Bladzijde 83

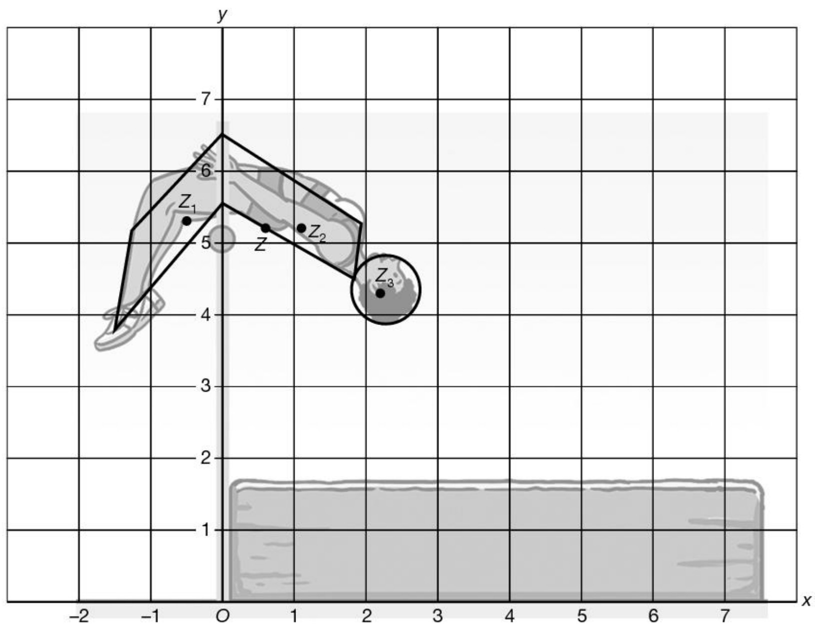
55 *A* heeft massa 4, *B* heeft massa 12, *C* heeft massa 8.



De totale oppervlakte van de figuur is 4 + 12 + 8 = 24.

$$\vec{z} = \tfrac{1}{24}\left(4\cdot\tfrac{1}{3}\cdot\begin{pmatrix}0+2+0\\0+4+4\end{pmatrix} + 12\cdot\begin{pmatrix}3\\5\end{pmatrix} + 8\cdot\begin{pmatrix}5\\2\end{pmatrix}\right) = \tfrac{1}{24}\left(\begin{pmatrix}2\frac{2}{3}\\10\frac{2}{3}\end{pmatrix} + \begin{pmatrix}36\\60\end{pmatrix} + \begin{pmatrix}40\\16\end{pmatrix}\right) = \begin{pmatrix}3\frac{5}{18}\\3\frac{11}{18}\end{pmatrix}$$

56



Stel de massa van de hoogspringer is 70 kg, verdeeld over

- de benen met massa 26 kg en zwaartepunt $Z_1(-0{,}5;\ 5{,}3)$
- de romp met massa 38 kg en zwaartepunt $Z_2(1{,}1;\ 5{,}2)$
- het hoofd met massa 6 kg en zwaartepunt $Z_3(2{,}7;\ 4{,}3)$.

$$\vec{z} = \tfrac{1}{70}\left(26\begin{pmatrix}-0{,}5\\5{,}3\end{pmatrix} + 38\begin{pmatrix}1{,}1\\5{,}2\end{pmatrix} + 6\begin{pmatrix}2{,}7\\4{,}3\end{pmatrix}\right) = \tfrac{1}{70}\begin{pmatrix}45\\361{,}2\end{pmatrix} \approx \begin{pmatrix}0{,}6\\5{,}2\end{pmatrix}$$

14.4 Bewegingsvergelijkingen onderzoeken

Bladzijde 85

57 **a** $t = 3$ geeft $x(3) = 3$ en $y(3) = -4$, dus $P(3, -4)$.

b $y = -1$ geeft $t^2 - 4t - 1 = -1$

$$t^2 - 4t = 0$$
$$t(t-4) = 0$$
$$t = 0 \vee t = 4$$

$t = 0$ geeft het punt $(6, -1)$ en $t = 4$ geeft het punt $\left(-\frac{2}{3}, -1\right)$.
De lengte van het lijnstuk is $6 - -\frac{2}{3} = 6\frac{2}{3}$.

c $x = 6$ geeft $\frac{1}{3}t^3 - 3t^2 + 5t + 6 = 6$

$$\tfrac{1}{3}t^3 - 3t^2 + 5t = 0$$
$$\tfrac{1}{3}t(t^2 - 9t + 15) = 0$$
$$t = 0 \vee t^2 - 9t + 15 = 0$$
$$t = 0 \vee D = (-9)^2 - 4 \cdot 1 \cdot 15 = 21$$
$$t = 0 \vee t = \frac{9 - \sqrt{21}}{2} \vee t = \frac{9 + \sqrt{21}}{2}$$

Bladzijde 86

58 **a** Evenwijdig aan de y-as, dus $x'(t) = 0 \wedge y'(t) \neq 0$

$$(t = 1 \vee t = 5) \wedge t \neq 2$$
$$t = 1 \vee t = 5$$

$t = 1$ geeft het punt $\left(8\frac{1}{3}, -4\right)$.

$t = 5$ geeft het punt $\left(-2\frac{1}{3}, 4\right)$.

b $y = 4$ geeft $t^2 - 4t - 1 = 4$

$$t^2 - 4t - 5 = 0$$
$$(t+1)(t-5) = 0$$
$$t = -1 \vee t = 5$$

$t = -1$ geeft $x = -2\frac{1}{3}$ en $t = 5$ geeft $x = -2\frac{1}{3}$.
Dus de baan snijdt zichzelf in C.

c $\vec{v}(-1) = \begin{pmatrix}(-1)^2 - 6 \cdot -1 + 5\\ 2 \cdot -1 - 4\end{pmatrix} = \begin{pmatrix}12\\-6\end{pmatrix}$

$\vec{v}(5) = \begin{pmatrix}5^2 - 6 \cdot 5 + 5\\ 2 \cdot 5 - 4\end{pmatrix} = \begin{pmatrix}0\\6\end{pmatrix}$

Stel de hoek waaronder de baan zichzelf snijdt α.

$$\cos(\alpha) = \frac{\left|\begin{pmatrix}12\\-6\end{pmatrix} \cdot \begin{pmatrix}0\\6\end{pmatrix}\right|}{\left|\begin{pmatrix}12\\-6\end{pmatrix}\right| \cdot \left|\begin{pmatrix}0\\6\end{pmatrix}\right|} = \frac{|-36|}{6\sqrt{5} \cdot 6} = \frac{36}{36\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$\alpha = 63{,}43...^\circ$
Dus de baan snijdt zichzelf onder een hoek van 63°.

d $\vec{v}(6) = \begin{pmatrix}6^2 - 6 \cdot 6 + 5\\ 2 \cdot 6 - 4\end{pmatrix} = \begin{pmatrix}5\\8\end{pmatrix}$

$\text{rc}_l = \frac{8}{5} = 1\frac{3}{5}$
$t = 6$ geeft $D(0, 11)$
Dus l: $y = 1\frac{3}{5}x + 11$.

e De baansnelheid in D is $|\vec{v}(6)| = \sqrt{(x'(6))^2 + (y'(6))^2} = \sqrt{5^2 + 8^2} = \sqrt{89}$

f $x'(t) = t^2 - 6t + 5$ geeft $x''(t) = 2t - 6$
$y'(t) = 2t - 4$ geeft $y''(t) = 2$

De baanversnelling in D is $a_b(6) = \dfrac{\begin{pmatrix}5\\8\end{pmatrix}\cdot\begin{pmatrix}2\cdot 6-6\\2\end{pmatrix}}{\left|\begin{pmatrix}5\\8\end{pmatrix}\right|} = \dfrac{\begin{pmatrix}5\\8\end{pmatrix}\cdot\begin{pmatrix}6\\2\end{pmatrix}}{\left|\begin{pmatrix}5\\8\end{pmatrix}\right|} = \dfrac{30+16}{\sqrt{25+64}} = \dfrac{46}{\sqrt{89}} = \frac{46}{89}\sqrt{89}.$

Bladzijde 87

59 **a** $t = 1$ geeft $P(-4, -4)$ en $Q(2, 5)$.
$PQ = \sqrt{(2 - -4)^2 + (5 - -4)^2} = \sqrt{117}$

b $x_P(t) = x_Q(t) \wedge y_P(t) = y_Q(t)$
$t^2 - 2t - 3 = -t + 3 \wedge t^2 + t - 6 = 3t + 2$
$t^2 - t - 6 = 0 \wedge t^2 - 2t - 8 = 0$
$(t + 2)(t - 3) = 0 \wedge (t + 2)(t - 4) = 0$
$(t = -2 \vee t = 3) \wedge (t = -2 \vee t = 4)$
$t = -2$
$\begin{cases}x_P'(t) = 2t - 2\\ y_P'(t) = 2t + 1\end{cases}$ en $\begin{cases}x_Q'(t) = -1\\ y_Q'(t) = 3\end{cases}$

Voor de baan van P geldt $\left[\dfrac{dy}{dx}\right]_{t=-2} = \dfrac{2\cdot -2+1}{2\cdot -2-2} = \frac{1}{2}$.

Voor de baan van Q geldt $\left[\dfrac{dy}{dx}\right]_{t=-2} = \dfrac{3}{-1} = -3$.

$\tan(\alpha) = \frac{1}{2}$ geeft $\alpha = 26{,}56...°$
$\tan(\beta) = -3$ geeft $\beta = -71{,}56...°$
$\alpha - \beta = 98{,}13...°$
Dus de hoek tussen de banen is ongeveer $180° - 98° = 82°$.

c $\left[\dfrac{dy}{dx}\right]_P = \left[\dfrac{dy}{dx}\right]_Q$ geeft $\dfrac{2t+1}{2t-2} = \dfrac{3}{-1}$
$6t - 6 = -2t - 1$
$8t = 5$
$t = \frac{5}{8}$
$t = \frac{5}{8}$ geeft het punt $\left(-3\frac{55}{64}, -4\frac{63}{64}\right)$.

d $\vec{v}(t) = \begin{pmatrix}2t-2\\2t+1\end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix}2\\2\end{pmatrix}$

$\vec{v}(t) \perp \vec{a}(t)$ geeft $\begin{pmatrix}2t-2\\2t+1\end{pmatrix}\cdot\begin{pmatrix}2\\2\end{pmatrix} = 0$
$4t - 4 + 4t + 2 = 0$
$8t = 2$
$t = \frac{1}{4}$
$t = \frac{1}{4}$ geeft het punt $\left(-3\frac{7}{16}, -5\frac{11}{16}\right)$.

e $y_P(t) = 0$ geeft $t^2 + t - 6 = 0$
$(t - 2)(t + 3) = 0$
$t = 2 \vee t = -3$
$t = 2$ geeft het punt $(-3, 0)$.
$\vec{v}(2) = \begin{pmatrix}2\cdot 2-2\\2\cdot 2+1\end{pmatrix} = \begin{pmatrix}2\\5\end{pmatrix}$ en $\vec{a}(2) = \begin{pmatrix}2\\2\end{pmatrix}$

$a_b(2) = \dfrac{\begin{pmatrix}2\\5\end{pmatrix}\cdot\begin{pmatrix}2\\2\end{pmatrix}}{\left|\begin{pmatrix}2\\5\end{pmatrix}\right|} = \dfrac{14}{\sqrt{29}} = \frac{14}{29}\sqrt{29}$

60 **a** $y = 4\frac{1}{2}$ geeft $3 + 3\cos(t) = 4\frac{1}{2}$
$3\cos(t) = 1\frac{1}{2}$
$\cos(t) = \frac{1}{2}$
$t = \frac{1}{3}\pi + k\cdot 2\pi \vee t = -\frac{1}{3}\pi + k\cdot 2\pi$
t op $[-2\pi, 2\pi]$ geeft $t = -1\frac{2}{3}\pi \vee t = -\frac{1}{3}\pi \vee t = \frac{1}{3}\pi \vee t = 1\frac{2}{3}\pi$

$t = -1\frac{2}{3}\pi$ geeft $B\left(-1\frac{2}{3}\pi + 1\frac{1}{2}\sqrt{3}, 4\frac{1}{2}\right)$

$t = -\frac{1}{3}\pi$ geeft $A\left(-\frac{1}{3}\pi - 1\frac{1}{2}\sqrt{3}, 4\frac{1}{2}\right)$

$t = \frac{1}{3}\pi$ geeft $D\left(\frac{1}{3}\pi + 1\frac{1}{2}\sqrt{3}, 4\frac{1}{2}\right)$

$t = 1\frac{2}{3}\pi$ geeft $C\left(1\frac{2}{3}\pi - 1\frac{1}{2}\sqrt{3}, 4\frac{1}{2}\right)$

$AD = x_D - x_A = \frac{1}{3}\pi + 1\frac{1}{2}\sqrt{3} - \left(-\frac{1}{3}\pi - 1\frac{1}{2}\sqrt{3}\right) = \frac{2}{3}\pi + 3\sqrt{3}$

14

b $\begin{cases} x(t) = t + 3\sin(t) \\ y(t) = 3 + 3\cos(t) \end{cases}$ geeft $\begin{cases} x'(t) = 1 + 3\cos(t) \\ y'(t) = -3\sin(t) \end{cases}$

raaklijn verticaal geeft $x'(t) = 0 \wedge y'(t) \neq 0$

$1 + 3\cos(t) = 0 \wedge -3\sin(t) \neq 0$

$\cos(t) = -\frac{1}{3}$

$\cos(t) = -\frac{1}{3}$ geeft $y = 3 + 3 \cdot -\frac{1}{3} = 2$

Dus de y-coördinaat van de punten met verticale raaklijn is 2.

c $|\vec{v}(t)| = \sqrt{(1 + 3\cos(t))^2 + (-3\sin(t))^2} = \sqrt{1 + 6\cos(t) + 9\cos^2(t) + 9\sin^2(t)} = \sqrt{10 + 6\cos(t)}$

d De maximale baansnelheid krijg je als $10 + 6\cos(t)$ maximaal is. Dat is voor $\cos(t) = 1$.

De maximale baansnelheid is $\sqrt{10 + 6 \cdot 1} = \sqrt{16} = 4$.

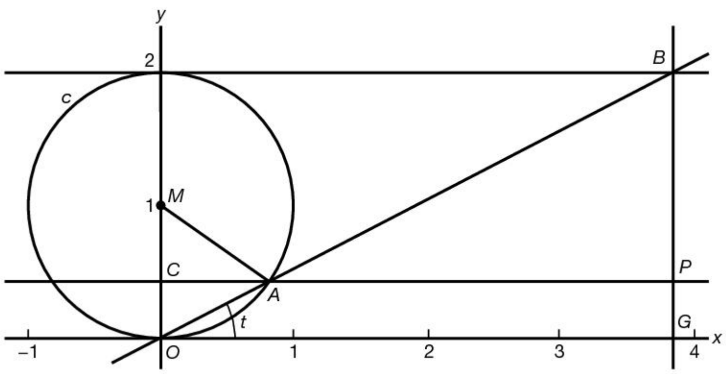
e $\begin{cases} x'(t) = 1 + 3\cos(t) \\ y'(t) = -3\sin(t) \end{cases}$ geeft $\begin{cases} x''(t) = -3\sin(t) \\ y''(t) = -3\cos(t) \end{cases}$

$$a_b\left(\tfrac{1}{2}\pi\right) = \frac{\begin{pmatrix} 1 + 3\cos\left(\frac{1}{2}\pi\right) \\ -3\sin\left(\frac{1}{2}\pi\right) \end{pmatrix} \cdot \begin{pmatrix} -3\sin\left(\frac{1}{2}\pi\right) \\ -3\cos\left(\frac{1}{2}\pi\right) \end{pmatrix}}{\left|\begin{pmatrix} 1 + 3\cos\left(\frac{1}{2}\pi\right) \\ -3\sin\left(\frac{1}{2}\pi\right) \end{pmatrix}\right|} = \frac{\begin{pmatrix} 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ -3 \end{pmatrix}\right|} = \frac{-3}{\sqrt{10}} = -\frac{3}{10}\sqrt{10}$$

Bladzijde 88

61 **a** $\left.\begin{array}{l} \sin(t) = \dfrac{2}{OB} \text{ ofwel } OB = \dfrac{2}{\sin(t)} \\ \cos(t) = \dfrac{x_P}{OB} \text{ ofwel } x_P = OB\cos(t) \end{array}\right\} x_P = \dfrac{2}{\sin(t)} \cdot \cos(t) = \dfrac{2\cos(t)}{\sin(t)}$

b



$\angle GOB = t$ geeft $\angle MOA = \frac{1}{2}\pi - t$

$\triangle OAM$ is gelijkbenig, dus $\angle MOA = \angle MAO = \frac{1}{2}\pi - t$.

De hoekensom driehoek in $\triangle OAM$ geeft $\angle OMA = \pi - \left(\frac{1}{2}\pi - t\right) - \left(\frac{1}{2}\pi - t\right) = 2t$.

$\cos(\angle OMA) = \dfrac{CM}{AM} = \dfrac{1 - y_P}{1} = 1 - y_P$

$y_P = 1 - \cos(2t)$

c $y = \frac{1}{2}$ geeft $1 - \cos(2t) = \frac{1}{2}$

$\cos(2t) = \frac{1}{2}$

$2t = \frac{1}{3}\pi + k \cdot 2\pi \vee 2t = -\frac{1}{3}\pi + k \cdot 2\pi$

$t = \frac{1}{6}\pi + k \cdot \pi \vee t = -\frac{1}{6}\pi + k \cdot \pi$

$0 < t < \pi$ geeft $t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$

$t = \frac{1}{6}\pi$ geeft $x = \dfrac{2\cos\left(\frac{1}{6}\pi\right)}{\sin\left(\frac{1}{6}\pi\right)} = \dfrac{2 \cdot \frac{1}{2}\sqrt{3}}{\frac{1}{2}} = 2\sqrt{3}$

$t = \frac{5}{6}\pi$ geeft $x = \dfrac{2\cos\left(\frac{5}{6}\pi\right)}{\sin\left(\frac{5}{6}\pi\right)} = \dfrac{2 \cdot -\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = -2\sqrt{3}$

$AB = 2\sqrt{3} - -2\sqrt{3} = 4\sqrt{3}$

d $\begin{cases} x(t) = \dfrac{2\cos(t)}{\sin(t)} \\ y(t) = 1 - \cos(2t) \end{cases}$ geeft $\begin{cases} x'(t) = \dfrac{\sin(t) \cdot -2\sin(t) - 2\cos(t) \cdot \cos(t)}{\sin^2(t)} = \dfrac{-2}{\sin^2(t)} \\ y'(t) = 2\sin(2t) \end{cases}$

$t = \frac{1}{6}\pi$ geeft $\vec{r}_k = \begin{pmatrix} \dfrac{-2}{(\frac{1}{2})^2} \\ 2 \cdot \frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -8 \\ \sqrt{3} \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} \sqrt{3} \\ 8 \end{pmatrix}$.

$\left.\begin{array}{l} x\sqrt{3} + 8y = c \\ \text{door } \left(2\sqrt{3}, \frac{1}{2}\right) \end{array}\right\} c = 2\sqrt{3} \cdot \sqrt{3} + 8 \cdot \frac{1}{2} = 10$

Dus k: $x\sqrt{3} + 8y = 10$.

e $\begin{cases} x(t) = \dfrac{2\cos(t)}{\sin(t)} \\ y(t) = 1 - \cos(2t) \end{cases}$ invullen in $y = \dfrac{8}{x^2 + 4}$ geeft $1 - \cos(2t) = \dfrac{8}{\left(\dfrac{2\cos(t)}{\sin(t)}\right)^2 + 4}$

$$1 - (1 - 2\sin^2(t)) = \frac{8}{\dfrac{4\cos^2(t)}{\sin^2(t)} + 4}$$

$$2\sin^2(t) = \frac{2}{\dfrac{\cos^2(t)}{\sin^2(t)} + 1}$$

$$2\cos^2(t) + 2\sin^2(t) = 2$$
$$2(\cos^2(t) + \sin^2(t)) = 2$$
$$2 \cdot 1 = 2$$

Dit klopt voor elke t, dus bij de baan hoort de formule $y = \dfrac{8}{x^2 + 4}$.

f $y = \dfrac{8}{x^2 + 4}$ geeft $y' = \dfrac{0 - 8 \cdot 2x}{(x^2 + 4)^2} = \dfrac{-16x}{(x^2 + 4)^2}$ en

$$y'' = \frac{(x^2 + 4)^2 \cdot -16 - -16x \cdot 2(x^2 + 4) \cdot 2x}{(x^2 + 4)^4} = \frac{-16(x^2 + 4) + 64x^2}{(x^2 + 4)^3} = \frac{48x^2 - 64}{(x^2 + 4)^3}$$

$y'' = 0$ geeft $48x^2 - 64 = 0$

$48x^2 = 64$

$x^2 = \frac{4}{3}$

$x = \sqrt{\frac{4}{3}} = \frac{2}{3}\sqrt{3} \vee x = -\frac{2}{3}\sqrt{3}$

$x = \frac{2}{3}\sqrt{3}$ geeft $y = \dfrac{8}{\frac{4}{3} + 4} = \dfrac{24}{4 + 12} = \frac{24}{16} = 1\frac{1}{2}$ en $x = -\frac{2}{3}\sqrt{3}$ geeft $y = 1\frac{1}{2}$.

Dus de buigpunten zijn $\left(\frac{2}{3}\sqrt{3}, 1\frac{1}{2}\right)$ en $\left(-\frac{2}{3}\sqrt{3}, 1\frac{1}{2}\right)$.

g $\overrightarrow{QP} = \vec{p} - \vec{q} = \begin{pmatrix} \dfrac{2\cos(t)}{\sin(t)} \\ 1 - \cos(2t) \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} \dfrac{2\cos(t)}{\sin(t)} \\ -1 - \cos(2t) \end{pmatrix}$

$$\overrightarrow{OT} = \overrightarrow{OQ} + \tfrac{1}{2}\overrightarrow{QP} + \tfrac{1}{2}\overrightarrow{QP}_L = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} \dfrac{\cos(t)}{\sin(t)} \\ -\frac{1}{2} - \frac{1}{2}\cos(2t) \end{pmatrix} + \begin{pmatrix} \frac{1}{2} + \frac{1}{2}\cos(2t) \\ \dfrac{\cos(t)}{\sin(t)} \end{pmatrix} = \begin{pmatrix} \dfrac{\cos(t)}{\sin(t)} + \frac{1}{2} + \frac{1}{2}\cos(2t) \\ 1\frac{1}{2} - \frac{1}{2}\cos(2t) + \dfrac{\cos(t)}{\sin(t)} \end{pmatrix}$$

Dus $\begin{cases} x_T(t) = \dfrac{\cos(t)}{\sin(t)} + \frac{1}{2} + \frac{1}{2}\cos(2t) \\ y_T(t) = \dfrac{\cos(t)}{\sin(t)} + 1\frac{1}{2} - \frac{1}{2}\cos(2t) \end{cases}$

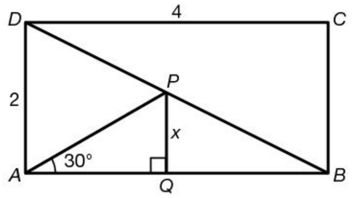
Diagnostische toets

Bladzijde 90

1 **a** $\dfrac{3}{\sqrt{7} - 1} = \dfrac{3}{\sqrt{7} - 1} \cdot \dfrac{\sqrt{7} + 1}{\sqrt{7} + 1} = \dfrac{3\sqrt{7} + 3}{7 - 1} = \dfrac{3 + 3\sqrt{7}}{6} = \frac{1}{2} + \frac{1}{2}\sqrt{7}$

b $\dfrac{5\sqrt{3}}{\sqrt{5} - \sqrt{3}} = \dfrac{5\sqrt{3}}{\sqrt{5} - \sqrt{3}} \cdot \dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \dfrac{5\sqrt{15} + 15}{5 - 3} = 7\frac{1}{2} + 2\frac{1}{2}\sqrt{15}$

2 Stel $PQ = x$.



$PQ = x$ geeft $AQ = x\sqrt{3}$ en $AP = 2x$

$$\triangle ABD \backsim \triangle QBP \text{ geeft } \frac{AD}{QP} = \frac{AB}{QB}$$
$$\frac{2}{x} = \frac{4}{4 - x\sqrt{3}}$$
$$4x = 8 - 2x\sqrt{3}$$
$$4x + 2x\sqrt{3} = 8$$
$$x(4 + 2\sqrt{3}) = 8$$
$$x = \frac{8}{4 + 2\sqrt{3}} = \frac{8}{4 + 2\sqrt{3}} \cdot \frac{4 - 2\sqrt{3}}{4 - 2\sqrt{3}} = \frac{32 - 16\sqrt{3}}{16 - 12} = 8 - 4\sqrt{3}$$

Dus $AP = 2x = 16 - 8\sqrt{3}$.

3 Stel $NP = x$.

$$\triangle MPD \backsim \triangle NPC \text{ geeft } \frac{MP}{NP} = \frac{MD}{NC}$$
$$\frac{x + 4}{x} = \frac{2}{1}$$
$$2x = x + 4$$
$$x = 4$$

$$\left.\begin{array}{l} \sin\left(\frac{1}{2}\angle(k, l)\right) = \frac{NC}{NP} = \frac{1}{4} \\ \cos^2\left(\frac{1}{2}\angle(k, l)\right) + \sin^2\left(\frac{1}{2}\angle(k, l)\right) = 1 \end{array}\right\} \cos^2\left(\tfrac{1}{2}\angle(k, l)\right) + \tfrac{1}{16} = 1$$
$$\cos^2\left(\tfrac{1}{2}\angle(k, l)\right) = \tfrac{15}{16}$$
$$\cos\left(\tfrac{1}{2}\angle(k, l)\right) = \tfrac{1}{4}\sqrt{15} \vee \cos\left(\tfrac{1}{2}\angle(k, l)\right) = -\tfrac{1}{4}\sqrt{15}$$

vold. / vold. niet

$$\sin(\angle(k, l)) = 2\sin\left(\tfrac{1}{2}\angle(k, l)\right) \cdot \cos\left(\tfrac{1}{2}\angle(k, l)\right) = 2 \cdot \tfrac{1}{4} \cdot \tfrac{1}{4}\sqrt{15} = \tfrac{1}{8}\sqrt{15}$$

4 **a** Het midden van AC is $(1, 4\frac{1}{2})$ en $\overrightarrow{AC} = \begin{pmatrix} 0 \\ 7 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, dus mll_{AC}: $y = 4\frac{1}{2}$

Het midden van BC is $(3, 6)$ en $\overrightarrow{BC} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, dus mll_{BC}: $x - y = -3$

$$\left.\right\} x - 4\tfrac{1}{2} = -3$$
$$x = 1\tfrac{1}{2}$$

Dus het snijpunt van de middelloodlijnen van AC en BC is $M\left(1\frac{1}{2}, 4\frac{1}{2}\right)$.

b
$$\left.\begin{array}{l} \overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \\ \overrightarrow{AC} = \begin{pmatrix} 0 \\ 7 \end{pmatrix} \triangleq \begin{pmatrix} 0 \\ 5 \end{pmatrix} \end{array}\right\} \vec{r}_k = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\vec{r}_k = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ geeft $\vec{n}_k = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$$\left.\begin{array}{l} k\text{: } 2x - y = c \\ \text{door } A(1, 1) \end{array}\right\} c = 2 \cdot 1 - 1 = 1$$

Dus l: $2x - y = 1$.

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, dus BC: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

Snijden van l en BC geeft $2(5 + \lambda) - (4 - \lambda) = 1$
$$10 + 2\lambda - 4 + \lambda = 1$$
$$3\lambda = -5$$
$$\lambda = -1\tfrac{2}{3}$$

$\lambda = -1\frac{2}{3}$ geeft $D\left(5 - 1\frac{2}{3}, 4 + 1\frac{2}{3}\right)$ ofwel $D\left(3\frac{1}{3}, 5\frac{2}{3}\right)$.

c Stel $\vec{r}_l = \begin{pmatrix} 1 \\ p \end{pmatrix}$.

Uit $\angle(AC, l) = 60°$ volgt $\cos(60°) = \dfrac{|\vec{r}_{AC} \cdot \vec{r}_l|}{|\vec{r}_{AC}| \cdot |\vec{r}_l|}$

$$\tfrac{1}{2} = \frac{\left|\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ p \end{pmatrix}\right|}{\left|\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ p \end{pmatrix}\right|}$$

$$\tfrac{1}{2} = \frac{|p|}{1 \cdot \sqrt{1^2 + p^2}}$$

$$2 \cdot |p| = \sqrt{1 + p^2}$$
$$4p^2 = 1 + p^2$$
$$3p^2 = 1$$
$$p^2 = \tfrac{1}{3}$$
$$p = \tfrac{1}{3}\sqrt{3} \vee p = -\tfrac{1}{3}\sqrt{3}$$

$p = \tfrac{1}{3}\sqrt{3}$ geeft $\vec{r}_{l_1} = \begin{pmatrix} 1 \\ \frac{1}{3}\sqrt{3} \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix} \triangleq \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$, dus $\vec{n}_{l_1} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$.

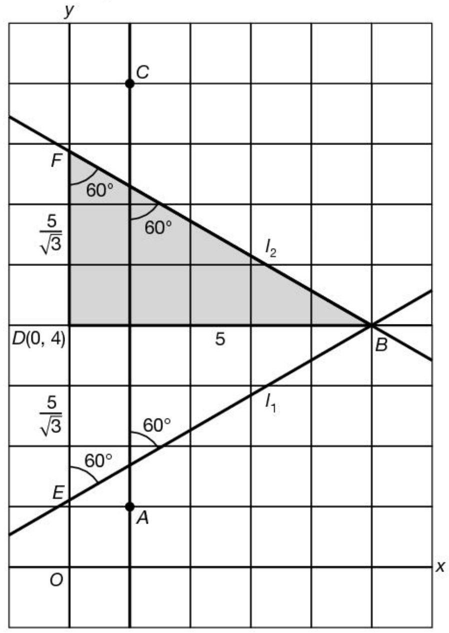
$p = -\tfrac{1}{3}\sqrt{3}$ geeft $\vec{r}_{l_2} = \begin{pmatrix} 1 \\ -\frac{1}{3}\sqrt{3} \end{pmatrix}$, dus $\vec{n}_{l_2} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$.

$\left.\begin{array}{l} l_1\colon x - y\sqrt{3} = c \\ B(5, 4) \end{array}\right\}$ $c = 5 - 4\sqrt{3}$, dus $l_1\colon x - y\sqrt{3} = 5 - 4\sqrt{3}$.

$\left.\begin{array}{l} l_2\colon x + y\sqrt{3} = c \\ B(5, 4) \end{array}\right\}$ $c = 5 + 4\sqrt{3}$, dus $l_2\colon x + y\sqrt{3} = 5 + 4\sqrt{3}$.

Alternatieve uitwerking

c Zie de figuur.



$BD = 5$ geeft $DE = DF = \dfrac{5}{\sqrt{3}} = 1\tfrac{2}{3}\sqrt{3}$, dus $E\left(0, 4 - 1\tfrac{2}{3}\sqrt{3}\right)$ en $F\left(0, 4 + 1\tfrac{2}{3}\sqrt{3}\right)$.

$\text{rc}_{l_1} = \tan(30°) = \tfrac{1}{3}\sqrt{3}$ en $\text{rc}_{l_2} = -\tan(30°) = -\tfrac{1}{3}\sqrt{3}$

Dus $l_1\colon y = \tfrac{1}{3}x\sqrt{3} + 4 - 1\tfrac{2}{3}\sqrt{3}$ en $l_2\colon y = -\tfrac{1}{3}x\sqrt{3} + 4 + 1\tfrac{2}{3}\sqrt{3}$.

5 **a** $d(M, k) = \dfrac{|2 \cdot 2 + 1 - 9|}{\sqrt{2^2 + 1^2}} = \dfrac{4}{\sqrt{5}}$

Dus $c_1\colon (x - 2)^2 + (y - 1)^2 = \tfrac{16}{5}$.

b $x^2 + y^2 - 2x - 4y - 5 = 0$
$x^2 - 2x + y^2 - 4y - 5 = 0$
$(x - 1)^2 - 1 + (y - 2)^2 - 4 - 5 = 0$
$(x - 1)^2 + (y - 2)^2 = 10$

Het middelpunt is $N(1, 2)$.

$\overrightarrow{NA} = \vec{a} - \vec{n} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\left.\begin{array}{l} x - 3y = c \\ A(2, -1) \end{array}\right\} c = 2 - 3 \cdot -1 = 5$

Dus l: $x - 3y = 5$.

c c_3: $(x + 2)^2 + (y - 3)^2 = 5$

Het middelpunt van c_3 is $B(-2, 3)$ en de straal is $\sqrt{5}$.

$\left.\begin{array}{l} \text{Stel } m\text{: } y = ax + b \\ P(1, 2) \end{array}\right\} \begin{array}{l} a \cdot 1 + b = 2 \\ b = 2 - a \end{array}$

Dus m: $y = ax + 2 - a$ ofwel m: $ax - y + 2 - a = 0$.

$d(B, m) = \sqrt{5}$ geeft $\frac{|-2a - 3 + 2 - a|}{\sqrt{a^2 + 1}} = \sqrt{5}$

$|-3a - 1| = \sqrt{5(a^2 + 1)}$

$9a^2 + 6a + 1 = 5a^2 + 5$

$4a^2 + 6a - 4 = 0$

$2a^2 + 3a - 2 = 0$

$D = 3^2 - 4 \cdot 2 \cdot -2 = 25$

$a = \frac{-3 + 5}{4} = \frac{1}{2} \vee a = \frac{-3 - 5}{4} = -2$

Dus m_1: $y = \frac{1}{2} + 1\frac{1}{2}$ en m_2: $y = -2x + 4$.

d $x^2 + y^2 + 8x - 6y + 20 = 0$

$x^2 + 8x + y^2 - 6y + 20 = 0$

$(x + 4)^2 - 16 + (y - 3)^2 - 9 + 20 = 0$

$(x + 4)^2 + (y - 3)^2 = 5$

Het middelpunt van c_4 is $C(-4, 3)$ en de straal is $\sqrt{5}$.

Stel n: $y = 2x + b$ ofwel $2x - y + b = 0$.

$d(C, n) = \sqrt{5}$ geeft $\frac{|2 \cdot -4 - 3 + b|}{\sqrt{2^2 + 1^2}} = \sqrt{5}$

$|b - 11| = 5$

$b - 11 = 5 \vee b - 11 = -5$

$b = 16 \vee b = 6$

Dus n_1: $y = 2x + 16$ en n_2: $y = 2x + 6$.

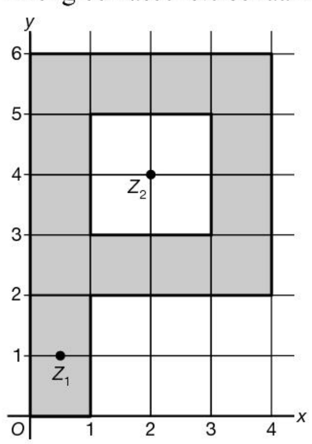
Bladzijde 91

6 $M = m_1 + m_2 + m_3 + m_4 = 5 + 2 + 8 + 12 = 27$

$\vec{z} = \frac{1}{27}\left(5 \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} + 8 \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} + 12 \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix}\right) = \frac{1}{27}\left(\begin{pmatrix} 5 \\ 15 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 32 \\ 16 \end{pmatrix} + \begin{pmatrix} 36 \\ 48 \end{pmatrix}\right) = \frac{1}{27}\begin{pmatrix} 79 \\ 81 \end{pmatrix} = \begin{pmatrix} \frac{79}{27} \\ 3 \end{pmatrix}$

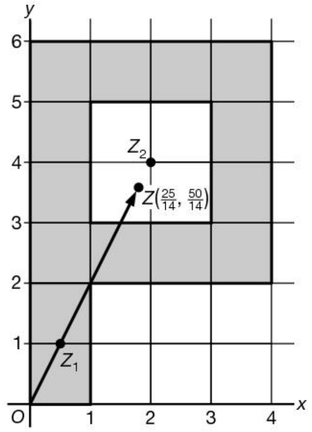
Dus $Z\left(2\frac{25}{27}, 3\right)$.

7 Breng een assenstelsel aan en verdeel de letter P in twee delen zoals hieronder.



$M = m_1 + m_2 = 2 + 12 = 14$

$$\vec{z} = \tfrac{1}{14}(2 \cdot \vec{z}_1 + 12 \cdot \vec{z}_2) = \tfrac{1}{14}\left(2 \cdot \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} + 12 \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix}\right) = \tfrac{1}{14}\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 24 \\ 48 \end{pmatrix}\right) = \tfrac{1}{14}\begin{pmatrix} 25 \\ 50 \end{pmatrix} = \tfrac{25}{14}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



8 **a** $\begin{cases} x(t) = t^2 \\ y(t) = -t^3 + 3t^2 + 9t \end{cases}$ geeft $\begin{cases} x'(t) = 2t \\ y'(t) = -3t^2 + 6t + 9 \end{cases}$

Raaklijn evenwijdig aan de x-as, dus $y'(t) = 0 \wedge x'(t) \neq 0$

$$-3t^2 + 6t + 9 = 0 \wedge 2t \neq 0$$
$$t^2 - 2t - 3 = 0 \wedge t \neq 0$$
$$(t + 1)(t - 3) = 0 \wedge t \neq 0$$
$$(t = -1 \vee t = 3) \wedge t \neq 0$$
$$t = -1 \vee t = 3$$

$t = -1$ geeft het punt $(1, -5)$ en $t = 3$ geeft het punt $(9, 27)$.
Dus de raaklijn is evenwijdig aan de x-as in de punten $(1, -5)$ en $(9, 27)$.
Raaklijn evenwijdig aan de y-as, dus $x'(t) = 0 \wedge y'(t) \neq 0$

$$t = 0 \wedge t \neq -1 \wedge t \neq 3$$
$$t = 0$$

$t = 0$ geeft het punt $(0, 0)$.
Dus de raaklijn is evenwijdig aan de y-as in het punt $(0, 0)$.

b $t = 2$ geeft $\begin{pmatrix} x'(2) \\ y'(2) \end{pmatrix} = \begin{pmatrix} 2 \cdot 2 \\ -3 \cdot 2^2 + 6 \cdot 2 + 9 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 9 \\ -4 \end{pmatrix}$.

$t = 2$ geeft $A(4, 22)$

$\left.\begin{matrix} 9x - 4y = b \\ A(4, 22) \end{matrix}\right\} b = 9 \cdot 4 - 4 \cdot 22 = -52$

Dus k: $9x - 4y = -52$.

c $x = 9$ geeft $t^2 = 9$

$$t = -3 \vee t = 3$$

Zie vraag a, $t = 3$ geeft een horizontale raaklijn.

$t = -3$ geeft $\text{rc}_{\text{raaklijn}} = \dfrac{y'(-3)}{x'(-3)} = \dfrac{-36}{-6} = 6$

$\tan(\alpha) = 6$ geeft $\alpha = 80{,}53...^\circ$
Dus de hoek waaronder de baan zichzelf snijdt is ongeveer 80,5°.

d $\vec{v}(t) = \begin{pmatrix} 2t \\ -3t^2 + 6t + 9 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 2 \\ -6t + 6 \end{pmatrix}$

$|\vec{v}(-1)| = \sqrt{(-2)^2 + (-3 - 6 + 9)^2} = \sqrt{4 + 0} = 2$

Dus de baansnelheid is 2 cm/s.

$$a_b(-1) = \frac{\vec{v}(-1) \cdot \vec{a}(-1)}{|\vec{v}(-1)|} = \frac{\begin{pmatrix} -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 12 \end{pmatrix}}{\left|\begin{pmatrix} -2 \\ 0 \end{pmatrix}\right|} = \frac{-4}{2} = -2$$

Dus de baanversnelling is -2 cm/s^2.

9 **a** $\begin{cases} x(t) = \sin(2t) \\ y(t) = t + \cos(t) \end{cases}$ geeft $\begin{cases} x'(t) = 2\cos(2t) \\ y'(t) = 1 - \sin(t) \end{cases}$

Raaklijn evenwijdig aan de x-as, dus $y'(t) = 0 \wedge x'(t) \neq 0$

$$1 - \sin(t) = 0 \wedge 2\cos(2t) \neq 0$$
$$\sin(t) = 1 \wedge 2t \neq \tfrac{1}{2}\pi + k \cdot \pi$$
$$t = \tfrac{1}{2}\pi + k \cdot 2\pi \wedge t \neq \tfrac{1}{4}\pi + k \cdot \tfrac{1}{2}\pi$$
$$t \text{ op } [-\pi, \pi] \text{ geeft } t = \tfrac{1}{2}\pi$$

$t = \tfrac{1}{2}\pi$ geeft het punt $B\left(0, \tfrac{1}{2}\pi\right)$.
Dus de raaklijn is horizontaal in het punt $B\left(0, \tfrac{1}{2}\pi\right)$.
Raaklijn evenwijdig aan de y-as, dus $x'(t) = 0 \wedge y'(t) \neq 0$

$$t = \tfrac{1}{4}\pi + k \cdot \tfrac{1}{2}\pi \wedge t \neq \tfrac{1}{2}\pi + k \cdot 2\pi$$
$$t \text{ op } [-\pi, \pi] \text{ geeft } t = -\tfrac{3}{4}\pi \vee t = -\tfrac{1}{4}\pi \vee t = \tfrac{1}{4}\pi \vee t = \tfrac{3}{4}\pi$$

$t = -\tfrac{3}{4}\pi$ geeft $\left(1, -\tfrac{3}{4}\pi - \tfrac{1}{2}\sqrt{2}\right)$

$t = -\tfrac{1}{4}\pi$ geeft $\left(-1, -\tfrac{1}{4}\pi + \tfrac{1}{2}\sqrt{2}\right)$

$t = \tfrac{1}{4}\pi$ geeft $\left(1, \tfrac{1}{4}\pi + \tfrac{1}{2}\sqrt{2}\right)$

$t = \tfrac{3}{4}\pi$ geeft $\left(-1, \tfrac{3}{4}\pi - \tfrac{1}{2}\sqrt{2}\right)$
Dus de raaklijn is verticaal in de punten $\left(1, -\tfrac{3}{4}\pi - \tfrac{1}{2}\sqrt{2}\right)$, $\left(-1, -\tfrac{1}{4}\pi + \tfrac{1}{2}\sqrt{2}\right)$, $\left(1, \tfrac{1}{4}\pi + \tfrac{1}{2}\sqrt{2}\right)$ en $\left(-1, \tfrac{3}{4}\pi - \tfrac{1}{2}\sqrt{2}\right)$.

b $x = 0$ geeft $\sin(2t) = 0$

$$2t = k \cdot \pi$$
$$t = k \cdot \tfrac{1}{2}\pi$$

t op $[-\pi, \pi]$ geeft $t = -\pi$, $t = -\tfrac{1}{2}\pi$, $t = 0$, $t = \tfrac{1}{2}\pi$ en $t = \pi$.

$t = -\tfrac{1}{2}\pi$ geeft $A\left(0, -\tfrac{1}{2}\pi\right)$

$\begin{cases} x'\left(-\tfrac{1}{2}\pi\right) = 2\cos(-\pi) = -2 \\ y'\left(-\tfrac{1}{2}\pi\right) = 1 - \sin\left(-\tfrac{1}{2}\pi\right) = 2 \end{cases}$

Dus $\vec{n}_k = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$\left.\begin{array}{l} k\colon x + y = b \\ A\left(0, -\tfrac{1}{2}\pi\right) \end{array}\right\} b = 0 - \tfrac{1}{2}\pi = -\tfrac{1}{2}\pi$

Dus $k\colon x + y = -\tfrac{1}{2}\pi$.

c $\vec{v}(t) = \begin{pmatrix} 2\cos(2t) \\ 1 - \sin(t) \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} -4\sin(2t) \\ -\cos(t) \end{pmatrix}$

$v\left(\tfrac{1}{2}\pi\right) = \sqrt{(2\cos(\pi))^2 + \left(1 - \sin\left(\tfrac{1}{2}\pi\right)\right)^2} = \sqrt{(-2)^2 + (1-1)^2} = \sqrt{4 + 0} = 2$

Dus de baansnelheid is 2 cm/s.

$$a_b\left(\tfrac{1}{2}\pi\right) = \frac{\vec{v}\left(\tfrac{1}{2}\pi\right) \cdot \vec{a}\left(\tfrac{1}{2}\pi\right)}{\left|\vec{v}\left(\tfrac{1}{2}\pi\right)\right|} = \frac{\begin{pmatrix} -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} -2 \\ 0 \end{pmatrix}\right|} = \frac{0}{2} = 0$$

Dus de baanversnelling is 0 cm/s^2.

d $v(t) = \sqrt{(2\cos(2t))^2 + (1 - \sin(t))^2}$
Voer in $y_1 = \sqrt{(2\cos(2x))^2 + (1 - \sin(x))^2}$.
De optie maximum geeft $x = -1{,}57079...$ en $y = 2{,}828...$
$\tfrac{1}{2}\pi = 1{,}57079...$
Dus de baansnelheid is maximaal in het punt A.

15 Afgeleiden en primitieven

Voorkennis Differentiëren en integreren

Bladzijde 95

1 **a** $f(x) = x^2 \cdot \sqrt{5x}$ geeft $f'(x) = 2x \cdot \sqrt{5x} + x^2 \cdot \frac{5}{2\sqrt{5x}} = 2x\sqrt{5x} + \frac{5x^2}{2\sqrt{5x}}$

b $f(x) = 10x \cdot 2^x$ geeft $f'(x) = 10 \cdot 2^x + 10x \cdot 2^x \cdot \ln(2) = 10 \cdot 2^x + 10x\ln(2) \cdot 2^x$

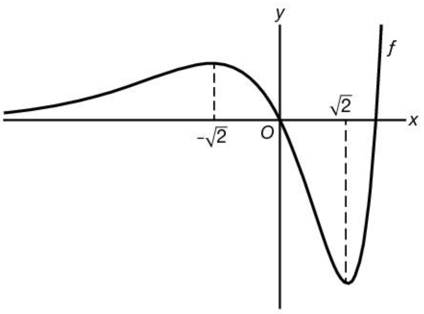
c $f(x) = x^2 - x\ln(x)$ geeft $f'(x) = 2x - 1 \cdot \ln(x) - x \cdot \frac{1}{x} = 2x - 1 - \ln(x)$

d $f(x) = \frac{e^x - 1}{x^2}$ geeft $f'(x) = \frac{x^2 \cdot e^x - (e^x - 1) \cdot 2x}{x^4} = \frac{xe^x - 2(e^x - 1)}{x^3} = \frac{xe^x - 2e^x + 2}{x^3} = \frac{(x-2)e^x + 2}{x^3}$

e $f(x) = {}^2\log(x^2 + 4)$ geeft $f'(x) = \frac{1}{\ln(2) \cdot (x^2 + 4)} \cdot 2x = \frac{2x}{\ln(2) \cdot (x^2 + 4)}$

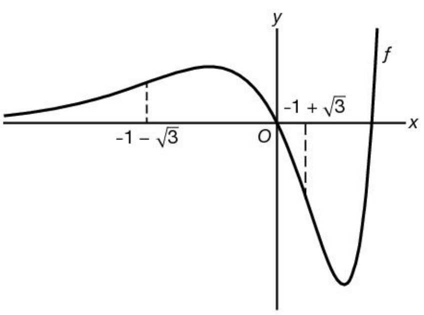
f $f(x) = 3\sin(x)\sin(2x)$ geeft
$f'(x) = 3\cos(x) \cdot \sin(2x) + 3\sin(x) \cdot 2\cos(2x) = 3\cos(x)\sin(2x) + 6\sin(x)\cos(2x)$

2 **a** $f(x) = (x^2 - 2x)e^x$ geeft $f'(x) = (2x - 2) \cdot e^x + (x^2 - 2x) \cdot e^x = (x^2 - 2)e^x$
$f'(x) = 0$ geeft $(x^2 - 2)e^x = 0$
$x^2 - 2 = 0$
$x^2 = 2$
$x = \sqrt{2} \vee x = -\sqrt{2}$



max. is $f(-\sqrt{2}) = (2 + 2\sqrt{2})e^{-\sqrt{2}}$ en min. is $f(\sqrt{2}) = (2 - 2\sqrt{2})e^{\sqrt{2}}$.

b $f'(x) = (x^2 - 2)e^x$ geeft $f''(x) = 2x \cdot e^x + (x^2 - 2) \cdot e^x = (x^2 + 2x - 2)e^x$
$f''(x) = 0$ geeft $(x^2 + 2x - 2)e^x = 0$
$x^2 + 2x - 2 = 0$
$(x + 1)^2 - 1 - 2 = 0$
$(x + 1)^2 = 3$
$x + 1 = -\sqrt{3} \vee x + 1 = \sqrt{3}$
$x = -1 - \sqrt{3} \vee x = -1 + \sqrt{3}$



Dus de x-coördinaten van de buigpunten zijn $-1 - \sqrt{3}$ en $-1 + \sqrt{3}$.

3 $f(x) = \cos^2(x) - \sin(x)$ geeft $f'(x) = 2\cos(x) \cdot -\sin(x) - \cos(x) = -2\cos(x)\sin(x) - \cos(x)$

$f'(x) = 0$ geeft $-2\cos(x)\sin(x) - \cos(x) = 0$

$$\cos(x)(-2\sin(x) - 1) = 0$$
$$\cos(x) = 0 \vee \sin(x) = -\tfrac{1}{2}$$
$$x = \tfrac{1}{2}\pi + k \cdot \pi \vee x = -\tfrac{1}{6}\pi + k \cdot 2\pi \vee x = 1\tfrac{1}{6}\pi + k \cdot 2\pi$$

x op $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{1}{2}\pi \vee x = 1\frac{5}{6}\pi$

Dus $x_A = \frac{1}{2}\pi$ en $x_B = 1\frac{5}{6}\pi$.

$f\left(\frac{1}{2}\pi\right) = 0 - 1 = -1$ en $f\left(1\frac{5}{6}\pi\right) = \left(\frac{1}{2}\sqrt{3}\right)^2 - -\frac{1}{2} = \frac{3}{4} + \frac{1}{2} = 1\frac{1}{4}$

Dus $A\left(\frac{1}{2}\pi, -1\right)$ en $B\left(1\frac{5}{6}\pi, 1\frac{1}{4}\right)$.

Stel k: $y = ax + b$ met $a = \dfrac{1\frac{1}{4} - -1}{1\frac{5}{6}\pi - \frac{1}{2}\pi} = \dfrac{2\frac{1}{4}}{1\frac{1}{3}\pi} = \dfrac{27}{16\pi}$.

$$\left.\begin{array}{l} y = \dfrac{27}{16\pi}x + b \\ \text{door } A\left(\frac{1}{2}\pi, -1\right) \end{array}\right\} \dfrac{27}{16\pi} \cdot \tfrac{1}{2}\pi + b = -1$$
$$\tfrac{27}{32} + b = -1$$
$$b = -1\tfrac{27}{32}$$

Dus $y_C = -1\frac{27}{32}$.

Bladzijde 97

4 **a** $I(N) = \pi \displaystyle\int_{1\frac{1}{2}}^{3\frac{1}{2}} (f(x) + 1)^2 \,dx - \pi \cdot 1^2 \cdot \left(3\tfrac{1}{2} - 1\tfrac{1}{2}\right) = \pi \int_{1\frac{1}{2}}^{3\frac{1}{2}} \left(\sqrt{2x - 3} + 1\right)^2 dx - \pi \cdot 2$

$$= \pi \int_{1\frac{1}{2}}^{3\frac{1}{2}} \left(2x - 3 + 2\sqrt{2x - 3} + 1\right) dx - 2\pi = \pi \int_{1\frac{1}{2}}^{3\frac{1}{2}} \left(2x - 2 + 2(2x - 3)^{\frac{1}{2}}\right) dx - 2\pi$$
$$= \pi\left[x^2 - 2x + \tfrac{2}{3}(2x - 3)^{1\frac{1}{2}}\right]_{1\frac{1}{2}}^{3\frac{1}{2}} - 2\pi$$
$$= \pi\left(12\tfrac{1}{4} - 7 + \tfrac{2}{3}(7 - 3)\sqrt{7 - 3} - \left(2\tfrac{1}{4} - 3 + \tfrac{2}{3}(3 - 3)\sqrt{3 - 3}\right)\right) - 2\pi$$
$$= \pi\left(5\tfrac{1}{4} + 5\tfrac{1}{3} - \left(-\tfrac{3}{4}\right)\right) - 2\pi$$
$$= 11\tfrac{1}{3}\pi - 2\pi = 9\tfrac{1}{3}\pi$$

b $f(x) = \sqrt{2x - 3}$

$\downarrow$ translatie $\left(-3\frac{1}{2}, 0\right)$

$$y = \sqrt{2\left(x + 3\tfrac{1}{2}\right) - 3}$$
$$y = \sqrt{2x + 4}$$
$$2x + 4 = y^2$$
$$2x = y^2 - 4$$
$$x = \tfrac{1}{2}y^2 - 2$$

$$I(P) = \pi \int_0^2 \left(\tfrac{1}{2}y^2 - 2\right)^2 dy = \pi \int_0^2 \left(\tfrac{1}{4}y^4 - 2y^2 + 4\right) dy = \pi\left[\tfrac{1}{20}y^5 - \tfrac{2}{3}y^3 + 4y\right]_0^2$$
$$= \pi\left(\tfrac{1}{20} \cdot 2^5 - \tfrac{2}{3} \cdot 2^3 + 4 \cdot 2 - 0\right) = 4\tfrac{4}{15}\pi$$

Bladzijde 98

5 **a** $f(x) = g(x)$ geeft $\dfrac{10}{2x - 1} = -2x + 12$

$$-4x^2 + 26x - 12 = 10$$
$$-4x^2 + 26x - 22 = 0$$
$$2x^2 - 13x + 11 = 0$$
$$D = (-13)^2 - 4 \cdot 2 \cdot 11 = 81$$
$$x = \frac{13 + 9}{4} = 5\tfrac{1}{2} \vee x = \frac{13 - 9}{4} = 1$$

$$O(V) = \int_1^{5\frac{1}{2}} (g(x) - f(x))dx = \int_1^{5\frac{1}{2}} \left(-2x + 12 - \frac{10}{2x - 1}\right) dx = \left[-x^2 + 12x - 5\ln(2x - 1)\right]_1^{5\frac{1}{2}}$$
$$= -\left(5\tfrac{1}{2}\right)^2 + 12 \cdot 5\tfrac{1}{2} - 5\ln\left(2 \cdot 5\tfrac{1}{2} - 1\right) - (-1^2 + 12 \cdot 1 - 5\ln(2 \cdot 1 - 1))$$
$$= -30\tfrac{1}{4} + 66 - 5\ln(10) - (-1 + 12 - 0) = 24\tfrac{3}{4} - 5\ln(10)$$

b $I(L) = \pi\int_{1}^{5\frac{1}{2}} (g(x))^2\,\mathrm{d}x - \pi\int_{1}^{5\frac{1}{2}} (f(x))^2\,\mathrm{d}x = \pi\int_{1}^{5\frac{1}{2}} (-2x+12)^2\,\mathrm{d}x - \pi\int_{1}^{5\frac{1}{2}} \frac{100}{(2x-1)^2}\,\mathrm{d}x$

$= \pi\int_{1}^{5\frac{1}{2}} (4x^2 - 48x + 144)\,\mathrm{d}x - \pi\int_{1}^{5\frac{1}{2}} 100(2x-1)^{-2}\,\mathrm{d}x$

$= \pi\left[1\tfrac{1}{3}x^3 - 24x^2 + 144x\right]_{1}^{5\frac{1}{2}} - \pi\left[-50(2x-1)^{-1}\right]_{1}^{5\frac{1}{2}} = \pi\left[1\tfrac{1}{3}x^3 - 24x^2 + 144x\right]_{1}^{5\frac{1}{2}} - \pi\left[\frac{-50}{2x-1}\right]_{1}^{5\frac{1}{2}}$

$= \pi\left(1\tfrac{1}{3}\cdot\left(5\tfrac{1}{2}\right)^3 - 24\cdot\left(5\tfrac{1}{2}\right)^2 + 144\cdot 5\tfrac{1}{2} - \left(1\tfrac{1}{3}\cdot 1^3 - 24\cdot 1^2 + 144\cdot 1\right)\right) - \pi\left(\frac{-50}{10} - \frac{-50}{1}\right)$

$= 287\tfrac{5}{6}\pi - 121\tfrac{1}{3}\pi - 45\pi = 121\tfrac{1}{2}\pi$

c $f(1) = 10$ en $f\left(5\tfrac{1}{2}\right) = 1$

$y = \frac{10}{2x-1}$ geeft $2xy - y = 10$

$2xy = y + 10$

$x = \tfrac{1}{2} + \frac{1}{2y}$

$y = -2x + 12$ geeft $2x = 12 - y$

$x = 6 - \tfrac{1}{2}y$

$I(M) = \pi\int_{1}^{10}\left(6 - \tfrac{1}{2}y\right)^2\mathrm{d}y - \pi\int_{1}^{10}\left(\tfrac{1}{2} + \frac{1}{2y}\right)^2\mathrm{d}y = \pi\int_{1}^{10}\left(36 - 6y + \tfrac{1}{4}y^2\right)\mathrm{d}y - \pi\int_{1}^{10}\left(\tfrac{1}{4} + \frac{1}{2y} + \frac{1}{4y^2}\right)\mathrm{d}y$

$= \pi\int_{1}^{10}\left(36 - 6y + \tfrac{1}{4}y^2\right)\mathrm{d}y - \pi\int_{1}^{10}\left(\tfrac{1}{4} + \tfrac{1}{2}\cdot\frac{1}{y} + \tfrac{1}{4}y^{-2}\right)\mathrm{d}y$

$= \pi\left[36y - 3y^2 + \tfrac{1}{12}y^3\right]_{1}^{10} - \pi\left[\tfrac{1}{4}y + \tfrac{1}{2}\ln(y) - \tfrac{1}{4}y^{-1}\right]_{1}^{10}$

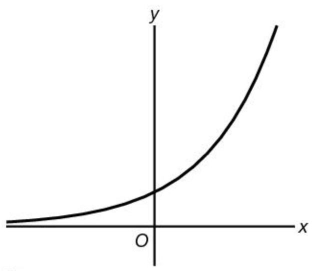
$= \pi\left(\left(360 - 300 + 83\tfrac{1}{3}\right) - \left(36 - 3 + \tfrac{1}{12}\right)\right) - \pi\left(\left(2\tfrac{1}{2} + \tfrac{1}{2}\ln(10) - \tfrac{1}{40}\right) + \left(\tfrac{1}{4} + 0 - \tfrac{1}{4}\right)\right)$

$= 143\tfrac{1}{3}\pi - 33\tfrac{1}{12}\pi - 2\tfrac{19}{40}\pi - \tfrac{1}{2}\pi\ln(10) = 107\tfrac{31}{40}\pi - \tfrac{1}{2}\pi\ln(10)$

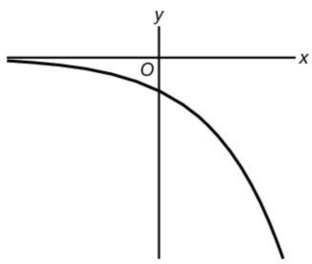
15.1 Hellingen, buigpunten en toppen

Bladzijde 99

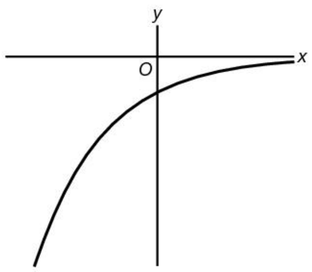
1 **a** I



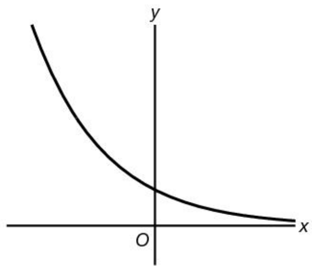
III



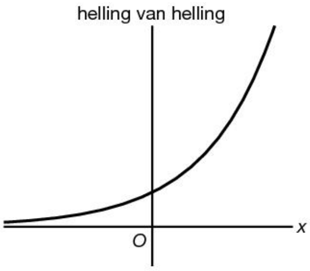
II



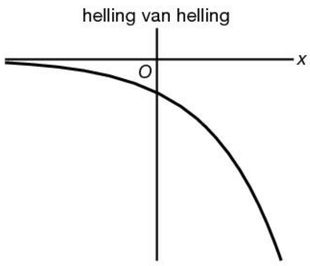
IV



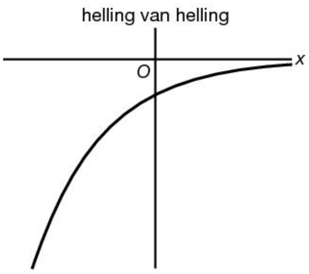
b I



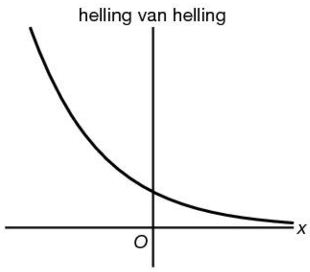
III



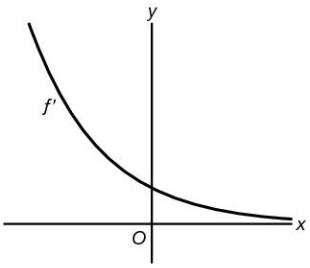
II

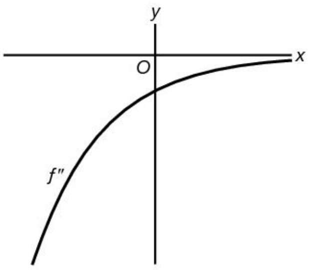


IV

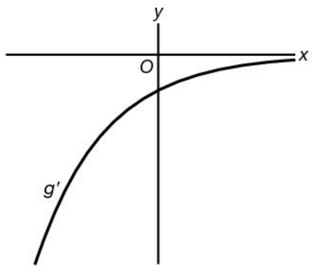


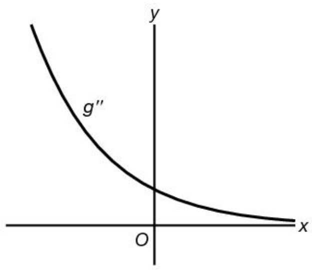
2 **a**





b





3 $f(x) = x + \frac{1}{2}e^x$ geeft $f'(x) = 1 + \frac{1}{2}e^x > 0$ voor elke x, dus f is stijgend.
$f'(x) = 1 + \frac{1}{2}e^x$ geeft $f''(x) = \frac{1}{2}e^x > 0$ voor elke x, dus f' is stijgend ofwel f is toenemend stijgend.

Bladzijde 101

4 $T = 20 + 80e^{-0,2t}$ geeft $\frac{dT}{dt} = 80e^{-0,2t} \cdot -0,2 = -16e^{-0,2t}$

$\frac{d}{dt}\left(\frac{dT}{dt}\right) = -16e^{-0,2t} \cdot -0,2 = 3,2e^{-0,2t}$

$$\left.\begin{array}{l} \frac{dT}{dt} = -16e^{-0,2t} < 0 \\ \frac{d}{dt}\left(\frac{dT}{dt}\right) = 3,2e^{-0,2t} > 0 \end{array}\right\} T \text{ is afnemend dalend}$$

Het afkoelingsproces verloopt dus steeds langzamer.

Bladzijde 102

5 **a** Voer in $y_1 = 100e^{-0,01x^2}$ en $y_2 = 50$.
Intersect geeft $x = 8,325...$
Na ongeveer 8,325... minuten ≈ 500 seconden is de helft weggestroomd.

b $\frac{dV}{dt} = 100 \cdot e^{-0,01t^2} \cdot -0,02t = -2te^{-0,01t^2}$

$\frac{d}{dt}\left(\frac{dV}{dt}\right) = -2e^{-0,01t^2} - 2t \cdot e^{-0,01t^2} \cdot -0,02t = -2e^{-0,01t^2} + 0,04t^2e^{-0,01t^2} = (0,04t^2 - 2)e^{-0,01t^2}$

$\frac{d}{dt}\left(\frac{dV}{dt}\right) = 0$ geeft $(0{,}04t^2 - 2)e^{-0{,}01t^2} = 0$

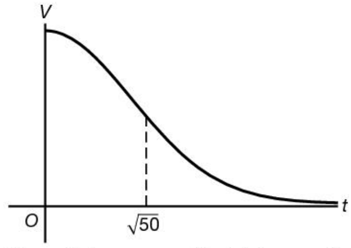
$0{,}04t^2 - 2 = 0$

$0{,}04t^2 = 2$

$t^2 = 50$

$t = \sqrt{50} \vee t = -\sqrt{50}$

vold. vold. niet



De uitstroomsnelheid is maximaal na $\sqrt{50}$ minuten, dus na ongeveer 424 seconden.

c $\left[\frac{dV}{dt}\right]_{t=\sqrt{50}} = -8{,}577...$

De maximale uitstroomsnelheid is dus 8,577... l/minuut.
De helft van de maximale snelheid is 4,288... l/minuut.
Voer in $y_1 = -2x\,e^{-0{,}01x^2}$ en $y_2 = -4{,}288...$
Voor $t > \sqrt{50}$ geeft intersect $x \approx 13{,}586...$
Na $13{,}586... - \sqrt{50} = 6{,}51...$ minuten ≈ 391 seconden is de uitstroomsnelheid afgenomen tot de helft van de maximale snelheid.

6 $f(x) = (x^2 - 3)(x^2 - 5) = x^4 - 8x^2 + 15$
$f'(x) = 4x^3 - 16x$
$f''(x) = 12x^2 - 16$
$f'(1) = 4 - 16 = -12 < 0$ dus dalend
$f''(1) = 12 - 16 = -4 < 0$
Dus toenemende daling.

7 $f(x) = \frac{2x+1}{x^2+1}$ geeft $f'(x) = \frac{(x^2+1)\cdot 2 - (2x+1)\cdot 2x}{(x^2+1)^2} = \frac{2x^2 + 2 - 4x^2 - 2x}{(x^2+1)^2} = \frac{-2x^2 - 2x + 2}{(x^2+1)^2}$

$$f''(x) = \frac{(x^2+1)^2\cdot(-4x-2) - (-2x^2-2x+2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$$

$$= \frac{(x^2+1)(-4x-2) - 4x(-2x^2-2x+2)}{(x^2+1)^3}$$

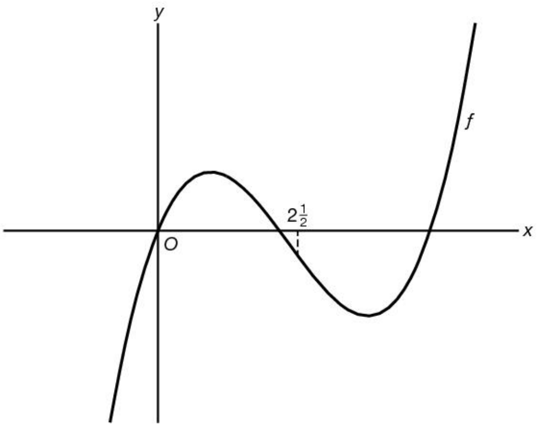
$$= \frac{-4x^3 - 2x^2 - 4x - 2 + 8x^3 + 8x^2 - 8x}{(x^2+1)^3} = \frac{4x^3 + 6x^2 - 12x - 2}{(x^2+1)^3}$$

$f'(-3) = \frac{-18+6+2}{10^2} < 0$ en $f''(-3) = \frac{-108+54+36-2}{10^3} < 0$, dus in A toenemend dalend.

$f'(0) = \frac{0+0+2}{1^2} > 0$ en $f''(0) = \frac{0+0+0-2}{1^3} < 0$, dus in B afnemend stijgend.

$f'(1) = \frac{-2-2+2}{2^2} < 0$ en $f''(1) = \frac{4+6-12-2}{2^3} < 0$, dus in C toenemend dalend.

8 $f(x) = 2x^3 - 15x^2 + 24x$
$f'(x) = 6x^2 - 30x + 24$
$f''(x) = 12x - 30$
$f''\left(2\frac{1}{2}\right) = 0$ en $f'\left(2\frac{1}{2}\right) = -13\frac{1}{2} < 0$



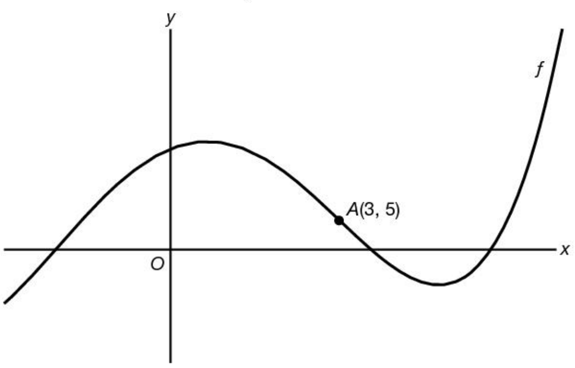
Dus de grafiek van f gaat bij $x = 2\frac{1}{2}$ over van toenemend dalend naar afnemend dalend.

Bladzijde 103

9 **a** $f(x) = ax^4 - \frac{1}{6}x^3 - 3x^2 + 4x + b$
$f'(x) = 4ax^3 - \frac{1}{2}x^2 - 6x + 4$
$f''(x) = 12ax^2 - x - 6$
$f''(3) = 0$ geeft $12a \cdot 3^2 - 3 - 6 = 0$
$108a = 9$
$a = \frac{1}{12}$

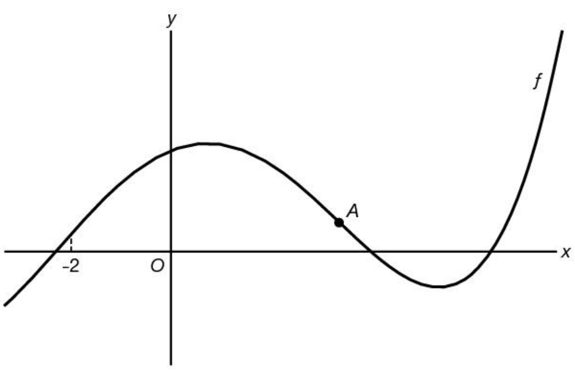
Dus $f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 - 3x^2 + 4x + b$.

$f(3) = 5$ geeft $\frac{1}{12} \cdot 3^4 - \frac{1}{6} \cdot 3^3 - 3 \cdot 3^2 + 4 \cdot 3 + b = 5$
$6\frac{3}{4} - 4\frac{1}{2} - 27 + 12 + b = 5$
$b = 17\frac{3}{4}$



De grafiek van f gaat in A over van toenemend dalend naar afnemend dalend.
Dus $a = \frac{1}{12}$ en $b = 17\frac{3}{4}$.

b $f''(x) = 0$ geeft $x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x = -2 \vee x = 3$



$f(-2) = \frac{5}{12}$, dus in het punt $\left(-2, \frac{5}{12}\right)$ gaat de grafiek van f over van toenemend stijgend naar afnemend stijgend.

10 $f(-2) = -\frac{4}{e}$ geeft $-2a\,e^{-2b} = -\frac{4}{e}$

$f(x) = ax\,e^{bx}$ geeft $f'(x) = a \cdot e^{bx} + ax \cdot e^{bx} \cdot b = (abx + a)e^{bx}$ en

$f''(x) = ab \cdot e^{bx} + (abx + a) \cdot e^{bx} \cdot b = (ab^2x + 2ab)e^{bx}$

$f''(-4) = 0$ geeft $(-4ab^2 + 2ab)e^{-4b} = 0$

$ab(-4b + 2)e^{-4b} = 0$

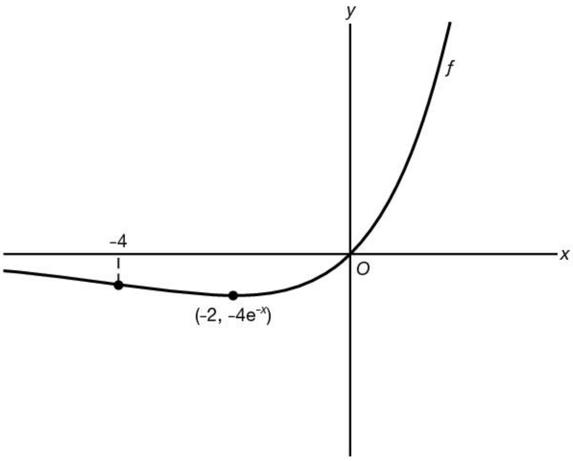
$a = 0 \quad \vee \quad b = 0 \quad \vee \quad -4b + 2 = 0$

vold. niet $\quad$ vold. niet $\quad -4b = -2$

$b = \frac{1}{2}$

$$\left.\begin{array}{l} -2ae^{-2b} = -\frac{4}{e} \\ b = \frac{1}{2} \end{array}\right\} \begin{array}{l} -2ae^{-1} = -4e^{-1} \\ a = 2 \end{array}$$

$f'(-2) = \left(2 \cdot \frac{1}{2} \cdot -2 + 2\right)e^{\frac{1}{2} \cdot -2} = 0$



De grafiek van f gaat voor $x = -4$ over van toenemend dalend naar afnemend dalend en heeft een top voor $x = -2$.
Dus $a = 2$ en $b = \frac{1}{2}$.

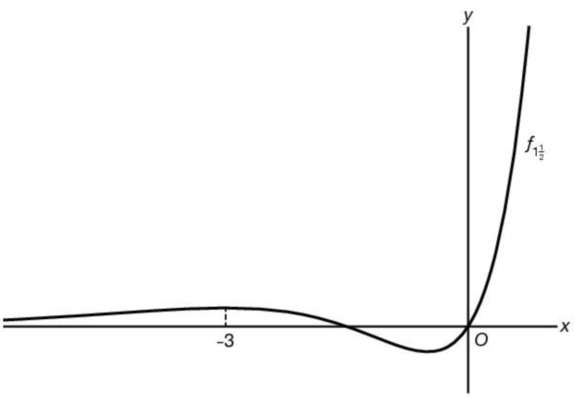
11 **a** $f_a(x) = (x^2 + ax)e^x$ geeft $f_a'(x) = (2x + a) \cdot e^x + (x^2 + ax) \cdot e^x = (x^2 + (a + 2)x + a)e^x$

$f_a'(-3) = 0$ geeft $(9 - 3(a + 2) + a)e^{-3} = 0$

$9 - 3a - 6 + a = 0$

$-2a = -3$

$a = 1\frac{1}{2}$



Dus voor $a = 1\frac{1}{2}$ heeft f_a een maximum voor $x = -3$.

$f_{1\frac{1}{2}}'(x) = \left(x^2 + 3\frac{1}{2}x + 1\frac{1}{2}\right)e^x$

$f_{1\frac{1}{2}}'(x) = 0$ geeft $\left(x^2 + 3\frac{1}{2}x + 1\frac{1}{2}\right)e^x = 0$

$x^2 + 3\frac{1}{2}x + 1\frac{1}{2} = 0$

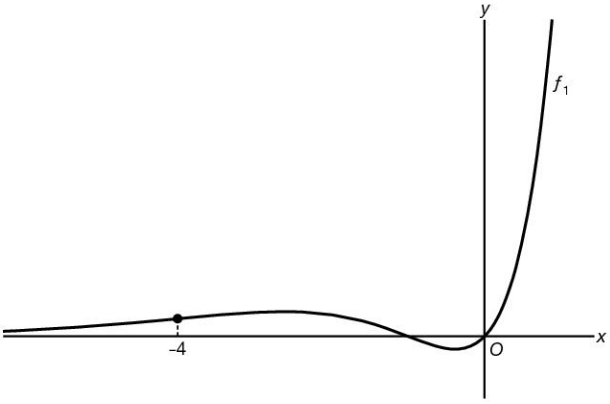
$(x + 3)\left(x + \frac{1}{2}\right) = 0$

$x = -3 \vee x = -\frac{1}{2}$

min. is $f_{1\frac{1}{2}}\left(-\frac{1}{2}\right) = \left(\left(-\frac{1}{2}\right)^2 + 1\frac{1}{2} \cdot -\frac{1}{2}\right)e^{-\frac{1}{2}} = -\frac{1}{2}e^{-\frac{1}{2}} = -\frac{1}{2\sqrt{e}}$

15

b $f_a'(x) = (x^2 + (a+2)x + a)\mathrm{e}^x$ geeft
$f_a''(x) = (2x + a + 2)\mathrm{e}^x + (x^2 + (a+2)x + a)\mathrm{e}^x = (x^2 + (a+4)x + 2a + 2)\mathrm{e}^x$
$f_a''(-4) = 0$ geeft $((-4)^2 - 4(a+4) + 2a + 2)\mathrm{e}^{-4} = 0$
$16 - 4a - 16 + 2a + 2 = 0$
$-2a + 2 = 0$
$-2a = -2$
$a = 1$



Dus voor $a = 1$ gaat de grafiek van f_a bij $x = -4$ over van toenemend stijgend naar afnemend stijgend.

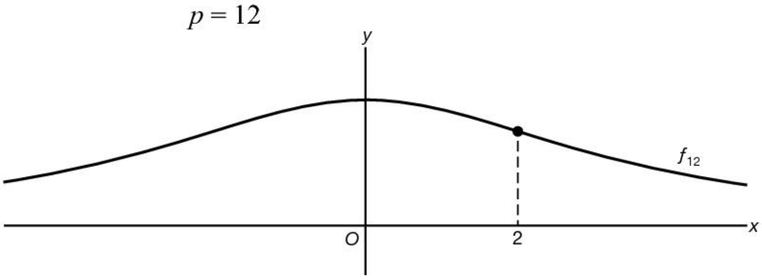
12 $f_p(x) = \dfrac{10}{x^2 + p}$

$f_p'(x) = \dfrac{(x^2+p) \cdot 0 - 10 \cdot 2x}{(x^2+p)^2} = \dfrac{-20x}{(x^2+p)^2}$

$f_p''(x) = \dfrac{(x^2+p)^2 \cdot -20 - -20x \cdot 2(x^2+p) \cdot 2x}{(x^2+p)^4} = \dfrac{(x^2+p) \cdot -20 - -20x \cdot 2 \cdot 2x}{(x^2+p)^3}$

$= \dfrac{-20x^2 - 20p + 80x^2}{(x^2+p)^3} = \dfrac{60x^2 - 20p}{(x^2+p)^3}$

$f_p''(2) = 0$ geeft $\dfrac{60 \cdot 4 - 20p}{(4+p)^3} = 0$
$240 - 20p = 0$
$-20p = -240$
$p = 12$



Dus voor $p = 12$ gaat de grafiek van f_p bij $x = 2$ over van toenemend dalend naar afnemend dalend.

Bladzijde 104

13 **a** $s(t) = at^3 + bt^2$
$v(t) = 3at^2 + 2bt$
$a(t) = 6at + 2b$
Bij $t = 10$ gaat de snelheid over van toenemend stijgend naar afnemend stijgend, dus $a(10) = 0$.
Dit geeft $60a + 2b = 0$ ofwel $30a + b = 0$
$s(15) = 675$ geeft $3375a + 225b = 675$ ofwel $15a + b = 3$

$$\begin{cases} 30a + b = 0 \\ 15a + b = 3 \end{cases} -$$
$15a = -3$

$$\left.\begin{array}{l} a = -\frac{1}{5} \\ 30a + b = 0 \end{array}\right\} 30 \cdot -\tfrac{1}{5} + b = 0$$
$-6 + b = 0$
$b = 6$

Dus $a = -\frac{1}{5}$ en $b = 6$.

b $s(t) = -\frac{1}{5}t^3 + 6t^2$

$v(t) = -\frac{3}{5}t^2 + 12t$

$v(15) = -\frac{3}{5} \cdot 225 + 12 \cdot 15 = 45$

In de 15 seconden tussen $t = 15$ en $t = 30$ wordt $15 \cdot 45 = 675$ meter afgelegd.

Dus op $t = 30$ is in totaal $675 + 675 = 1350$ meter afgelegd.

c Om $2000 - 675 = 1325$ meter af te leggen met een snelheid van 45 m/s is $\frac{1325}{45} = 29\frac{4}{9}$ seconde nodig.

Dus er zijn $15 + 29\frac{4}{9} = 44\frac{4}{9} \approx 44$ seconden nodig.

14 **a** $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 4$ geeft $f'(x) = x^2 - 4x + 3$

$f'(x) = 0$ geeft $x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1 \vee x = 3$

$f(1) = 5\frac{1}{3}$, dus $A\left(1, 5\frac{1}{3}\right)$.

$f(3) = 4$, dus $B(3, 4)$.

b $OA = \sqrt{1^2 + \left(5\frac{1}{3}\right)^2} \approx 5{,}43$

$OB = \sqrt{3^2 + 4^2} = 5$

Bladzijde 105

15 $f_p(x) = p + \sin(x)$ geeft $f_p'(x) = \cos(x)$

$f_p'(x) = 0$ geeft $\cos(x) = 0$

$x = \frac{1}{2}\pi + k \cdot \pi$

Dus $A\left(\frac{1}{2}\pi, p + 1\right)$ en $B\left(1\frac{1}{2}\pi, p - 1\right)$.

$OA = OB$ geeft $OA^2 = OB^2$

$\left(\frac{1}{2}\pi\right)^2 + (p + 1)^2 = \left(1\frac{1}{2}\pi\right)^2 + (p - 1)^2$

$\frac{1}{4}\pi^2 + p^2 + 2p + 1 = 2\frac{1}{4}\pi^2 + p^2 - 2p + 1$

$4p = 2\pi^2$

$p = \frac{1}{2}\pi^2$

16 Door op de grafiek van f de translatie $(0, -p)$ toe te passen ontstaat de grafiek van de functie

$g_p(x) = \frac{3x^2 + 10x - 3}{x^2 + 4} - p.$

$g_p'(x) = \frac{(x^2 + 4)(6x + 10) - (3x^2 + 10x - 3) \cdot 2x}{(x^2 + 4)^2} = \frac{6x^3 + 10x^2 + 24x + 40 - 6x^3 - 20x^2 + 6x}{(x^2 + 4)^2}$

$= \frac{-10x^2 + 30x + 40}{(x^2 + 4)^2}$

$g_p'(x) = 0$ geeft $-10x^2 + 30x + 40 = 0$

$x^2 - 3x - 4 = 0$

$(x + 1)(x - 4) = 0$

$x = -1 \vee x = 4$

$g_p(-1) = -2 - p$ en $g_p(4) = 4\frac{1}{4} - p$, dus $C(-1, -2 - p)$ en $D\left(4, 4\frac{1}{4} - p\right)$.

$OC = OD$ geeft $OC^2 = OD^2$

$(-1)^2 + (-2 - p)^2 = 4^2 + \left(4\frac{1}{4} - p\right)^2$

$1 + 4 + 4p + p^2 = 16 + 18\frac{1}{16} - 8\frac{1}{2}p + p^2$

$12\frac{1}{2}p = 29\frac{1}{16}$

$p = 2\frac{13}{40}$

17 $f_p(x) = -0{,}03x^4 + 0{,}08x^3 + 0{,}48x^2 + p$ geeft $f_p'(x) = -0{,}12x^3 + 0{,}24x^2 + 0{,}96x$

$f_p'(x) = 0$ geeft $-0{,}12x^3 + 0{,}24x^2 + 0{,}96x = 0$

$x^3 - 2x^2 - 8x = 0$

$x(x^2 - 2x - 8) = 0$

$x(x + 2)(x - 4) = 0$

$x = 0 \vee x = -2 \vee x = 4$

$f_p(-2) = 0{,}8 + p$ en $f_p(4) = 5{,}12 + p$, dus $A(-2;\ 0{,}8 + p)$ en $B(4;\ 5{,}12 + p)$.
$OB = 2OA$ geeft $OB^2 = 4OA^2$

$$4^2 + (5{,}12 + p)^2 = 4((-2)^2 + (0{,}8 + p)^2)$$
$$16 + 26{,}2144 + 10{,}24p + p^2 = 4(4 + 0{,}64 + 1{,}6p + p^2)$$
$$16 + 26{,}2144 + 10{,}24p + p^2 = 16 + 2{,}56 + 6{,}4p + 4p^2$$
$$-3p^2 + 3{,}84p + 23{,}6544 = 0$$
$$p^2 - 1{,}28p - 7{,}8848 = 0$$
$$D = (-1{,}28)^2 - 4 \cdot 1 \cdot -7{,}8848 = 33{,}1776$$
$$p = \frac{1{,}28 - 5{,}76}{2} = -2{,}24 \vee p = \frac{1{,}28 + 5{,}76}{2} = 3{,}52$$

vold. niet — vold.

Dus $p = 3{,}52$.

15.2 Raakproblemen

Bladzijde 107

18 **a** $rc_k = 1$, dus de richtingshoek van k is 45°.

b $f(x) = x^2$ geeft $f'(x) = 2x$

$f(x) = g(x)$ geeft $x^2 = x + 2$
$$x^2 - x - 2 = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1 \vee x = 2$$

$f'(-1) = -2$, dus $\tan(\alpha) = -2$
$$\alpha = -63{,}4...°$$
$f'(2) = 4$, dus $\tan(\beta) = 4$
$$\beta = 75{,}9...°$$
Dus de richtingshoek van l is 63° en de richtingshoek van m is 76°.

Bladzijde 108

19 $f(x) = g(x)$ geeft $\sqrt{x} = -2x + 6$

kwadrateren geeft
$$x = 4x^2 - 24x + 36$$
$$4x^2 - 25x + 36 = 0$$
$$D = (-25)^2 - 4 \cdot 4 \cdot 36 = 49$$
$$x = \frac{25 - 7}{8} = 2\tfrac{1}{4} \vee x = \frac{25 + 7}{8} = 4$$

vold. — vold. niet

$f(x) = \sqrt{x}$ geeft $f'(x) = \dfrac{1}{2\sqrt{x}}$

k is de raaklijn van de grafiek van f voor $x = 2\frac{1}{4}$.

$\tan(\alpha) = rc_k = f'\left(2\tfrac{1}{4}\right) = \dfrac{1}{2\sqrt{2\frac{1}{4}}} = \dfrac{1}{2 \cdot 1\frac{1}{2}} = \tfrac{1}{3}$ geeft $\alpha = 18{,}43...°$

l is de grafiek van g.

$\tan(\beta) = rc_l = -2$ geeft $\beta = -63{,}43...°$

$\alpha - \beta = 18{,}43...° - -63{,}43...° = 81{,}86...°$

De gevraagde hoek is ongeveer 82°.

20 k is de raaklijn van de grafiek van f in A.

$f(x) = -\frac{1}{2}x^2 + 3$ geeft $f'(x) = -x$, dus $rc_k = f'(1) = -1$.

$\tan(\alpha) = rc_k = -1$ geeft $\alpha = -45°$

l is de raaklijn van de grafiek van g in A.

$g(x) = 2\frac{1}{2}\sqrt{x}$ geeft $g'(x) = 2\frac{1}{2} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{5}{4\sqrt{x}}$, dus $g'(1) = \dfrac{5}{4\sqrt{1}} = 1\frac{1}{4}$.

$\tan(\beta) = rc_l = 1\frac{1}{4}$ geeft $\beta = 51{,}34...°$

$\beta - \alpha = 51{,}34...° - -45° = 96{,}34...°$

De gevraagde hoek is $180° - 96{,}34...° \approx 83{,}7°$.

21 $f(x) = g(x)$ geeft $x^2 - 4x = x^2 - 10x + 24$

$6x = 24$

$x = 4$

k is de raaklijn van de grafiek van f in het punt $A(4, 0)$.
$f(x) = x^2 - 4x$ geeft $f'(x) = 2x - 4$, dus $f'(4) = 2 \cdot 4 - 4 = 4$.
$\tan(\alpha) = \text{rc}_k = 4$ geeft $\alpha = 75{,}96...°$
l is de raaklijn van de grafiek van g in het punt $A(4, 0)$.
$g(x) = x^2 - 10x + 24$ geeft $g'(x) = 2x - 10$, dus $g'(4) = 2 \cdot 4 - 10 = -2$.
$\tan(\beta) = \text{rc}_l = -2$ geeft $\beta = -63{,}43...°$
$\alpha - \beta = 75{,}96...° - -63{,}43° = 139{,}39...°$
De gevraagde hoek is $180° - 139{,}39...° \approx 40{,}6°$.

22 $x^2 = -x^2 + 4x + 6$
$2x^2 - 4x - 6 = 0$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1 \vee x = 3$
$y = x^2$ geeft $\frac{dy}{dx} = 2x$ en $\left[\frac{dy}{dx}\right]_{x=-1} = 2 \cdot -1 = -2$.
k is de raaklijn aan p_1 voor $x = -1$.
$\tan(\alpha) = \text{rc}_k = -2$ geeft $\alpha = -63{,}43...°$
$y = -x^2 + 4x + 6$ geeft $\frac{dy}{dx} = -2x + 4$ en $\left[\frac{dy}{dx}\right]_{x=-1} = -2 \cdot -1 + 4 = 6$.
l is de raaklijn aan p_2 voor $x = -1$.
$\tan(\beta) = \text{rc}_l = 6$ geeft $\beta = 80{,}53...°$
$\beta - \alpha = 80{,}53...° - -63{,}43...° = 143{,}97...°$
De hoek waaronder de parabolen elkaar snijden voor $x = -1$ is $180° - 143{,}97...° \approx 36°$.
$y = x^2$ geeft $\frac{dy}{dx} = 2x$ en $\left[\frac{dy}{dx}\right]_{x=3} = 2 \cdot 3 = 6$.
$y = -x^2 + 4x + 6$ geeft $\frac{dy}{dx} = -2x + 4$ en $\left[\frac{dy}{dx}\right]_{x=3} = -2 \cdot 3 + 4 = -2$.
De richtingscoëfficiënten van de raaklijnen voor $x = 3$ zijn gelijk aan die voor $x = -1$, dus de gevraagde hoeken zijn beide 36°.

Bladzijde 109

23 **a** $f(1) = \sqrt{4 \cdot 1 + 5} = \sqrt{9} = 3$ en $g(1) = -1^2 + 6\frac{1}{2} \cdot 1 - 2\frac{1}{2} = -1 + 6\frac{1}{2} - 2\frac{1}{2} = 3$
$f(5) = \sqrt{4 \cdot 5 + 5} = \sqrt{25} = 5$ en $g(5) = -5^2 + 6\frac{1}{2} \cdot 5 - 2\frac{1}{2} = -25 + 32\frac{1}{2} - 2\frac{1}{2} = 5$
Dus de grafieken van f en g snijden elkaar in de punten $A(1, 3)$ en $B(5, 5)$.

b $f(x) = \sqrt{4x + 5}$ geeft $f'(x) = 4 \cdot \frac{1}{2\sqrt{4x + 5}} = \frac{2}{\sqrt{4x + 5}}$
k is de raaklijn van de grafiek van f in A.
$\tan(\alpha) = \text{rc}_k = f'(1) = \frac{2}{\sqrt{4 \cdot 1 + 5}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$ geeft $\alpha = 33{,}69...°$
$g(x) = -x^2 + 6\frac{1}{2}x - 2\frac{1}{2}$ geeft $g'(x) = -2x + 6\frac{1}{2}$
l is de raaklijn van de grafiek van g in A.
$\tan(\beta) = \text{rc}_l = g'(1) = -2 \cdot 1 + 6\frac{1}{2} = 4\frac{1}{2}$ geeft $\beta = 77{,}47...°$
$\beta - \alpha = 77{,}47...° - 33{,}69...° = 43{,}78...°$
Dus de hoek waaronder de grafieken van f en g elkaar in A snijden is ongeveer 44°.
m is de raaklijn van de grafiek van f in B.
$\tan(\alpha) = \text{rc}_m = f'(5) = \frac{2}{\sqrt{4 \cdot 5 + 5}} = \frac{2}{\sqrt{25}} = \frac{2}{5}$ geeft $\alpha = 21{,}80...°$
n is de raaklijn van de grafiek van g in B.
$\tan(\beta) = \text{rc}_n = g'(5) = -2 \cdot 5 + 6\frac{1}{2} = -3\frac{1}{2}$ geeft $\beta = -74{,}05...°$
$\alpha - \beta = 21{,}80...° - -74{,}05...° = 95{,}85...°$.
Dus de hoek waaronder de grafieken van f en g elkaar in B snijden is $180° - 96{,}85...° \approx 84°$.

24 $f(x) = g(x)$ geeft $\left(\frac{1}{2}x - 1\right)^3 = \dfrac{1}{\left(\frac{1}{2}x - 1\right)^2}$

$$\left(\tfrac{1}{2}x - 1\right)^5 = 1$$
$$\tfrac{1}{2}x - 1 = 1$$
$$\tfrac{1}{2}x = 2$$
$$x = 4$$

$f(x) = \left(\frac{1}{2}x - 1\right)^3$ geeft $f'(x) = \frac{1}{2} \cdot 3 \cdot \left(\frac{1}{2}x - 1\right)^2 = 1\frac{1}{2}\left(\frac{1}{2}x - 1\right)^2$
k is de raaklijn van de grafiek van f voor $x = 4$.

$\tan(\alpha) = \text{rc}_k = f'(4) = 1\frac{1}{2}\left(\frac{1}{2} \cdot 4 - 1\right)^2 = 1\frac{1}{2}(2 - 1)^2 = 1\frac{1}{2} \cdot 1 = 1\frac{1}{2}$ geeft $\alpha = 56{,}30...°$

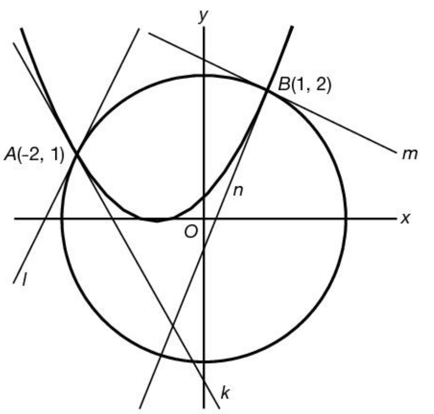
$g(x) = \dfrac{1}{\left(\frac{1}{2}x - 1\right)^2} = \left(\frac{1}{2}x - 1\right)^{-2}$ geeft $g'(x) = \frac{1}{2} \cdot -2 \cdot \left(\frac{1}{2}x - 1\right)^{-3} = -\dfrac{1}{\left(\frac{1}{2}x - 1\right)^3}$

l is de raaklijn van de grafiek van g voor $x = 4$.

$\tan(\beta) = \text{rc}_l = g'(4) = -\dfrac{1}{\left(\frac{1}{2} \cdot 4 - 1\right)^3} = -\dfrac{1}{(2 - 1)^3} = -\dfrac{1}{1} = -1$ geeft $\beta = -45°$

$\alpha - \beta = 56{,}30...° - -45° = 101{,}30...°$
De gevraagde hoek is $180° - 101{,}30...° \approx 78{,}7°$.

25 $y = \frac{2}{3}x^2 + x + \frac{1}{3}$ substitueren in $x^2 + y^2 = 5$ geeft $x^2 + \left(\frac{2}{3}x^2 + x + \frac{1}{3}\right)^2 = 5$.
Voer in $y_1 = x^2 + \left(\frac{2}{3}x^2 + x + \frac{1}{3}\right)^2$ en $y_2 = 5$.
Intersect geeft $x = -2$ en $x = 1$.
$x = -2$ geeft $y = \frac{2}{3} \cdot (-2)^2 - 2 + \frac{1}{3} = 1$ en $x = 1$ geeft $y = \frac{2}{3} \cdot 1 + 1 + \frac{1}{3} = 2$.



Dus $A(-2, 1)$ en $B(1, 2)$.
$y = \frac{2}{3}x^2 + x + \frac{1}{3}$ geeft $\dfrac{dy}{dx} = 1\frac{1}{3}x + 1$

$\text{rc}_k = \left[\dfrac{dy}{dx}\right]_{x = -2} = -1\frac{2}{3}$

$\tan(\alpha) = -1\frac{2}{3}$ geeft $\alpha = -59{,}0...°$

$\text{rc}_{OA} = \dfrac{-1}{2} = -\frac{1}{2}$, dus $\text{rc}_l = 2$.

$\tan(\beta) = 2$ geeft $\beta = 63{,}4...°$
$\beta - \alpha = 122{,}4...°$, dus de hoek waaronder p en c elkaar snijden in A is $180° - 122{,}4...° \approx 58°$.

$\text{rc}_n = \left[\dfrac{dy}{dx}\right]_{x = 1} = 2\frac{1}{3}$

$\tan(\gamma) = 2\frac{1}{3}$ geeft $\gamma = 66{,}8...°$

$\text{rc}_{OM} = \dfrac{2}{1} = 2$, dus $\text{rc}_m = -\frac{1}{2}$.

$\tan(\delta) = -\frac{1}{2}$ geeft $\delta = -26{,}5...°$
$\gamma - \delta = 93{,}3...°$, dus de hoek waaronder p en c elkaar snijden in B is $180° - 93{,}3...° \approx 87°$.

26 **a** $f(1) = 0{,}5 \cdot 1^2 + 1 + 1{,}5 = 3$ en $g(1) = -1^2 + 4 \cdot 1 = 3$
Dus $A(1, 3)$ ligt op de grafiek van f en op de grafiek van g.
b $f(x) = 0{,}5x^2 + x + 1{,}5$ geeft $f'(x) = x + 1$
Stel k: $y = ax + b$ met $a = f'(1) = 2$.
$$\left.\begin{array}{l} k: y = 2x + b \\ \text{door } A(1, 3) \end{array}\right\} \begin{array}{l} 2 + b = 3 \\ b = 1 \end{array}$$
Dus k: $y = 2x + 1$.
$g(x) = -x^2 + 4x$ geeft $g'(x) = -2x + 4$
Stel l: $y = ax + b$ met $a = g'(1) = 2$.
$$\left.\begin{array}{l} l: y = 2x + b \\ \text{door } A(1, 3) \end{array}\right\} \begin{array}{l} 2 + b = 3 \\ b = 1 \end{array}$$
Dus l: $y = 2x + 1$.
c A is het punt waar de raaklijnen aan de grafieken van f en g samenvallen.

Bladzijde 110

27 De grafieken hebben voor $x = -3$ geen punt gemeenschappelijk, dus ze raken elkaar niet. Wel geldt dat voor $x = 3$ de raaklijn aan de grafiek van f en de raaklijn aan de grafiek van g evenwijdig zijn.

28 **a** $f(x) = x^3 + 4x^2 + 2x + 1$ geeft $f'(x) = 3x^2 + 8x + 2$
$g(x) = x^2 + 11x + 28$ geeft $g'(x) = 2x + 11$
$f(x) = g(x) \wedge f'(x) = g'(x)$
$x^3 + 4x^2 + 2x + 1 = x^2 + 11x + 28 \wedge 3x^2 + 8x + 2 = 2x + 11$
$x^3 + 3x^2 - 9x - 27 = 0 \wedge 3x^2 + 6x - 9 = 0$
$3x^2 + 6x - 9 = 0$ geeft $x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
$x = -3 \vee x = 1$
Substitutie van $x = -3$ in $x^3 + 3x^2 - 9x - 27 = 0$ geeft
$-27 + 27 + 27 - 27 = 0$ klopt, dus de grafieken raken elkaar.
Substitutie van $x = 1$ in $x^3 + 3x^2 - 9x - 27 = 0$ geeft
$1 + 3 - 9 - 27 = 0$ klopt niet.
b $g'(-3) = -6 + 11 = 5$
$$\left.\begin{array}{l} k: y = 5x + b \\ g(-3) = 9 - 33 + 28 = 4 \end{array}\right\} \begin{array}{l} 5 \cdot -3 + b = 4 \\ b = 19 \end{array}$$
Dus de gemeenschappelijke raaklijn is k: $y = 5x + 19$.

29 **a** $f(x) = \sqrt{2x}$ geeft $f'(x) = \dfrac{1}{2\sqrt{2x}} \cdot 2 = \dfrac{1}{\sqrt{2x}}$
en $g_1(x) = x^2 + 1$ geeft $g_1'(x) = 2x$
Voor raken moet gelden $f(x) = g_1(x) \wedge f'(x) = g_1'(x)$.
$\sqrt{2x} = x^2 + 1 \wedge \dfrac{1}{\sqrt{2x}} = 2x$
$\sqrt{2x} = x^2 + 1 \wedge 2x\sqrt{2x} = 1$
$2x\sqrt{2x} = 1$ kwadrateren geeft $(2x)^3 = 1$
$2x = 1$
$x = \frac{1}{2}$
Substitutie van $x = \frac{1}{2}$ in $\sqrt{2x} = x^2 + 1$ geeft
$\sqrt{1} = \frac{1}{4} + 1$ klopt niet
Dus de grafieken van f en g_1 raken elkaar niet.
b $f(x) = g_p(x) \wedge f'(x) = g_p'(x)$
$\sqrt{2x} = x^2 + p \wedge \dfrac{1}{\sqrt{2x}} = 2x$
$x = \frac{1}{2}$
Substitutie van $x = \frac{1}{2}$ in $\sqrt{2x} = x^2 + p$ geeft $\sqrt{1} = \frac{1}{4} + p$
$p = \frac{3}{4}$
Dus de grafieken raken elkaar voor $p = \frac{3}{4}$.

Bladzijde 111

30 **a** $f(x) = x - \ln(x)$ geeft $f'(x) = 1 - \frac{1}{x}$

$g_p(x) = px$ geeft $g_p'(x) = p$

Voor raken geldt $f(x) = g_p(x) \wedge f'(x) = g_p'(x)$

$$x - \ln(x) = px \wedge 1 - \frac{1}{x} = p$$

b Substitutie van $p = 1 - \frac{1}{x}$ in $x - \ln(x) = px$ geeft $x - \ln(x) = \left(1 - \frac{1}{x}\right) \cdot x$

$$x - \ln(x) = x - 1$$

c $x - \ln(x) = x - 1$

$-\ln(x) = -1$

$\ln(x) = 1$

$x = \mathrm{e}$

$x = \mathrm{e}$ geeft $p = 1 - \frac{1}{\mathrm{e}}$ en het raakpunt $(\mathrm{e}, \mathrm{e} - 1)$.

31 **a** $f(x) = -x^2 + 8x - 12$ geeft $f'(x) = -2x + 8$ en $g_p(x) = x^2 + px$ geeft $g_p'(x) = 2x + p$

$f(x) = g_p(x) \wedge f'(x) = g_p'(x)$

$-x^2 + 8x - 12 = x^2 + px \wedge -2x + 8 = 2x + p$

$-x^2 + 8x - 12 = x^2 + px \wedge -4x + 8 = p$

$-x^2 + 8x - 12 = x^2 + (-4x + 8)x$

$-x^2 + 8x - 12 = x^2 - 4x^2 + 8x$

$2x^2 = 12$

$x^2 = 6$

$x = \sqrt{6} \vee x = -\sqrt{6}$

$\left.\begin{array}{l} x = \sqrt{6} \\ p = -4x + 8 \end{array}\right\} p = -4\sqrt{6} + 8$

$\left.\begin{array}{l} x = -\sqrt{6} \\ p = -4x + 8 \end{array}\right\} p = 4\sqrt{6} + 8$

b $f(x) = x - \mathrm{e}^x$ geeft $f'(x) = 1 - \mathrm{e}^x$ en $g_p(x) = x^2 + px$ geeft $g_p'(x) = 2x + p$

$f(x) = g_p(x) \wedge f'(x) = g_p'(x)$

$x - \mathrm{e}^x = x^2 + px \wedge 1 - \mathrm{e}^x = 2x + p$

$px = x - \mathrm{e}^x - x^2 \wedge p = 1 - \mathrm{e}^x - 2x$

$p = 1 - \frac{\mathrm{e}^x}{x} - x \wedge p = 1 - \mathrm{e}^x - 2x$

Voer in $y_1 = 1 - \frac{\mathrm{e}^x}{x} - x$ en $y_2 = 1 - \mathrm{e}^x - 2x$.

Intersect geeft $x \approx -0{,}883$ en $y \approx 2{,}351$ en $x \approx 0{,}739$ en $y \approx -2{,}572$.

Dus $p \approx 2{,}351 \vee p \approx -2{,}572$.

Alternatieve uitwerking

$f(x) = g_p(x) \wedge f'(x) = g_p'(x)$

$x - \mathrm{e}^x = x^2 + px \wedge 1 - \mathrm{e}^x = 2x + p$

$x - \mathrm{e}^x = x^2 + px \wedge 1 - \mathrm{e}^x - 2x = p$

$p = 1 - \mathrm{e}^x - 2x$ substitueren in $x - \mathrm{e}^x = x^2 + px$ geeft

$x - \mathrm{e}^x = x^2 + (1 - \mathrm{e}^x - 2x) \cdot x$

$x - \mathrm{e}^x = x^2 + x - x\mathrm{e}^x - 2x^2$

$x^2 = \mathrm{e}^x - x\mathrm{e}^x$

Voer in $y_1 = x^2$ en $y_2 = \mathrm{e}^x - x\mathrm{e}^x$.

Intersect geeft $x \approx -0{,}883$ en $x \approx 0{,}739$.

$\left.\begin{array}{l} x \approx -0{,}883 \\ p = 1 - \mathrm{e}^x - 2x \end{array}\right\} p \approx 2{,}351$

$\left.\begin{array}{l} x \approx -0{,}739 \\ p = 1 - \mathrm{e}^x - 2x \end{array}\right\} p \approx -2{,}572$

32 **a** $f_3(x) = 2\ln(x) + 3x$ geeft $f_3'(x) = \frac{2}{x} + 3$ en $g_q(x) = x^2 + q$ geeft $g_q'(x) = 2x$

$f_3(x) = g_q(x) \wedge f_3'(x) = g_q'(x)$

$2\ln(x) + 3x = x^2 + q \wedge \frac{2}{x} + 3 = 2x$

$2\ln(x) + 3x - x^2 = q \wedge 2 + 3x = 2x^2$

$2x^2 - 3x - 2 = 0$

$x^2 - 1\frac{1}{2}x - 1 = 0$

$(x-2)\left(x + \frac{1}{2}\right) = 0$

$x = 2 \vee x = -\frac{1}{2}$

vold. vold. niet

$\left.\begin{array}{l} x = 2 \\ q = 2\ln(x) + 3x - x^2 \end{array}\right\} q = 2 + 2\ln(2)$

b $f_p(x) = g_2(x) \wedge f_p'(x) = g_2'(x)$

$2\ln(x) + px = x^2 + 2 \wedge \frac{2}{x} + p = 2x$

$px = x^2 + 2 - 2\ln(x) \wedge p = 2x - \frac{2}{x}$

$p = x + \frac{2}{x} - \frac{2\ln(x)}{x} \wedge p = 2x - \frac{2}{x}$

Voer in $y_1 = x + \frac{2}{x} - \frac{2\ln(x)}{x}$ en $y_2 = 2x - \frac{2}{x}$.

Intersect geeft $x \approx 1{,}711$ en $y \approx 2{,}252$.

Dus $p \approx 2{,}252$.

Alternatieve uitwerking

$f_p(x) = g_2(x) \wedge f_p'(x) = g_2'(x)$

$2\ln(x) + px = x^2 + 2 \wedge \frac{2}{x} + p = 2x$

$2\ln(x) + px = x^2 + 2 \wedge p = 2x - \frac{2}{x}$

$2\ln(x) + \left(2x - \frac{2}{x}\right) \cdot x = x^2 + 2$

$2\ln(x) + 2x^2 - 2 = x^2 + 2$

$2\ln(x) = 4 - x^2$

Voer in $y_1 = 2\ln(x)$ en $y_2 = 4 - x^2$.

Intersect geeft $x = 1{,}7106....$

$\left.\begin{array}{l} x = 1{,}7106... \\ p = 2x - \frac{2}{x} \end{array}\right\} p \approx 2{,}252$

33 $f_p(x) = 2x + p\ln(x)$ geeft $f_p'(x) = 2 + \frac{p}{x}$ en $g_p(x) = px^2$ geeft $g_p'(x) = 2px$

$f_p(x) = g_p(x) \wedge f_p'(x) = g_p'(x)$

$2x + p\ln(x) = px^2 \wedge 2 + \frac{p}{x} = 2px$

$px^2 - p\ln(x) = 2x \wedge 2x + p = 2px^2$

$p(x^2 - \ln(x)) = 2x \wedge 2px^2 - p = 2x$

$p = \frac{2x}{x^2 - \ln(x)} \wedge p(2x^2 - 1) = 2x$

$p = \frac{2x}{x^2 - \ln(x)} \wedge p = \frac{2x}{2x^2 - 1}$

Voer in $y_1 = \frac{2x}{x^2 - \ln(x)}$ en $y_2 = \frac{2x}{2x^2 - 1}$.

Intersect geeft $x = 1$ en $y = 2$.

Dus $p = 2$.

34 $rc_l = 2$ geeft $rc_k = -\frac{1}{2}$

$$\left.\begin{array}{l} y = -\frac{1}{2}x + b \\ \text{door } A(4, 3) \end{array}\right\} \begin{array}{l} -\frac{1}{2} \cdot 4 + b = 3 \\ -2 + b = 3 \\ b = 5 \end{array}$$

Dus k: $y = -\frac{1}{2}x + 5$.

Bladzijde 112

35 **a** $f(x) = \sqrt{x}$ geeft $f'(x) = \dfrac{1}{2\sqrt{x}}$ en $g(x) = -\frac{1}{2}x^2 + 10$ geeft $g'(x) = -x$

$f(x) = g(x) \wedge f'(x) \cdot g'(x) = -1$

$\sqrt{x} = -\frac{1}{2}x^2 + 10 \wedge \dfrac{1}{2\sqrt{x}} \cdot -x = -1$

$\dfrac{1}{2\sqrt{x}} \cdot -x = -1$ geeft $\dfrac{x}{2\sqrt{x}} = 1$

$\frac{1}{2}\sqrt{x} = 1$

$\sqrt{x} = 2$

$x = 4$

Substitutie van $x = 4$ in $\sqrt{x} = -\frac{1}{2}x^2 + 10$ geeft $\sqrt{4} = -\frac{1}{2} \cdot 4^2 + 10$

$2 = -8 + 10$

$2 = 2$

Dit klopt, dus de grafieken snijden elkaar loodrecht.

b $f(x) = \sqrt{2} \cdot \sin(x)$ geeft $f'(x) = \sqrt{2} \cdot \cos(x)$

$g(x) = \sqrt{2} \cdot \cos(x)$ geeft $g'(x) = -\sqrt{2} \cdot \sin(x)$

Voor loodrecht snijden geldt $f(x) = g(x) \wedge f'(x) \cdot g'(x) = -1$.

$\sqrt{2} \cdot \sin(x) = \sqrt{2} \cdot \cos(x) \wedge \sqrt{2} \cdot \cos(x) \cdot -\sqrt{2} \cdot \sin(x) = -1$

$\sin(x) = \cos(x) \wedge -2\sin(x) \cdot \cos(x) = -1$

$\cos\left(x - \frac{1}{2}\pi\right) = \cos(x) \wedge \sin(2x) = 1$

$\left(x - \frac{1}{2}\pi = x + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -x + k \cdot 2\pi\right) \wedge 2x = \frac{1}{2}\pi + k \cdot 2\pi$

$\left(\text{geen opl.} \qquad 2x = \frac{1}{2}\pi + k \cdot 2\pi\right) \qquad \wedge 2x = \frac{1}{2}\pi + k \cdot 2\pi$

Dit klopt, dus de grafieken snijden elkaar loodrecht.

36 $f_p(x) = p\sqrt{x}$ geeft $f_p'(x) = \dfrac{p}{2\sqrt{x}}$ en $g(x) = \dfrac{8}{x} = 8x^{-1}$ geeft $g'(x) = -8x^{-2} = \dfrac{-8}{x^2}$

$f_p(x) = g(x) \wedge f_p'(x) \cdot g'(x) = -1$ geeft $p\sqrt{x} = \dfrac{8}{x} \wedge \dfrac{p}{2\sqrt{x}} \cdot \dfrac{-8}{x^2} = -1$

$p = \dfrac{8}{x\sqrt{x}} \wedge p = \dfrac{x^2 \cdot \sqrt{x}}{4}$

$\dfrac{8}{x\sqrt{x}} = \dfrac{x^2 \cdot \sqrt{x}}{4}$

$x^4 = 32$

$x = \sqrt[4]{32} = 2^{1\frac{1}{4}} = 2 \cdot \sqrt[4]{2} \vee x = -\sqrt[4]{32}$
vold. niet

$x = 2^{1\frac{1}{4}}$ geeft $p = \dfrac{8}{x\sqrt{x}} = \dfrac{8}{x^{1\frac{1}{2}}} = \dfrac{2^3}{\left(2^{1\frac{1}{4}}\right)^{1\frac{1}{2}}} = \dfrac{2^3}{2^{\frac{15}{8}}} = 2^{1\frac{1}{8}} = 2 \cdot \sqrt[8]{2}$

$g\left(2^{1\frac{1}{4}}\right) = \dfrac{8}{2^{1\frac{1}{4}}} = \dfrac{2^3}{2^{1\frac{1}{4}}} = 2^{1\frac{3}{4}} = 2 \cdot \sqrt[4]{8}$

Het snijpunt is $\left(2 \cdot \sqrt[4]{2}, 2 \cdot \sqrt[4]{8}\right)$.

37 **a** $f(x) = x^2 - 4x$ geeft $f'(x) = 2x - 4$

$f'(5) = 10 - 4 = 6$

$\text{rc}_k \cdot 6 = -1$ dus $\text{rc}_k = -\frac{1}{6}$

$$\left.\begin{array}{l} k\colon y = -\frac{1}{6}x + b \\ \text{door } A(5, 5) \end{array}\right\} \begin{array}{l} -\frac{5}{6} + b = 5 \\ b = 5\frac{5}{6} \end{array}$$

Dus $k\colon y = -\frac{1}{6}x + 5\frac{5}{6}$.

b $g(x) = \dfrac{2x - 1}{x + 2}$ geeft $g'(x) = \dfrac{(x + 2) \cdot 2 - (2x - 1) \cdot 1}{(x + 2)^2} = \dfrac{2x + 4 - 2x + 1}{(x + 2)^2} = \dfrac{5}{(x + 2)^2}$

$g(x) = -5x + p \wedge g'(x) \cdot \text{rc}_l = -1$

$\dfrac{2x - 1}{x + 2} = -5x + p \wedge \dfrac{5}{(x + 2)^2} \cdot -5 = -1$

$p = 5x + \dfrac{2x - 1}{x + 2} \wedge \dfrac{25}{(x + 2)^2} = 1$

$(x + 2)^2 = 25$

$x + 2 = 5 \vee x + 2 = -5$

$x = 3 \vee x = -7$

$$\left.\begin{array}{l} x = 3 \\ p = 5x + \dfrac{2x - 1}{x + 2} \end{array}\right\} p = 15 + \frac{5}{5} = 16$$

$$\left.\begin{array}{l} x = -7 \\ p = 5x + \dfrac{2x - 1}{x + 2} \end{array}\right\} p = -35 + \frac{-15}{-5} = -32$$

Dus $p = 16 \vee p = -32$.

Bladzijde 113

38 **a** $f(x) = e^{1 - x^2}$

$f'(x) = e^{1 - x^2} \cdot -2x = -2x\,e^{1 - x^2}$

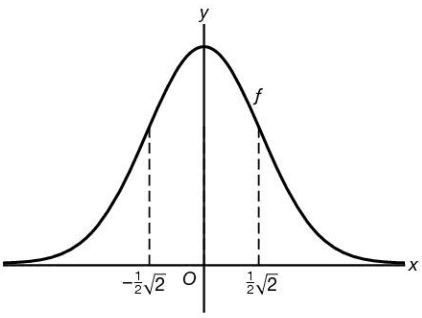
$f''(x) = -2\,e^{1 - x^2} - 2x \cdot e^{1 - x^2} \cdot -2x = -2\,e^{1 - x^2} + 4x^2 e^{1 - x^2} = (4x^2 - 2)\,e^{1 - x^2}$

$f''(x) = 0$ geeft $4x^2 - 2 = 0$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \frac{1}{2}\sqrt{2} \vee x = -\frac{1}{2}\sqrt{2}$



$f\left(-\frac{1}{2}\sqrt{2}\right) = e^{1 - \frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$ en $f\left(\frac{1}{2}\sqrt{2}\right) = \sqrt{e}$.

Dus de buigpunten zijn $\left(\frac{1}{2}\sqrt{2}, \sqrt{e}\right)$ en $\left(-\frac{1}{2}\sqrt{2}, \sqrt{e}\right)$.

b De lijn $y = ax$ snijdt de grafiek van f loodrecht als geldt

$e^{1 - x^2} = ax \wedge a \cdot -2x\,e^{1 - x^2} = -1$

$a = \dfrac{e^{1 - x^2}}{x} \wedge a = \dfrac{1}{2x\,e^{1 - x^2}}$

Voer in $y_1 = \dfrac{e^{1 - x^2}}{x}$ en $y_2 = \dfrac{1}{2x\,e^{1 - x^2}}$.

Intersect geeft $x \approx -1{,}16$ met $y \approx -0{,}61$ en $x \approx 1{,}16$ met $y \approx 0{,}61$.

Dus $a \approx -0{,}61 \vee a \approx 0{,}61$.

39 **a** $f(x) = x^2 - x$ geeft $f'(x) = 2x - 1$ en $g_p(x) = \dfrac{p}{x} = px^{-1}$ geeft $g_p'(x) = -px^{-2} = \dfrac{-p}{x^2}$

$f(x) = g_p(x) \wedge f'(x) = g_p'(x)$
$x^2 - x = \dfrac{p}{x} \wedge 2x - 1 = \dfrac{-p}{x^2}$
$x^3 - x^2 = p \wedge 2x^3 - x^2 = -p$
$p = x^3 - x^2 \wedge p = -2x^3 + x^2$
$x^3 - x^2 = -2x^3 + x^2$
$3x^3 - 2x^2 = 0$
$x^2(3x - 2) = 0$
$x^2 = 0 \vee \quad 3x = 2$
$x = 0 \quad \vee \quad x = \frac{2}{3}$
vold. niet voldoet

$\left.\begin{array}{l} x = \frac{2}{3} \\ p = x^3 - x^2 \end{array}\right\} p = \frac{8}{27} - \frac{4}{9} = -\frac{4}{27}$

$f\left(\frac{2}{3}\right) = \frac{4}{9} - \frac{2}{3} = -\frac{2}{9}$

Dus $p = -\frac{4}{27}$ en $A\left(\frac{2}{3}, -\frac{2}{9}\right)$.

b $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$
$x^2 - x = \dfrac{p}{x} \wedge (2x - 1) \cdot \dfrac{-p}{x^2} = -1$
$x^3 - x^2 = p \wedge p(2x - 1) = x^2$
$(x^3 - x^2)(2x - 1) = x^2$
$(x - 1)(2x - 1) = 1$
$2x^2 - 3x + 1 = 1$
$2x^2 - 3x = 0$
$x(2x - 3) = 0$
$x = 0 \quad \vee \quad x = 1\frac{1}{2}$
vold. niet voldoet

$\left.\begin{array}{l} x = \frac{3}{2} \\ p = x^3 - x^2 \end{array}\right\} p = \frac{27}{8} - \frac{9}{4} = \frac{9}{8}$

$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{3}{2} = \frac{3}{4}$

Dus $p = 1\frac{1}{8}$ en $B\left(1\frac{1}{2}, \frac{3}{4}\right)$.

15.3 Optimaliseringsproblemen

Bladzijde 115

40 **a** $PQ = y_Q = f\left(1\frac{1}{2}\right) = \sqrt{5 - 2 \cdot 1\frac{1}{2}} = \sqrt{2}$

$O(OPQR) = OP \cdot PQ = 1\frac{1}{2} \cdot \sqrt{2} = 1\frac{1}{2}\sqrt{2}$

b $PQ = y_Q = f(p) = \sqrt{5 - 2p}$

$A = O(OPQR) = OP \cdot PQ = p \cdot \sqrt{5 - 2p} = p\sqrt{5 - 2p}$

c Voer in $y_1 = x\sqrt{5 - 2x}$.
De optie maximum geeft $x \approx 1{,}67$ en $y \approx 2{,}15$.
Dus de maximale waarde van A is ongeveer 2,15.

Bladzijde 117

41 **a** $OQ = x_P = p$ en $PQ = y_P = f(p) = \sqrt{3 - p}$

$A = O(\triangle OPQ) = \frac{1}{2} \cdot p \cdot \sqrt{3 - p} = \frac{1}{2}p\sqrt{3 - p}$

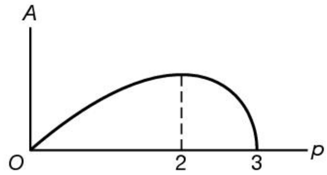
b $\dfrac{dy}{dp} = \dfrac{1}{2\sqrt{3 - p}} \cdot -1 = -\dfrac{1}{2\sqrt{3 - p}}$

$\dfrac{dA}{dp} = \frac{1}{2} \cdot \sqrt{3 - p} + \frac{1}{2}p \cdot -\dfrac{1}{2\sqrt{3 - p}} = \dfrac{\sqrt{3 - p}}{2} - \dfrac{p}{4\sqrt{3 - p}} = \dfrac{2(3 - p) - p}{4\sqrt{3 - p}} = \dfrac{6 - 3p}{4\sqrt{3 - p}}$

c $\frac{dA}{dp} = 0$ geeft $6 - 3p = 0$

$-3p = -6$

$p = 2$



De maximale oppervlakte is $\frac{1}{2} \cdot 2 \cdot \sqrt{3-2} = 1$.

d $OQ = x_P = p$ en $PQ = y_P = f(p) = \sqrt{3-p}$

In $\triangle OPQ$ is $OP^2 = OQ^2 + PQ^2$

$OP^2 = p^2 + \left(\sqrt{3-p}\right)^2$

$OP^2 = p^2 + 3 - p$

$OP^2 = p^2 - p + 3$

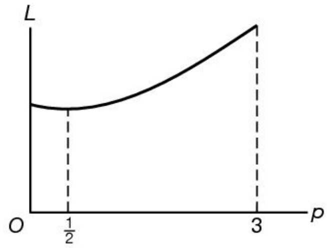
$L = OP = \sqrt{p^2 - p + 3}$

e $L = \sqrt{p^2 - p + 3}$ geeft $\frac{dL}{dp} = \frac{1}{2\sqrt{p^2-p+3}} \cdot (2p-1) = \frac{2p-1}{2\sqrt{p^2-p+3}}$

$\frac{dL}{dp} = 0$ geeft $2p - 1 = 0$

$2p = 1$

$p = \frac{1}{2}$



De minimale lengte van OP is $\sqrt{\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 3} = \sqrt{2\frac{3}{4}} = \sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{2} = \frac{1}{2}\sqrt{11}$.

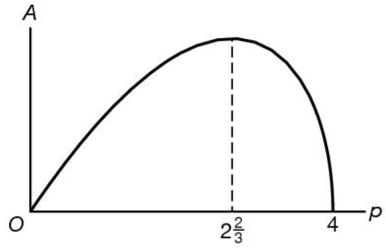
42 **a** $A = O(\triangle OSP) = \frac{1}{2} \cdot OS \cdot y_P = \frac{1}{2} \cdot 4 \cdot p\sqrt{8-2p} = 2p\sqrt{8-2p}$

$\frac{dA}{dp} = 2 \cdot \sqrt{8-2p} + 2p \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2 = 2\sqrt{8-2p} - \frac{2p}{\sqrt{8-2p}} = \frac{2(8-2p)-2p}{\sqrt{8-2p}} = \frac{16-6p}{\sqrt{8-2p}}$

$\frac{dA}{dp} = 0$ geeft $16 - 6p = 0$

$-6p = -16$

$p = 2\frac{2}{3}$



De maximale oppervlakte is $2 \cdot 2\frac{2}{3}\sqrt{8 - 2 \cdot 2\frac{2}{3}} = 5\frac{1}{3}\sqrt{2\frac{2}{3}} = 5\frac{1}{3}\sqrt{\frac{8}{3}} = 5\frac{1}{3} \cdot \frac{2\sqrt{2}}{\sqrt{3}} = 5\frac{1}{3} \cdot \frac{2\sqrt{6}}{3} = 3\frac{5}{9}\sqrt{6}$.

b $QS = x_S - x_Q = 4 - p$ en $PQ = y_P = f(p) = p\sqrt{8-2p}$

$A = O(\triangle QSP) = \frac{1}{2} \cdot QS \cdot PQ = \frac{1}{2} \cdot (4-p) \cdot p\sqrt{8-2p} = \left(2p - \frac{1}{2}p^2\right)\sqrt{8-2p}$

c $\frac{dA}{dp} = (2-p)\sqrt{8-2p} + \left(2p - \frac{1}{2}p^2\right) \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2 = (2-p)\sqrt{8-2p} - \frac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}}$

$= \frac{(2-p)(8-2p)}{\sqrt{8-2p}} - \frac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}} = \frac{16 - 4p - 8p + 2p^2 - \left(2p - \frac{1}{2}p^2\right)}{\sqrt{8-2p}} = \frac{2\frac{1}{2}p^2 - 14p + 16}{\sqrt{8-2p}}$

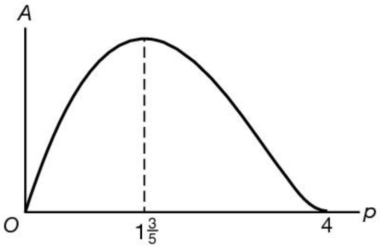
$= \frac{5p^2 - 28p + 32}{2\sqrt{8-2p}}$

d $\frac{dA}{dp} = 0$ geeft $5p^2 - 28p + 32 = 0$

$D = (-28)^2 - 4 \cdot 5 \cdot 32 = 144$

$p = \frac{28 - 12}{10} = 1\frac{3}{5} \vee p = \frac{28 + 12}{10} = 4$

vold. vold.



De maximale oppervlakte is $\left(2 \cdot 1\frac{3}{5} - \frac{1}{2}\left(1\frac{3}{5}\right)^2\right)\sqrt{8 - 2 \cdot 1\frac{3}{5}} \approx 4{,}21$.

43 **a** $PQ = x_P - x_Q = p - -p = 2p$ en $y_P = 3 - \frac{1}{2}p^2$

$A = O(\triangle OPQ) = \frac{1}{2} \cdot PQ \cdot y_P = \frac{1}{2} \cdot 2p \cdot \left(3 - \frac{1}{2}p^2\right) = 3p - \frac{1}{2}p^3$

b $\frac{dA}{dp} = 3 - 1\frac{1}{2}p^2$

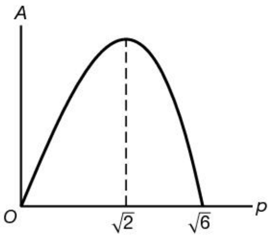
$\frac{dA}{dp} = 0$ geeft $3 - 1\frac{1}{2}p^2 = 0$

$-1\frac{1}{2}p^2 = -3$

$p^2 = 2$

$p = \sqrt{2} \vee p = -\sqrt{2}$

vold. vold. niet



Dus de maximale oppervlakte is $3\sqrt{2} - \frac{1}{2}\left(\sqrt{2}\right)^3 = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$.

c $OP^2 = x_P^2 + y_P^2 = p^2 + \left(3 - \frac{1}{2}p^2\right)^2 = p^2 + 9 - 3p^2 + \frac{1}{4}p^4 = \frac{1}{4}p^4 - 2p^2 + 9$

Stel de lengte van $OP = L$.

$L = \sqrt{\frac{1}{4}p^4 - 2p^2 + 9}$ geeft $\frac{dL}{dp} = \frac{1}{2\sqrt{\frac{1}{4}p^4 - 2p^2 + 9}} \cdot (p^3 - 4p) = \frac{p^3 - 4p}{2\sqrt{\frac{1}{4}p^4 - 2p^2 + 9}}$

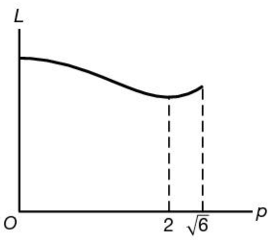
$\frac{dL}{dp} = 0$ geeft $p^3 - 4p = 0$

$p(p^2 - 4) = 0$

$p = 0 \vee p^2 = 4$

$p = 0 \vee p = 2 \vee p = -2$

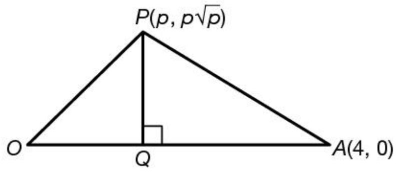
vold. vold. vold. niet



De minimale lengte van OP is $\sqrt{\frac{1}{4} \cdot 2^4 - 2 \cdot 2^2 + 9} = \sqrt{5}$.

Bladzijde 118

44 **a** $x_P = p$ en $y_P = f(p) = p\sqrt{p}$



$PQ = y_P = p\sqrt{p}$ en $AQ = x_A - x_Q = 4 - p$

In $\triangle APQ$ is $AP^2 = AQ^2 + PQ^2$

$$AP^2 = (4-p)^2 + \left(p\sqrt{p}\right)^2$$
$$AP^2 = 16 - 8p + p^2 + p^3$$
$$AP = \sqrt{p^3 + p^2 - 8p + 16}$$

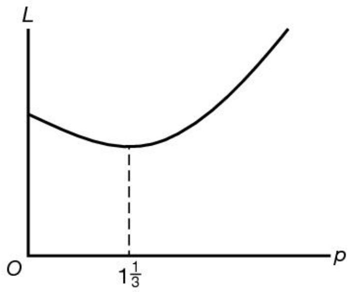
b $L = AP = \sqrt{p^3 + p^2 - 8p + 16}$ geeft

$$\frac{dL}{dp} = \frac{1}{2\sqrt{p^3 + p^2 - 8p + 16}} \cdot (3p^2 + 2p - 8) = \frac{3p^2 + 2p - 8}{2\sqrt{p^3 + p^2 - 8p + 16}}$$

$\frac{dL}{dp} = 0$ geeft $3p^2 + 2p - 8 = 0$

$$D = 2^2 - 4 \cdot 3 \cdot -8 = 100$$
$$p = \frac{-2 - 10}{6} = -2 \vee p = \frac{-2 + 10}{6} = 1\tfrac{1}{3}$$

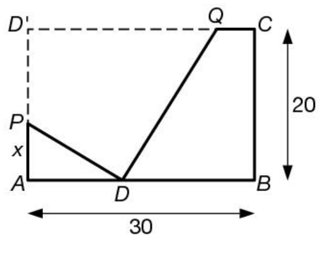
vold. niet   vold.



Het punt op de grafiek van f het dichtst bij A is

$$\left(1\tfrac{1}{3}, 1\tfrac{1}{3}\sqrt{1\tfrac{1}{3}}\right) = \left(1\tfrac{1}{3}, 1\tfrac{1}{3}\sqrt{\tfrac{4}{3}}\right) = \left(1\tfrac{1}{3}, 1\tfrac{1}{3} \cdot \frac{2}{\sqrt{3}}\right) = \left(1\tfrac{1}{3}, \tfrac{4}{3} \cdot \frac{2\sqrt{3}}{3}\right) = \left(1\tfrac{1}{3}, \tfrac{8}{9}\sqrt{3}\right).$$

45 Stel $AP = x$.



$$\left.\begin{array}{l} AP + PD' = 20 \\ AP = x \\ PD' = PD \end{array}\right\} x + PD = 20, \text{ dus } PD = 20 - x$$

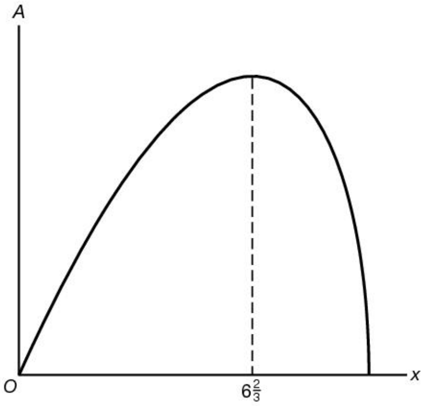
In $\triangle ADP$ is $AP^2 + AD^2 = PD^2$

$$x^2 + AD^2 = (20 - x)^2$$
$$x^2 + AD^2 = 400 - 40x + x^2$$
$$AD^2 = 400 - 40x$$
$$AD = \sqrt{400 - 40x}$$

$A = O(\triangle ADP) = \frac{1}{2} \cdot AD \cdot AP = \frac{1}{2} \cdot \sqrt{400 - 40x} \cdot x = \frac{1}{2}x\sqrt{400 - 40x}$

$$\frac{dA}{dx} = \tfrac{1}{2}\sqrt{400 - 40x} + \tfrac{1}{2}x \cdot \frac{1}{2\sqrt{400 - 40x}} \cdot -40 = \frac{\frac{1}{2}(400 - 40x)}{\sqrt{400 - 40x}} - \frac{10x}{\sqrt{400 - 40x}} = \frac{200 - 20x - 10x}{\sqrt{400 - 40x}}$$
$$= \frac{200 - 30x}{\sqrt{400 - 40x}}$$

$$\frac{dA}{dx} = 0 \text{ geeft } 200 - 30x = 0$$
$$-30x = -200$$
$$x = 6\tfrac{2}{3}$$



Dus $O(\triangle ADP)$ is maximaal voor $x = 6\frac{2}{3}$ en is $\frac{1}{2} \cdot 6\frac{2}{3} \cdot \sqrt{400 - 40 \cdot 6\frac{2}{3}} \approx 38{,}49$ cm^2.

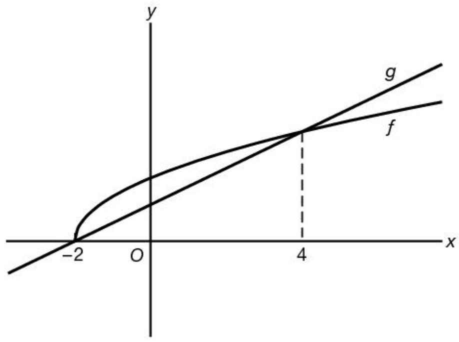
46 **a** $p = -3$ geeft $L = y_A - y_B = f(-3) - g(-3) = \sqrt{2 \cdot -3 + 15} - \frac{1}{2} \cdot -3 = 4\frac{1}{2}$

b $L = y_A - y_B = f(p) - g(p) = l = y_A - y_B = f(p) - g(p) = \sqrt{2p + 15} - \frac{1}{2}p$

c Voer in $y_1 = \sqrt{2x + 15} - \frac{1}{2}x$.
De optie maximum geeft $x = -5{,}5$.
Dus voor $p = -5{,}5$ is de lengte van AB maximaal.

Bladzijde 120

47 **a**



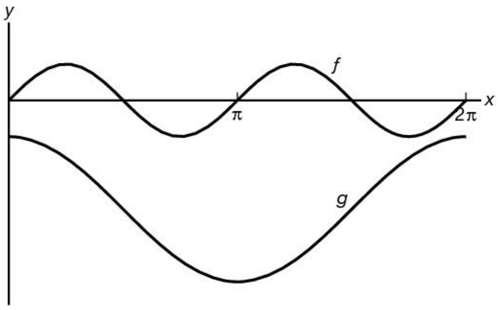
$L = f(p) - g(p) = \sqrt{6p + 12} - (p + 2) = \sqrt{6p + 12} - p - 2$

b $\frac{dL}{dp} = \frac{1}{2\sqrt{6p + 12}} \cdot 6 - 1 = \frac{3}{\sqrt{6p + 12}} - 1$

$$\frac{dL}{dp} = 0 \text{ geeft } \frac{3}{\sqrt{6p + 12}} - 1 = 0$$
$$\frac{3}{\sqrt{6p + 12}} = 1$$
$$\sqrt{6p + 12} = 3$$
$$6p + 12 = 9$$
$$6p = -3$$
$$p = -\tfrac{1}{2}$$

De maximale waarde van L is $\sqrt{6 \cdot -\frac{1}{2} + 12} - -\frac{1}{2} - 2 = 1\frac{1}{2}$.

48



$L = AB = f(p) - g(p) = \frac{1}{2}\sin(2p) - \left(\cos(p) - 1\frac{1}{2}\right) = \frac{1}{2}\sin(2p) - \cos(p) + 1\frac{1}{2}$

$\frac{dL}{dp} = \frac{1}{2} \cdot 2\cos(2p) + \sin(p) = \cos(2p) + \sin(p)$

$\frac{dL}{dp} = 0$ geeft $\cos(2p) + \sin(p) = 0$

$1 - 2\sin^2(p) + \sin(p) = 0$

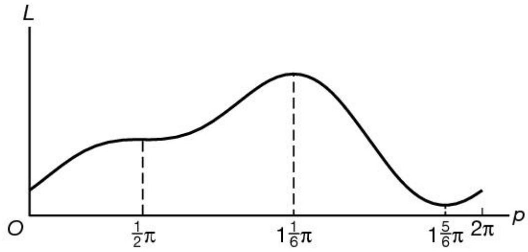
$-2\sin^2(p) + \sin(p) + 1 = 0$

$(-\sin(p) + 1)(2\sin(p) + 1) = 0$

$\sin(p) = 1 \vee \sin(p) = -\frac{1}{2}$

$p = \frac{1}{2}\pi + k \cdot 2\pi \vee p = -\frac{1}{6}\pi + k \cdot 2\pi \vee p = 1\frac{1}{6}\pi + k \cdot 2\pi$

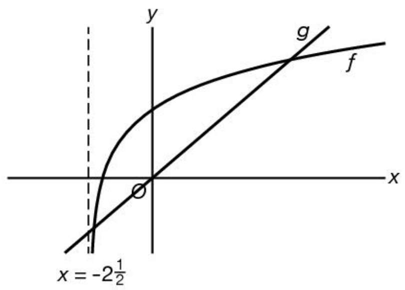
p op $[0, 2\pi]$ geeft $p = \frac{1}{2}\pi \vee p = 1\frac{5}{6}\pi \vee p = 1\frac{1}{6}\pi$



De maximale lengte van AB is

$\frac{1}{2}\sin\left(2 \cdot 1\frac{1}{6}\pi\right) - \cos\left(1\frac{1}{6}\pi\right) + 1\frac{1}{2} = \frac{1}{2}\sin\left(2\frac{1}{3}\pi\right) - \cos\left(1\frac{1}{6}\pi\right) + 1\frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} + 1\frac{1}{2} = \frac{3}{4}\sqrt{3} + 1\frac{1}{2}$.

49



$L = CD = f(p) - g(p) = \ln(2p + 5) - \frac{1}{2}p$

$\frac{dL}{dp} = \frac{1}{2p + 5} \cdot 2 - \frac{1}{2} = \frac{2}{2p + 5} - \frac{1}{2}$

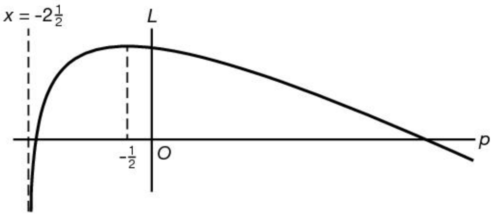
$\frac{dL}{dp} = 0$ geeft $\frac{2}{2p + 5} - \frac{1}{2} = 0$

$\frac{2}{2p + 5} = \frac{1}{2}$

$2p + 5 = 4$

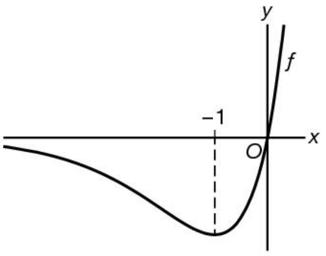
$2p = -1$

$p = -\frac{1}{2}$



De maximale lengte van CD is $\ln\left(2 \cdot -\frac{1}{2} + 5\right) - \frac{1}{2} \cdot -\frac{1}{2} = \ln(4) + \frac{1}{4}$.

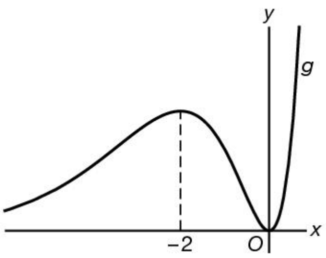
50 **a** $f(x) = 5x\,\mathrm{e}^x$ geeft $f'(x) = 5 \cdot \mathrm{e}^x + 5x \cdot \mathrm{e}^x = (5x + 5)\,\mathrm{e}^x$
$f'(x) = 0$ geeft $5x + 5 = 0$
$5x = -5$
$x = -1$



min. is $f(-1) = 5 \cdot -1 \cdot \mathrm{e}^{-1} = \dfrac{-5}{\mathrm{e}}$

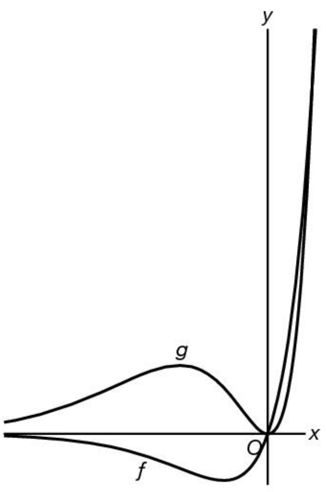
Dus $B_f = \left[-\dfrac{5}{\mathrm{e}}, \rightarrow\right\rangle$.

b $g(x) = 5x^2\,\mathrm{e}^x$ geeft $g'(x) = 10x \cdot \mathrm{e}^x + 5x^2 \cdot \mathrm{e}^x = (5x^2 + 10x)\,\mathrm{e}^x$
$g'(x) = 0$ geeft $5x^2 + 10x = 0$
$5x(x + 2) = 0$
$x = 0 \vee x = -2$



max. is $g(-2) = 5 \cdot (-2)^2 \cdot \mathrm{e}^{-2} = \dfrac{20}{\mathrm{e}^2}$ en min. is $g(0) = 5 \cdot 0^2 \cdot \mathrm{e}^0 = 0$.

c

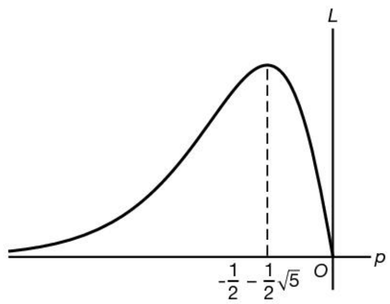


$L = AB = g(p) - f(p) = 5p^2\,e^p - 5p\,e^p = (5p^2 - 5p)\,e^p$

$\frac{dL}{dp} = (10p - 5)\,e^p + (5p^2 - 5p)\,e^p = (5p^2 + 5p - 5)\,e^p$

$\frac{dL}{dp} = 0$ geeft $(5p^2 + 5p - 5)\,e^p = 0$

$5p^2 + 5p - 5 = 0$
$p^2 + p - 1 = 0$
$\left(p + \frac{1}{2}\right)^2 - \frac{1}{4} - 1 = 0$
$\left(p + \frac{1}{2}\right)^2 = \frac{5}{4}$
$p + \frac{1}{2} = \frac{1}{2}\sqrt{5} \vee p + \frac{1}{2} = -\frac{1}{2}\sqrt{5}$
$p = -\frac{1}{2} + \frac{1}{2}\sqrt{5} \vee p = -\frac{1}{2} - \frac{1}{2}\sqrt{5}$
vold. niet vold.



Dus AB is maximaal voor $p = -\frac{1}{2} - \frac{1}{2}\sqrt{5}$.

d $L = CD = f(q) - g(q) = 5q\,e^q - 5q^2\,e^q$
Voer in $y_1 = 5x\,e^x - 5x^2\,e^x$.
De optie maximum geeft $x \approx 0{,}618$ en $y \approx 2{,}190$.
De maximale lengte van CD is ongeveer 2,190.

Bladzijde 121

51 **a** De kettingboog door (0, 10) geeft $\frac{1}{2}a(e^0 + e^0) + b = 10$
$\frac{1}{2}a(1 + 1) + b = 10$
$a + b = 10$
$b = 10 - a$

De kettingboog door (10, 0) geeft $\frac{1}{2}a\left(e^{\frac{10}{a}} + e^{-\frac{10}{a}}\right) + b = 0$
$b = -\frac{1}{2}a\left(e^{\frac{10}{a}} + e^{-\frac{10}{a}}\right)$

Voer in $y_1 = 10 - x$ en $y_2 = -\frac{1}{2}x\left(e^{\frac{10}{x}} + e^{-\frac{10}{x}}\right)$.
Intersect geeft $x = -6{,}1875...$ en $y = 16{,}1875...$
Dus $a \approx -6{,}188$ en $b \approx 16{,}188$.

b $\left.\begin{array}{l} y_p = mx^2 + n \\ \text{door } (0, 10) \end{array}\right\} \begin{array}{l} m \cdot 0^2 + n = 10 \\ n = 10 \end{array}$

$\left.\begin{array}{l} y_p = mx^2 + 10 \\ \text{door } (10, 0) \end{array}\right\} \begin{array}{l} m \cdot 10^2 + 10 = 0 \\ 100m = -10 \\ m = -0{,}1 \end{array}$

Dus $y_p = -0{,}1x^2 + 10$.
Het verschil tussen y_k en y_p is

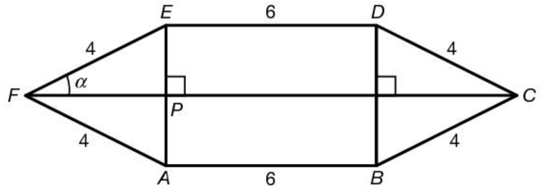
$|y_k - y_p| = \left| -3{,}0937... \cdot \left(e^{\frac{x}{-6{,}1875...}} + e^{-\frac{x}{-6{,}1875...}} \right) + 16{,}1875... - (-0{,}1x^2 + 10) \right|$.

Voer in $y_3 = \left| -3{,}0937... \cdot \left(e^{\frac{x}{-6{,}1875...}} + e^{-\frac{x}{-6{,}1875...}} \right) + 16{,}1875... - (-0{,}1x^2 + 10) \right|$.

De optie maximum geeft $x = 7{,}1448...$ en $y = 0{,}5005...$
Dus het maximale verschil tussen y_k en y_p is 50 cm.

52 **a** In $\triangle FPE$ is $\sin(\alpha) = \dfrac{EP}{4}$, dus $EP = 4\sin(\alpha)$.

$AE = 2EP = 8\sin(\alpha)$



b In $\triangle FPE$ is $\cos(\alpha) = \dfrac{FP}{4}$, dus $FP = 4\cos(\alpha)$.

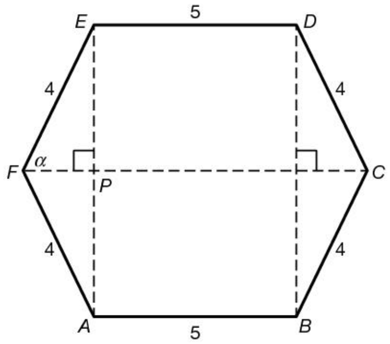
$O(\triangle AEF) = \frac{1}{2} FP \cdot AE = \frac{1}{2} \cdot 4\cos(\alpha) \cdot 8\sin(\alpha) = 16\sin(\alpha)\cos(\alpha)$

c $O(ABDE) = AB \cdot AE = 6 \cdot 8\sin(\alpha) = 48\sin(\alpha)$

d $O(ABCDEF) = 2\ O(AEF) + O(ABDE) = 32\sin(\alpha)\cos(\alpha) + 48\sin(\alpha) = 16\sin(2\alpha) + 48\sin(\alpha)$

Bladzijde 123

53 **a**



$\cos(\alpha) = \dfrac{FP}{EF}$ $\qquad$ $\sin(\alpha) = \dfrac{EP}{EF}$

$\cos(\alpha) = \dfrac{FP}{4}$ $\qquad$ $\sin(\alpha) = \dfrac{EP}{4}$

$FP = 4\cos(\alpha)$ $\qquad$ $EP = 4\sin(\alpha)$

$AE = 2AP = 8\sin(\alpha)$

$A = O(ABCDEF) = 2 \cdot O(AEF) + O(ABDE)$

$= 2 \cdot \frac{1}{2} \cdot AE \cdot FP + AB \cdot AE = 8\sin(\alpha) \cdot 4\cos(\alpha) + 5 \cdot 8\sin(\alpha) = 32\sin(\alpha)\cos(\alpha) + 40\sin(\alpha)$

b $A = 16 \cdot 2\sin(\alpha)\cos(\alpha) + 40\sin(\alpha) = 16\sin(2\alpha) + 40\sin(\alpha)$

$\dfrac{dA}{d\alpha} = 16 \cdot 2\cos(2\alpha) + 40\cos(\alpha) = 32\cos(2\alpha) + 40\cos(\alpha)$

$\dfrac{dA}{d\alpha} = 0$ geeft $32\cos(2\alpha) + 40\cos(\alpha) = 0$

$32(2\cos^2(\alpha) - 1) + 40\cos(\alpha) = 0$

$64\cos^2(\alpha) - 32 + 40\cos(\alpha) = 0$

$8\cos^2(\alpha) + 5\cos(\alpha) - 4 = 0$

Stel $\cos(\alpha) = u$.

$8u^2 + 5u - 4 = 0$

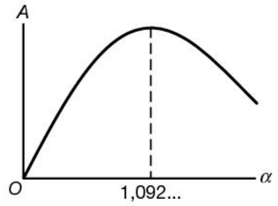
$D = 5^2 - 4 \cdot 8 \cdot -4 = 153$

$u = \dfrac{-5 - \sqrt{153}}{16} \vee u = \dfrac{-5 + \sqrt{153}}{16}$

$\cos(\alpha) = \dfrac{-5 - \sqrt{153}}{16} \vee \cos(\alpha) = \dfrac{-5 + \sqrt{153}}{16}$

geen opl. $\qquad$ $\alpha = 1{,}092... + k \cdot 2\pi \vee \alpha = -1{,}092... + k \cdot 2\pi$

α op $\left[0, \frac{1}{2}\pi\right]$ geeft $\alpha = 1{,}092...$



De oppervlakte is maximaal bij een hoek van $1{,}092... \cdot \dfrac{180^\circ}{\pi} \approx 63^\circ$.

54 **a** $\cos(\alpha) = \dfrac{AP}{AB} = \dfrac{AP}{1}$, dus $AP = \cos(\alpha)$.

$\sin(\alpha) = \dfrac{BP}{AB} = \dfrac{BP}{1}$, dus $BP = \sin(\alpha)$.

$\triangle APB$ is gelijkvormig met en even groot als $\triangle COA$, dus $OA = \sin(\alpha)$ en $OC = \cos(\alpha)$.

$V = OP \cdot OC = (\sin(\alpha) + \cos(\alpha)) \cdot \cos(\alpha) = \sin(\alpha)\cos(\alpha) + \cos^2(\alpha) = \frac{1}{2}\sin(2\alpha) + \cos^2(\alpha)$

b $V = \frac{1}{2}\sin(2\alpha) + \cos^2(\alpha)$ geeft $\dfrac{dV}{d\alpha} = \frac{1}{2} \cdot 2\cos(2\alpha) + 2\cos(\alpha) \cdot -\sin(\alpha) = \cos(2\alpha) - \sin(2\alpha)$

$\dfrac{dV}{d\alpha} = 0$ geeft $\cos(2\alpha) - \sin(2\alpha) = 0$

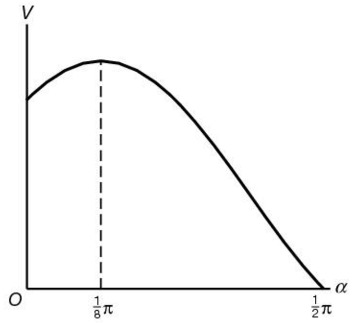
$\sin(2\alpha) = \cos(2\alpha)$

$\dfrac{\sin(2\alpha)}{\cos(2\alpha)} = 1$

$\tan(2\alpha) = 1$

$2\alpha = \frac{1}{4}\pi + k \cdot \pi$

$\alpha = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$



Dus de oppervlakte van rechthoek $OPQC$ is maximaal voor $\alpha = \frac{1}{8}\pi$.

Bladzijde 124

55 **a** $\cos(\alpha) = \dfrac{AP}{AB} = \dfrac{AP}{1}$, dus $AP = \cos(\alpha)$.

$\sin(\alpha) = \dfrac{BP}{AB} = \dfrac{BP}{1}$, dus $BP = \sin(\alpha)$.

$\triangle COA \backsim \triangle APB$, dus $\dfrac{CO}{AP} = \dfrac{CA}{AB}$

$\dfrac{CO}{\cos(\alpha)} = \dfrac{\sqrt{3}}{1}$

$CO = \sqrt{3} \cdot \cos(\alpha)$

Ook geldt $\dfrac{OA}{PB} = \dfrac{CA}{AB}$

$\dfrac{OA}{\sin(\alpha)} = \dfrac{\sqrt{3}}{1}$

$OA = \sqrt{3} \cdot \sin(\alpha)$

$V = OP \cdot OC = \left(\sqrt{3} \cdot \sin(\alpha) + \cos(\alpha)\right) \cdot \sqrt{3} \cdot \cos(\alpha) = 3\sin(\alpha)\cos(\alpha) + \sqrt{3} \cdot \cos^2(\alpha)$
$= 1\frac{1}{2}\sin(2\alpha) + \sqrt{3} \cdot \cos^2(\alpha)$

b $V = 1\frac{1}{2}\sin(2\alpha) + \sqrt{3} \cdot \cos^2(\alpha)$ geeft

$\dfrac{dV}{d\alpha} = 1\frac{1}{2} \cdot 2\cos(2\alpha) + 2\sqrt{3} \cdot \cos(\alpha) \cdot -\sin(\alpha) = 3\cos(2\alpha) - \sqrt{3} \cdot \sin(2\alpha)$

$\dfrac{dV}{d\alpha} = 0$ geeft $3\cos(2\alpha) - \sqrt{3} \cdot \sin(2\alpha) = 0$

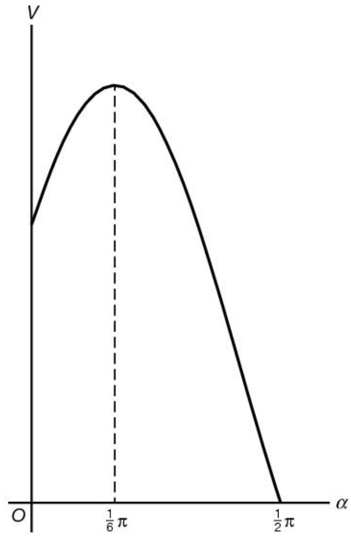
$\sqrt{3} \cdot \sin(2\alpha) = 3\cos(2\alpha)$

$\dfrac{\sin(2\alpha)}{\cos(2\alpha)} = \dfrac{3}{\sqrt{3}}$

$\tan(2\alpha) = \sqrt{3}$

$2\alpha = \frac{1}{3}\pi + k \cdot \pi$

$\alpha = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$



Dus de oppervlakte van rechthoek $OPQC$ is maximaal voor $\alpha = \frac{1}{6}\pi$.

c $Q\left(\sqrt{3} \cdot \sin(\alpha) + \cos(\alpha), \sqrt{3} \cdot \cos(\alpha)\right)$

Stel RQ: $y = ax + b$ met $a = \dfrac{0 - \sqrt{3} \cdot \cos(\alpha)}{4 - \left(\sqrt{3} \cdot \sin(\alpha) + \cos(\alpha)\right)} = \dfrac{-\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)}$

$$\left.\begin{array}{l} y = \dfrac{-\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)} \cdot x + b \\ \text{door } R(4, 0) \end{array}\right\} \dfrac{-\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)} \cdot 4 + b = 0$$

$$b = \frac{4\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)}$$

Dus RQ: $y = \dfrac{-\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)} \cdot x + \dfrac{4\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)}$.

Dus $OS = \dfrac{4\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)}$.

d $OS = \dfrac{4\sqrt{3} \cdot \cos(\alpha)}{4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)}$ geeft

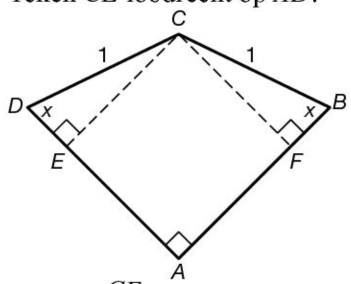
$$\frac{dOS}{d\alpha} = \frac{\left(4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)\right) \cdot -4\sqrt{3} \cdot \sin(\alpha) - 4\sqrt{3} \cdot \cos(\alpha) \cdot \left(-\sqrt{3} \cdot \cos(\alpha) + \sin(\alpha)\right)}{\left(4 - \sqrt{3} \cdot \sin(\alpha) - \cos(\alpha)\right)^2}$$

Voer in $y_1 = \dfrac{\left(4 - \sqrt{3} \cdot \sin(x) - \cos(x)\right) \cdot -4\sqrt{3} \cdot \sin(x) - 4\sqrt{3} \cdot \cos(x) \cdot \left(-\sqrt{3} \cdot \cos(x) + \sin(x)\right)}{\left(4 - \sqrt{3} \cdot \sin(x) - \cos(x)\right)^2}$.

De optie zero (TI) of ROOT (Casio) geeft $x = 0{,}4478...$

Dus $\alpha = 0{,}4478... \cdot \dfrac{180°}{\pi} \approx 26°$.

56 **a** Teken CE loodrecht op AD.



$\sin(x) = \dfrac{CE}{CD}$ $\qquad$ $\cos(x) = \dfrac{DE}{CD}$

$\sin(x) = \dfrac{CE}{1}$ $\qquad$ $\cos(x) = \dfrac{DE}{1}$

$CE = \sin(x)$ $\qquad$ $DE = \cos(x)$

$AE = CF = CE = \sin(x)$

$A = O(ABCD) = 2 \cdot O(\triangle CDE) + O(AFCE) = 2 \cdot \frac{1}{2} \cdot DE \cdot CE + AE \cdot CE$
$= \cos(x) \cdot \sin(x) + \sin(x) \cdot \sin(x) = \sin^2(x) + \sin(x)\cos(x)$

b $\frac{dA}{dx} = \cos(x)(\cos(x) + \sin(x)) + \sin(x)(-\sin(x) + \cos(x))$

$= \cos^2(x) + \sin(x)\cos(x) - \sin^2(x) + \sin(x)\cos(x)$

$= \cos^2(x) - \sin^2(x) + 2\sin(x)\cos(x)$

$= \cos(2x) + \sin(2x)$

$\frac{dA}{dx} = 0$ geeft $\cos(2x) + \sin(2x) = 0$

$\sin(2x) = -\cos(2x)$

$\cos\left(2x - \frac{1}{2}\pi\right) = \cos(2x + \pi)$

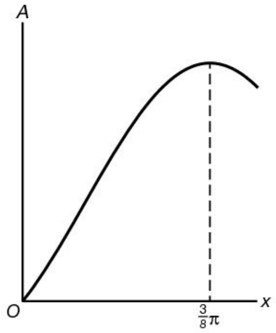
$2x - \frac{1}{2}\pi = 2x + \pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = -(2x + \pi) + k \cdot 2\pi$

geen opl. $\quad 2x - \frac{1}{2}\pi = -2x - \pi + k \cdot 2\pi$

$4x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = -\frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$

x op $\left[0, \frac{1}{2}\pi\right]$ geeft $x = \frac{3}{8}\pi$.



De oppervlakte is maximaal voor $x = \frac{3}{8}\pi$.

15.4 Integralen bij oppervlakte en inhoud

Bladzijde 126

57 **a** $f(x) = 2 + \frac{5}{2x-1} = \frac{2(2x-1)}{2x-1} + \frac{5}{2x-1} = \frac{4x-2+5}{2x-1} = \frac{4x+3}{2x-1}$

b $\int_1^3 \frac{4x+3}{2x-1}\,dx = \int_1^3 \left(2 + \frac{5}{2x-1}\right)dx = \left[2x + 5 \cdot \frac{1}{2}\ln|2x-1|\right]_1^3 = 6 + 2\frac{1}{2}\ln(5) - \left(2 + 2\frac{1}{2}\ln(1)\right)$

$= 6 + 2\frac{1}{2}\ln(5) - 2 = 4 + 2\frac{1}{2}\ln(5)$

Bladzijde 127

58 **a** $f(x) = \frac{2x+1}{x+1} = \frac{2(x+1)-2+1}{x+1} = 2 - \frac{1}{x+1}$ geeft $F(x) = 2x - \ln|x+1| + c$

b $f(x) = \frac{x}{x+1} = \frac{x+1-1}{x+1} = 1 - \frac{1}{x+1}$ geeft $F(x) = x - \ln|x+1| + c$

c $f(x) = \frac{x+1}{2x+1} = \frac{\frac{1}{2}(2x+1) - \frac{1}{2} + 1}{2x+1} = \frac{1}{2} + \frac{\frac{1}{2}}{2x+1} = \frac{1}{2} + \frac{1}{4x+2}$ geeft $F(x) = \frac{1}{2}x + \frac{1}{4}\ln|4x+2| + c$

d $f(x) = \frac{2-x}{x+1} = \frac{-(x+1)+1+2}{x+1} = -1 + \frac{3}{x+1}$ geeft $F(x) = -x + 3\ln|x+1| + c$

e $f(x) = \frac{3-4x}{2x+1} = \frac{-2(2x+1)+2+3}{2x+1} = -2 + \frac{5}{2x+1}$ geeft

$F(x) = -2x + 5 \cdot \frac{1}{2}\ln|2x+1| + c = -2x + 2\frac{1}{2}\ln|2x+1| + c$

f $f(x) = \frac{6x-1}{1-2x} = \frac{-3(1-2x)+3-1}{1-2x} = -3 + \frac{2}{1-2x}$ geeft

$F(x) = -3x + 2 \cdot -\frac{1}{2}\ln|1-2x| + c = -3x - \ln|1-2x| + c$

59 $f(x) = g(x)$ geeft $\frac{x+8}{x+1} = \frac{4x-4}{x+1}$

$x + 8 = 4x - 4 \wedge x \neq -1$

$-3x = -12 \wedge x \neq -1$

$x = 4 \wedge x \neq -1$

$x = 4$

$g(x) = 0$ geeft $4x - 4 = 0$

$x = 1$

$$f(x) = \frac{x+8}{x+1} = \frac{x+1-1+8}{x+1} = 1 + \frac{7}{x+1}$$

$$g(x) = \frac{4x-4}{x+1} = \frac{4(x+1)-4-4}{x+1} = 4 - \frac{8}{x+1}$$

$$O(V) = \int_0^1 f(x)\,dx + \int_1^4 (f(x) - g(x))\,dx = \int_0^1 \left(1 + \frac{7}{x+1}\right)dx + \int_1^4 \left(1 + \frac{7}{x+1} - 4 + \frac{8}{x+1}\right)dx$$

$$= \int_0^1 \left(1 + \frac{7}{x+1}\right)dx + \int_1^4 \left(-3 + \frac{15}{x+1}\right)dx = \left[x + 7\ln|x+1|\right]_0^1 + \left[-3x + 15\ln|x+1|\right]_1^4$$

$$= 1 + 7\ln(2) - (0 + 7\ln(1)) + -12 + 15\ln(5) - (-3 + 15\ln(2))$$

$$= 1 + 7\ln(2) - 12 + 15\ln(5) + 3 - 15\ln(2) = 15\ln(5) - 8\ln(2) - 8$$

60 $f_a(x) = \frac{ax+4}{2x-3} = \frac{\frac{1}{2}a(2x-3) + 1\frac{1}{2}a + 4}{2x-3} = \frac{1}{2}a + \frac{1\frac{1}{2}a + 4}{2x-3}$

$$O(V) = \int_2^4 \left(\frac{1}{2}a + \frac{1\frac{1}{2}a + 4}{2x-3}\right)dx = \left[\frac{1}{2}ax + \left(1\frac{1}{2}a + 4\right)\cdot\frac{1}{2}\ln|2x-3|\right]_2^4$$

$$= 2a + \left(1\frac{1}{2}a + 4\right)\cdot\frac{1}{2}\ln(5) - \left(a + \left(1\frac{1}{2}a + 4\right)\cdot\frac{1}{2}\ln(1)\right) = 2a + \left(1\frac{1}{2}a + 4\right)\cdot\frac{1}{2}\ln(5) - a - 0$$

$$= a + \frac{3}{4}a\ln(5) + 2\ln(5)$$

$O(V) = 10$ geeft $a + \frac{3}{4}a\ln(5) + 2\ln(5) = 10$

$$a\left(1 + \frac{3}{4}\ln(5)\right) = 10 - 2\ln(5)$$

$$a = \frac{10 - 2\ln(5)}{1 + \frac{3}{4}\ln(5)} \approx 3{,}07$$

61 **a** $O(V + W) = \int_0^p \sqrt{x}\,dx = \int_0^p x^{\frac{1}{2}}\,dx = \left[\frac{2}{3}x^{1\frac{1}{2}}\right]_0^p = \left[\frac{2}{3}x\sqrt{x}\right]_0^p = \frac{2}{3}p\sqrt{p} - 0 = \frac{2}{3}p\sqrt{p}$

b $O(W) = \frac{1}{2}\cdot p \cdot f(p) = \frac{1}{2}p\sqrt{p}$

c $O(V) = O(V + W) - O(W) = \frac{2}{3}p\sqrt{p} - \frac{1}{2}p\sqrt{p} = \frac{1}{6}p\sqrt{p}$

d $O(V) : O(W) = \frac{1}{6}p\sqrt{p} : \frac{1}{2}p\sqrt{p} = \frac{1}{6} : \frac{1}{2} = 1 : 3$

Bladzijde 128

62 $O(V) = O(W)$

$1 + 2\ln(p) = p^2 - (1 + 2\ln(p))$

$1 + 2\ln(p) = p^2 - 1 - 2\ln(p)$

$2 + 4\ln(p) = p^2$

Voer in $y_1 = 2 + 4\ln(x)$ en $y_2 = x^2$.

Intersect geeft $x = 0{,}681...$ en $x = 2{,}314...$

Dus $p \approx 2{,}31$.

63 $O(V + W) = \int_{-3}^{3} f(x)\,dx = \int_{-3}^{3} (9 - x^2)\,dx = \left[9x - \frac{1}{3}x^3\right]_{-3}^{3} = 27 - 9 - (-27 + 9) = 36$

$$O(W) = \int_{-\sqrt{a}}^{\sqrt{a}} g(x)\,dx = \int_{-\sqrt{a}}^{\sqrt{a}} (a - x^2)\,dx = \left[ax - \frac{1}{3}x^3\right]_{-\sqrt{a}}^{\sqrt{a}} = a\sqrt{a} - \frac{1}{3}a\sqrt{a} - \left(-a\sqrt{a} + \frac{1}{3}a\sqrt{a}\right) = 1\frac{1}{3}a\sqrt{a}$$

$O(V) = O(W)$ geeft $O(W) = \frac{1}{2}O(V + W)$

$$1\tfrac{1}{3}a\sqrt{a} = \tfrac{1}{2} \cdot 36$$
$$a\sqrt{a} = 13\tfrac{1}{2}$$
$$a^{1\frac{1}{2}} = \tfrac{27}{2}$$
$$a = \left(\tfrac{27}{2}\right)^{\frac{2}{3}}$$
$$a = \sqrt[3]{\left(\tfrac{27}{2}\right)^2} = \sqrt[3]{182\tfrac{1}{4}}$$

Bladzijde 129

15

64 **a** $f_{3\frac{1}{3}}(x) = -\frac{1}{3}x^3 + x^2 + 3\frac{1}{3}x$ geeft $f_{3\frac{1}{3}}'(x) = -x^2 + 2x + 3\frac{1}{3}$

$f_{3\frac{1}{3}}'(0) = 3\frac{1}{3}$, dus k: $y = 3\frac{1}{3}x$.

k snijden met de grafiek van $f_{3\frac{1}{3}}$ geeft $-\frac{1}{3}x^3 + x^2 + 3\frac{1}{3}x = 3\frac{1}{3}x$

$$-\tfrac{1}{3}x^3 + x^2 = 0$$
$$-\tfrac{1}{3}x^2(x - 3) = 0$$
$$x = 0 \vee x = 3$$

Het snijpunt is (3, 10).

$f_{3\frac{1}{3}}(x) = 0$ geeft $-\frac{1}{3}x^3 + x^2 + 3\frac{1}{3}x = 0$

$$-\tfrac{1}{3}x(x^2 - 3x - 10) = 0$$
$$-\tfrac{1}{3}x(x + 2)(x - 5) = 0$$
$$x = 0 \vee x = -2 \vee x = 5$$

$$O(W) = \int_0^3 3\tfrac{1}{3}x \, dx + \int_3^5 \left(-\tfrac{1}{3}x^3 + x^2 + 3\tfrac{1}{3}x\right) dx = \left[\tfrac{5}{3}x^2\right]_0^3 + \left[-\tfrac{1}{12}x^4 + \tfrac{1}{3}x^3 + \tfrac{5}{3}x^2\right]_3^5$$
$$= 15 - 0 - \tfrac{625}{12} + \tfrac{125}{3} + \tfrac{125}{3} - \left(-\tfrac{27}{4} + 9 + 15\right) = 15 - \tfrac{625}{12} + \tfrac{250}{3} + \tfrac{27}{4} - 24 = 29$$

b $f_a(x) = -\frac{1}{3}x^3 + x^2 + ax$ geeft $f_a'(x) = -x^2 + 2x + a$

$f_a'(0) = a$, dus k: $y = ax$.

k snijden met de grafiek van f_a geeft $-\frac{1}{3}x^3 + x^2 + ax = ax$

$$-\tfrac{1}{3}x^3 + x^2 = 0$$
$$-\tfrac{1}{3}x^2(x - 3) = 0$$
$$x = 0 \vee x = 3$$

$O(V) = \int_0^3 \left(-\frac{1}{3}x^3 + x^2 + ax - ax\right) dx = \int_0^3 \left(-\frac{1}{3}x^3 + x^2\right) dx$ en dit is onafhankelijk van a.

65 $f_a(x) = \frac{1}{4}x^2(x - a)^2 = \frac{1}{4}x^2(x^2 - 2ax + a^2) = \frac{1}{4}x^4 - \frac{1}{2}ax^3 + \frac{1}{4}a^2x^2$ geeft $f_a'(x) = x^3 - 1\frac{1}{2}ax^2 + \frac{1}{2}a^2x$

$f_a'(x) = 0$ geeft $x^3 - 1\frac{1}{2}ax^2 + \frac{1}{2}a^2x = 0$

$$x\left(x^2 - 1\tfrac{1}{2}ax + \tfrac{1}{2}a^2\right) = 0$$
$$x\left(x - \tfrac{1}{2}a\right)(x - a) = 0$$
$$x = 0 \vee x = \tfrac{1}{2}a \vee x = a$$

$f_a\left(\frac{1}{2}a\right) = \frac{1}{16}a^2\left(\frac{1}{2}a - a\right)^2 = \frac{1}{64}a^4$, dus $T\left(\frac{1}{2}a, \frac{1}{64}a^4\right)$.

$O(\text{rechthoek } OABC) = a \cdot \frac{1}{64}a^4 = \frac{1}{64}a^5$

$$O(V) = \int_0^a \left(\tfrac{1}{4}x^4 - \tfrac{1}{2}ax^3 + \tfrac{1}{4}a^2x^2\right) dx = \left[\tfrac{1}{20}x^5 - \tfrac{1}{8}ax^4 + \tfrac{1}{12}a^2x^3\right]_0^a = \tfrac{1}{20}a^5 - \tfrac{1}{8}a^5 + \tfrac{1}{12}a^5 - 0 = \tfrac{1}{120}a^5$$

De gevraagde verhouding is $\frac{1}{120}a^5 : \frac{1}{64}a^5 = 64 : 120 = 8 : 15$.

66 $f_a(x) = 0$ geeft $x = a$

$f_a(x) = 2\sqrt{x}$ geeft $\sqrt{a - x} = 2\sqrt{x}$

kwadrateren geeft

$a - x = 4x$

$-5x = -a$

$x = \frac{1}{5}a$

vold.

$f_a(x) = \frac{1}{2}\sqrt{x}$ geeft $\sqrt{a - x} = \frac{1}{2}\sqrt{x}$
kwadrateren geeft
$a - x = \frac{1}{4}x$
$-\frac{5}{4}x = -a$
$x = \frac{4}{5}a$
vold.

$$O(V+W) = \int_0^{\frac{1}{5}a} 2\sqrt{x}\,\mathrm{d}x + \int_{\frac{1}{5}a}^{a} \sqrt{a-x}\,\mathrm{d}x = \int_0^{\frac{1}{5}a} 2x^{\frac{1}{2}}\,\mathrm{d}x + \int_{\frac{1}{5}a}^{a} (a-x)^{\frac{1}{2}}\,\mathrm{d}x = \left[2 \cdot \tfrac{2}{3}x^{1\frac{1}{2}}\right]_0^{\frac{1}{5}a} + \left[-\tfrac{2}{3}(a-x)^{1\frac{1}{2}}\right]_{\frac{1}{5}a}^{a}$$

$$= \left[\tfrac{4}{3}x\sqrt{x}\right]_0^{\frac{1}{5}a} + \left[-\tfrac{2}{3}(a-x)\sqrt{a-x}\right]_{\frac{1}{5}a}^{a} = \tfrac{4}{3} \cdot \tfrac{1}{5}a\sqrt{\tfrac{1}{5}a} - 0 + 0 - -\tfrac{2}{3} \cdot \tfrac{4}{5}a\sqrt{\tfrac{4}{5}a}$$

$$= \tfrac{4}{15}a\sqrt{\tfrac{1}{5}a} + \tfrac{16}{15}a\sqrt{\tfrac{1}{5}a} = \tfrac{4}{3}a\sqrt{\tfrac{1}{5}a}$$

$$O(W) = \int_0^{\frac{4}{5}a} \tfrac{1}{2}\sqrt{x}\,\mathrm{d}x + \int_{\frac{4}{5}a}^{a} \sqrt{a-x}\,\mathrm{d}x = \int_0^{\frac{4}{5}a} \tfrac{1}{2}x^{\frac{1}{2}}\,\mathrm{d}x + \int_{\frac{4}{5}a}^{a} (a-x)^{\frac{1}{2}}\,\mathrm{d}x = \left[\tfrac{1}{2} \cdot \tfrac{2}{3}x^{1\frac{1}{2}}\right]_0^{\frac{4}{5}a} + \left[-\tfrac{2}{3}\left(a-x\right)^{1\frac{1}{2}}\right]_{\frac{4}{5}a}^{a}$$

$$= \left[\tfrac{1}{3}x\sqrt{x}\right]_0^{\frac{4}{5}a} + \left[-\tfrac{2}{3}(a-x)\sqrt{a-x}\right]_{\frac{4}{5}a}^{a} = \tfrac{1}{3} \cdot \tfrac{4}{5}a\sqrt{\tfrac{4}{5}a} + 0 - -\tfrac{2}{3} \cdot \tfrac{1}{5}a\sqrt{\tfrac{1}{5}a}$$

$$= \tfrac{8}{15}a\sqrt{\tfrac{1}{5}a} + \tfrac{2}{15}a\sqrt{\tfrac{1}{5}a} = \tfrac{2}{3}a\sqrt{\tfrac{1}{5}a}$$

$O(W) = \frac{1}{2}O(V+W)$, dus $O(V) = O(W)$.

67 **a** $I(L) = \pi \int_0^p \left(\sqrt{x}\right)^2\,\mathrm{d}x = \pi \int_0^p x\,\mathrm{d}x = \pi\left[\tfrac{1}{2}x^2\right]_0^p = \tfrac{1}{2}\pi p^2$

b $y = \sqrt{x}$ geeft $y^2 = x$ ofwel $x = y^2$

$$I(M) = \pi \int_0^{\sqrt{p}} x^2\,\mathrm{d}y = \pi \int_0^{\sqrt{p}} y^4\,\mathrm{d}y = \pi\left[\tfrac{1}{5}y^5\right]_0^{\sqrt{p}} = \tfrac{1}{5}\pi \cdot p^2\sqrt{p}$$

c $I(L) = I(M)$ geeft $\frac{1}{2}\pi p^2 = \frac{1}{5}\pi p^2\sqrt{p}$
$\frac{1}{2} = \frac{1}{5}\sqrt{p}$
$\sqrt{p} = \frac{5}{2}$
$p = 6\frac{1}{4}$

Bladzijde 131

68 **a** $I(M) = 1\frac{2}{5}I(L)$ geeft $\left(21 - \dfrac{21}{a^2}\right)\pi = \frac{7}{5} \cdot 15\frac{3}{4}\pi$
$21 - \dfrac{21}{a^2} = 22\frac{1}{20}$
$\dfrac{21}{a^2} = -\frac{21}{20}$
$a^2 = -20$
Dit is dus niet mogelijk.

b $I(M) = 1\frac{3}{10}I(L)$ geeft $\left(21 - \dfrac{21}{a^2}\right)\pi = \frac{13}{10} \cdot 15\frac{3}{4}\pi$
$21 - \dfrac{21}{a^2} = 20\frac{19}{40}$
$\dfrac{21}{a^2} = \frac{21}{40}$
$a^2 = 40$
Dit is dus wel mogelijk.

69 **a** $f(x) = g_a(x)$ geeft $x^2 = a\sqrt{x}$
$x^{1\frac{1}{2}} = a$
$x = a^{\frac{2}{3}}$
$x = \sqrt[3]{a^2}$

$$O(V) = \int_0^{\sqrt[3]{a^2}} \left(a\sqrt{x} - x^2\right) \mathrm{d}x = \int_0^{\sqrt[3]{a^2}} \left(ax^{\frac{1}{2}} - x^2\right) \mathrm{d}x = \left[\tfrac{2}{3}ax^{1\frac{1}{2}} - \tfrac{1}{3}x^3\right]_0^{\sqrt[3]{a^2}} = \tfrac{2}{3}a^2 - \tfrac{1}{3}a^2 = \tfrac{1}{3}a^2$$

$O(V) = 10$ geeft $\frac{1}{3}a^2 = 10$
$a^2 = 30$
$a = \sqrt{30} \vee a = -\sqrt{30}$
vold. vold. niet
Dus $a = \sqrt{30}$.

b $$I(L) = \pi \int_0^{\sqrt[3]{a^2}} (a^2x - x^4)\,\mathrm{d}x = \pi\left[\tfrac{1}{2}a^2x^2 - \tfrac{1}{5}x^5\right]_0^{\sqrt[3]{a^2}} = \pi\left(\tfrac{1}{2}a^2 \cdot a^{1\frac{1}{3}} - \tfrac{1}{5}a^{3\frac{1}{3}}\right) = \tfrac{3}{10}\pi a^{3\frac{1}{3}}$$

$I(L) = 30\pi$ geeft $\frac{3}{10}\pi a^{3\frac{1}{3}} = 30\pi$
$a^{3\frac{1}{3}} = 100$
$a = 100^{\frac{3}{10}}$
$a = \sqrt[10]{1\,000\,000}$

Bladzijde 132

70 **a** Is de top van p_2 het punt $T(t, t^2)$, dan is het andere snijpunt van p_2 met de x-as $(2t, 0)$.
Dus p_2: $y = a(x - 0)(x - 2t) = ax(x - 2t)$.

$\left.\begin{array}{l} y = ax(x - 2t) \\ \text{door } (t, t^2) \end{array}\right\}$ $at(t - 2t) = t^2$
$-at^2 = t^2$
$a = -1$
Dus p_2: $y = -x(x - 2t)$ ofwel p_2: $y = -x^2 + 2tx$.

b $$O(V) = \int_0^t (-x^2 + 2tx - x^2)\mathrm{d}x = \int_0^t (-2x^2 + 2tx)\,\mathrm{d}x = \left[-\tfrac{2}{3}x^3 + tx^2\right]_0^t = -\tfrac{2}{3}t^3 + t^3 = \tfrac{1}{3}t^3$$

$O(V) = 10$ geeft $\frac{1}{3}t^3 = 10$
$t^3 = 30$
$t = \sqrt[3]{30}$

c $$I(L) = \pi \int_0^t ((-x^2 + 2tx)^2 - (x^2)^2)\mathrm{d}x = \pi \int_0^t (x^4 - 4tx^3 + 4t^2x^2 - x^4)\,\mathrm{d}x = \pi \int_0^t (-4tx^3 + 4t^2x^2)\,\mathrm{d}x$$
$$= \pi\left[-tx^4 + \tfrac{4}{3}t^2x^3\right]_0^t = \pi(-t^5 + \tfrac{4}{3}t^5 - 0) = \tfrac{1}{3}\pi t^5$$

$I(L) = 100\pi$ geeft $\frac{1}{3}\pi t^5 = 100\pi$
$t^5 = 300$
$t = \sqrt[5]{300}$

71 **a** $F(x) = x\ln^2(x) - 2x\ln(x) + 2x$ geeft

$$F'(x) = 1 \cdot \ln^2(x) + x \cdot 2\ln(x) \cdot \frac{1}{x} - \left(2\ln(x) + 2x \cdot \frac{1}{x}\right) + 2 = \ln^2(x) + 2\ln(x) - 2\ln(x) - 2 + 2 = \ln^2(x)$$

Dus F is een primitieve van $f(x) = \ln^2(x)$.

b $\mathrm{e}^x = a$ geeft $x = \ln(a)$, dus k: $x = \ln(a)$.

$$O(V) = \int_0^{\ln(a)} \mathrm{e}^x\,\mathrm{d}x = \left[\mathrm{e}^x\right]_0^{\ln(a)} = \mathrm{e}^{\ln(a)} - \mathrm{e}^0 = a - 1$$

$O(W) = a \cdot \ln(a) - O(V) = a\ln(a) - (a - 1) = a\ln(a) - a + 1$

$O(V) = O(W)$ geeft $a - 1 = a\ln(a) - a + 1$
$2a - 2 = a\ln(a)$
Voer in $y_1 = 2x - 2$ en $y_2 = x\ln(x)$.
Intersect geeft $x = 1$ en $x \approx 4{,}92$.
Dus $a \approx 4{,}92$.

15

c $I(L) = \pi \int_0^{\ln(a)} (e^x)^2 \, dx = \pi \int_0^{\ln(a)} e^{2x} \, dx = \pi \left[\tfrac{1}{2}e^{2x}\right]_0^{\ln(a)} = \pi\left(\tfrac{1}{2}e^{2\ln(a)} - \tfrac{1}{2}e^0\right) = \pi\left(\tfrac{1}{2}(e^{\ln(a)})^2 - \tfrac{1}{2}\right) = \tfrac{1}{2}\pi a^2 - \tfrac{1}{2}\pi$

$I(M) = \pi \cdot a^2 \cdot \ln(a) - I(L) = \pi a^2 \ln(a) - \left(\tfrac{1}{2}\pi a^2 - \tfrac{1}{2}\pi\right) = \pi a^2 \ln(a) - \tfrac{1}{2}\pi a^2 + \tfrac{1}{2}\pi$

$I(L) = I(M)$ geeft $\tfrac{1}{2}\pi a^2 - \tfrac{1}{2}\pi = \pi a^2 \ln(a) - \tfrac{1}{2}\pi a^2 + \tfrac{1}{2}\pi$

$\pi a^2 - \pi = \pi a^2 \ln(a)$

$a^2 - 1 = a^2 \ln(a)$

Voer in $y_1 = x^2 - 1$ en $y_2 = x^2 \ln(x)$.

Intersect geeft $x = 1$ en $x \approx 2{,}22$.

Dus $a \approx 2{,}22$.

d $y = e^x$ geeft $x = \ln(y)$

$I(N) = \pi \int_1^a x^2 \, dy = \pi \int_1^a \ln^2(y) \, dy = \pi\left[y\ln^2(y) - 2y\ln(y) + 2y\right]_1^a$

$= \pi(a\ln^2(a) - 2a\ln(a) + 2a - (\ln^2(1) - 2\ln(1) + 2)) = \pi(a\ln^2(a) - 2a\ln(a) + 2a - 2)$

$I(L) = 6 \cdot I(N)$ geeft $\tfrac{1}{2}\pi a^2 - \tfrac{1}{2}\pi = 6\pi(a\ln^2(a) - 2a\ln(a) + 2a - 2)$

$a^2 - 1 = 12(a\ln^2(a) - 2a\ln(a) + 2a - 2)$

Voer in $y_1 = x^2 - 1$ en $y_2 = 12(x\ln^2(x) - 2x\ln(x) + 2x - 2)$.

Intersect geeft $x \approx 0{,}51$, $x = 1$ en $x \approx 2{,}28$.

Dus $a \approx 2{,}28$.

Diagnostische toets

Bladzijde 134

1 $f(x) = x^4 - x^3 - 9x^2 - 5x$

$f'(x) = 4x^3 - 3x^2 - 18x - 5$

$f''(x) = 12x^2 - 6x - 18$

$f'(1) = -22 < 0$, dus de grafiek van f is dalend.

$f''(1) = -12 < 0$, dus de grafiek van f is toenemend dalend.

2 **a** $f_a(x) = (x + a)e^x$ geeft $f_a'(x) = 1 \cdot e^x + (x + a)e^x = (x + a + 1)e^x$

$f'(0) = 0$ geeft $(a + 1)e^0 = 0$, dus $a = -1$.

b $f_a'(x) = (x + a + 1)e^x$ geeft $f_a''(x) = 1 \cdot e^x + (x + a + 1)e^x = (x + a + 2)e^x$

$f_a'(5) < 0 \wedge f_a''(5) = 0$ geeft $(a + 6)e^5 < 0 \wedge (a + 7)e^5 = 0$

$a + 6 < 0 \wedge a + 7 = 0$

$a < -6 \wedge a = -7$

$a = -7$

3 $f_p(x) = \dfrac{px^2 + 2x + p}{x^2 + 1}$ geeft

$f_p'(x) = \dfrac{(x^2 + 1)(2px + 2) - (px^2 + 2x + p) \cdot 2x}{(x^2 + 1)^2} = \dfrac{2px^3 + 2x^2 + 2px + 2 - 2px^3 - 4x^2 - 2px}{(x^2 + 1)^2}$

$= \dfrac{-2x^2 + 2}{(x^2 + 1)^2}$

$f_p'(x) = 0$ geeft $-2x^2 + 2 = 0$

$x^2 = 1$

$x = 1 \vee x = -1$

Dus $A\left(1, \dfrac{2p + 2}{2}\right)$ en $B\left(-1, \dfrac{2p - 2}{2}\right)$ ofwel $A(1, p + 1)$ en $B(-1, p - 1)$.

$OA = \sqrt{5} \cdot OB$ geeft $\sqrt{1^2 + (p + 1)^2} = \sqrt{5} \cdot \sqrt{1^2 + (p - 1)^2}$

$\sqrt{p^2 + 2p + 2} = \sqrt{5} \cdot \sqrt{p^2 - 2p + 2}$

kwadrateren geeft

$p^2 + 2p + 2 = 5(p^2 - 2p + 2)$

$p^2 + 2p + 2 = 5p^2 - 10p + 10$

$4p^2 - 12p + 8 = 0$

$p^2 - 3p + 2 = 0$

$(p - 1)(p - 2) = 0$

$p = 1 \vee p = 2$

$p = 1$ geeft $OA = \sqrt{5}$ en $OB = 1$, dus voldoet.

$p = 2$ geeft $OA = \sqrt{10}$ en $OB = \sqrt{2}$, dus voldoet.

4 $f(x) = \frac{1}{x} = x^{-1}$ geeft $f'(x) = -x^{-2} = -\frac{1}{x^2}$

$g(x) = 1 + \ln(x)$ geeft $g'(x) = \frac{1}{x}$

k is de raaklijn van de grafiek van f in A.
$\text{rc}_k = f'(1) = -1$
l is de raaklijn van de grafiek van g in A.
$\text{rc}_l = g'(1) = 1$
$\tan(\alpha) = \text{rc}_k = -1$ geeft $\alpha = -45°$
$\tan(\beta) = g'(1) = 1$, dus $\beta = 45°$.
De hoek tussen de grafieken is $\alpha - \beta = 45° - -45° = 90°$.

5 **a** $f(x) = x^3 - 3x$ geeft $f'(x) = 3x^2 - 3$
$g_p(x) = px + 16$ geeft $g_p'(x) = p$
Raken, dus $f(x) = g_p(x) \wedge f'(x) = g_p'(x)$
$x^3 - 3x = px + 16 \wedge 3x^2 - 3 = p$
$p = 3x^2 - 3$ substitueren in $x^3 - 3x = px + 16$ geeft $x^3 - 3x = x(3x^2 - 3) + 16$
$x^3 - 3x = 3x^3 - 3x + 16$
$-2x^3 = 16$
$x^3 = -8$
$x = -2$
$x = -2$ geeft $p = 12 - 3 = 9$

b $f(x) = \sqrt{x^2 + 5}$ geeft $f'(x) = \frac{1}{2\sqrt{x^2+5}} \cdot 2x = \frac{x}{\sqrt{x^2+5}}$

$y = -1\frac{1}{2}x + 6$ geeft $\frac{dy}{dx} = -1\frac{1}{2}$

$f(x) = -1\frac{1}{2}x + 6 \wedge f'(x) \cdot -1\frac{1}{2} = -1$

$\sqrt{x^2+5} = -1\frac{1}{2}x + 6 \wedge \frac{x}{\sqrt{x^2+5}} \cdot -1\frac{1}{2} = -1$

$\sqrt{x^2+5} = -1\frac{1}{2}x + 6 \wedge \frac{x}{\sqrt{x^2+5}} = \frac{2}{3}$

$\sqrt{x^2+5} = -1\frac{1}{2}x + 6 \wedge \sqrt{x^2+5} = 1\frac{1}{2}x$

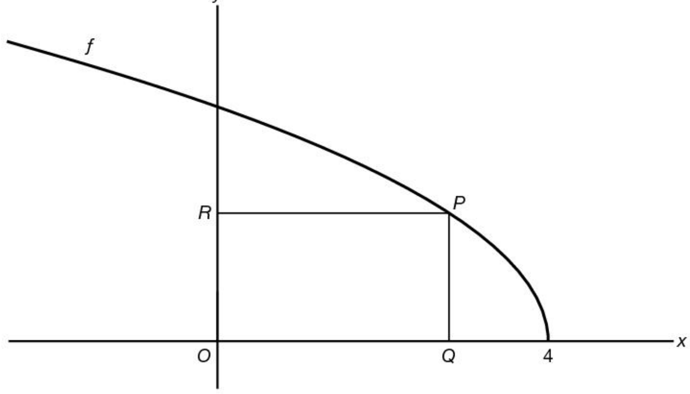
Hieruit volgt $-1\frac{1}{2}x + 6 = 1\frac{1}{2}x$
$-3x = -6$
$x = 2$
Dus de grafiek van f en de lijn snijden elkaar loodrecht.

6 **a** $P\left(p, \sqrt{8-2p}\right)$ geeft $L = \sqrt{p^2 + \left(\sqrt{8-2p}\right)^2} = \sqrt{p^2 - 2p + 8}$

b $L = \sqrt{p^2 - 2p + 8} = \sqrt{(p-1)^2 - 1 + 8} = \sqrt{(p-1)^2 + 7}$

$(p-1)^2 + 7$ is minimaal 7, dus L is minimaal $\sqrt{7}$.

c



$A(OQPR) = p \cdot f(p) = p\sqrt{8-2p}$

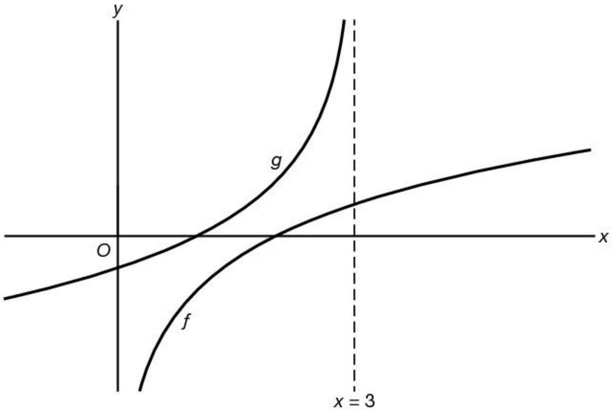
$\frac{dA}{dp} = 1 \cdot \sqrt{8-2p} + p \cdot \frac{-2}{2\sqrt{8-2p}} = \sqrt{8-2p} - \frac{p}{\sqrt{8-2p}}$

$$\frac{dA}{dp} = 0 \text{ geeft } \sqrt{8-2p} - \frac{p}{\sqrt{8-2p}} = 0$$
$$\sqrt{8-2p} = \frac{p}{\sqrt{8-2p}}$$
$$8 - 2p = p$$
$$-3p = -8$$
$$p = 2\tfrac{2}{3}$$
$$p = 2\tfrac{2}{3} \text{ geeft } A = 2\tfrac{2}{3}\sqrt{8-5\tfrac{1}{3}} = 2\tfrac{2}{3}\sqrt{2\tfrac{2}{3}} = \tfrac{8}{3}\sqrt{\tfrac{8}{3}} = \tfrac{8}{3}\cdot\tfrac{2}{3}\sqrt{6} = \tfrac{16}{9}\sqrt{6} = 1\tfrac{7}{9}\sqrt{6}$$

15

Bladzijde 135

7



$$L = AB = g(x) - f(x) = \ln\left(\frac{2}{3-x}\right) - \ln\left(\tfrac{1}{2}x\right) = \ln\left(\frac{2}{3-x}\cdot\frac{2}{x}\right) = \ln\left(\frac{4}{3x-x^2}\right) = \ln(4(3x-x^2)^{-1})$$
$$\frac{dL}{dx} = \frac{3x-x^2}{4}\cdot -4(3x-x^2)^{-2}\cdot(3-2x) = \frac{2x-3}{3x-x^2}$$
$$\frac{dL}{dx} = 0 \text{ geeft } 2x - 3 = 0, \text{ dus } x = 1\tfrac{1}{2}.$$
$$x = 1\tfrac{1}{2} \text{ geeft } L = \ln\left(\frac{4}{4\frac{1}{2}-2\frac{1}{4}}\right) = \ln\left(\frac{4}{2\frac{1}{4}}\right) = \ln\left(\tfrac{16}{9}\right) = \ln\left(1\tfrac{7}{9}\right)$$

Dus de minimale lengte van AB is $\ln\left(1\tfrac{7}{9}\right)$.

8 **a** $AB = x$ geeft $BC = 6 - x$

De stelling van Pythagoras geeft $AC^2 + x^2 = (6-x)^2$
$$AC^2 + x^2 = 36 - 12x + x^2$$
$$AC^2 = 36 - 12x$$
$$AC = \sqrt{36-12x}$$
$O(\triangle ABC) = \tfrac{1}{2}\cdot x\cdot\sqrt{36-12x}$

b $A = O(\triangle ABC) = x\cdot\sqrt{9-3x}$ geeft $\frac{dA}{dx} = 1\cdot\sqrt{9-3x} + x\cdot\frac{-3}{2\sqrt{9-3x}} = \sqrt{9-3x} - \frac{3x}{2\sqrt{9-3x}}$

$$\frac{dA}{dx} = 0 \text{ geeft } \sqrt{9-3x} - \frac{3x}{2\sqrt{9-3x}} = 0$$
$$\sqrt{9-3x} = \frac{3x}{2\sqrt{9-3x}}$$
$$2(9-3x) = 3x$$
$$18 - 6x = 3x$$
$$-9x = -18$$
$$x = 2$$

Dus $O(\triangle ABC) = 2\sqrt{9-6} = 2\sqrt{3}$.

9 **a** $\sin(\alpha) = \frac{AS}{5}$ geeft $AS = 5\sin(\alpha)$

$\cos(\alpha) = \frac{BS}{5}$ geeft $BS = 5\cos(\alpha)$

De stelling van Pythagoras in $\triangle ASD$ geeft $DS^2 = 6^2 - (5\sin(\alpha))^2$
$$DS^2 = 36 - 25\sin^2(\alpha)$$
$$DS = \sqrt{36 - 25\sin^2(\alpha)}$$

$BD = BS + DS = 5\cos(\alpha) + \sqrt{36-25\sin^2(\alpha)}$

b De oppervlakte van $ABCD$ is $2 \cdot O(\triangle ABD) = 2 \cdot \frac{1}{2} \cdot \left(5\cos(\alpha) + \sqrt{36 - 25\sin^2(\alpha)}\right) \cdot 5\sin(\alpha)$

Voer in $y_1 = \left(5\cos(x) + \sqrt{36 - 25\sin^2(x)}\right) \cdot 5\sin(x)$.
De optie maximum geeft $x = 50{,}194...°$.
Dus de waarde van α waarvoor de oppervlakte van $ABCD$ maximaal is, is 50°.

Alternatieve uitwerking
De oppervlakte van vierhoek $ABCD$ is maximaal als de oppervlakte van $\triangle ABD$ maximaal is.
Dat is het geval als de hoogte die bij de basis AB hoort maximaal is. En dat is het geval als $\angle A = 90°$.

Dit geeft $\tan(\alpha) = \frac{AD}{AB} = \frac{6}{5}$ en hieruit volgt $\alpha \approx 50°$.

15

10 **a** $f(x) = \frac{5 - x}{x + 3} = \frac{-(x + 3) + 3 + 5}{x + 3} = -1 + \frac{8}{x + 3}$ geeft $F(x) = -x + 8\ln|x + 3| + c$

b $g(x) = \frac{3x}{2x + 3} = \frac{1\frac{1}{2}(2x + 3) - 4\frac{1}{2}}{2x + 3} = 1\frac{1}{2} - \frac{4\frac{1}{2}}{2x + 3}$ geeft

$G(x) = 1\frac{1}{2}x - \frac{1}{2} \cdot 4\frac{1}{2}\ln|2x + 3| + c = 1\frac{1}{2}x - 2\frac{1}{4}\ln|2x + 3| + c$

11 **a** $y = 4\ln(x)$ geeft $\ln(x) = \frac{1}{4}y$

$x = e^{\frac{1}{4}y}$

$O(V) = \int_0^p e^{\frac{1}{4}y}\,dy = \left[4e^{\frac{1}{4}y}\right]_0^p = 4e^{\frac{1}{4}y} - 4$

$O(V) = O(W)$ ofwel $O(V) = \frac{1}{2}p^2$ geeft $4e^{\frac{1}{4}p} - 4 = \frac{1}{2}p^2$

Voer in $y_1 = 4e^{\frac{1}{4}x} - 4$ en $y_2 = \frac{1}{2}x^2$.

Intersect geeft $x = 2{,}9634...$, dus $p \approx 2{,}96$.

Alternatieve uitwerking
$f(x) = 0$ geeft $x = 1$
$f(x) = p$ geeft $4\ln(x) = p$

$\ln(x) = \frac{1}{4}p$

$x = e^{\frac{1}{4}p}$

$O(V) = 1 \cdot p + \int_1^{e^{\frac{1}{4}p}} (p - 4\ln(x))\,dx = p + \left[px - 4x\ln(x) + 4x\right]_1^{e^{\frac{1}{4}p}}$

$= p + pe^{\frac{1}{4}p} - 4e^{\frac{1}{4}p}\ln\left(e^{\frac{1}{4}p}\right) + 4e^{\frac{1}{4}p} - (p - 0 + 4) = p + pe^{\frac{1}{4}p} - 4e^{\frac{1}{4}p} \cdot \frac{1}{4}p + 4e^{\frac{1}{4}p} - p - 4$

$= 4e^{\frac{1}{4}p} - 4$

$O(V) = O(W)$ ofwel $O(V) = \frac{1}{2}p^2$ geeft $4e^{\frac{1}{4}p} - 4 = \frac{1}{2}p^2$

Voer in $y_1 = 4e^{\frac{1}{4}x} - 4$ en $y_2 = \frac{1}{2}x^2$.
Intersect geeft $x = 2{,}9634...$, dus $p \approx 2{,}96$.

b $y = 4\ln(x)$ geeft $\ln(x) = \frac{1}{4}y$

$x = e^{\frac{1}{4}y}$
$x^2 = e^{\frac{1}{2}y}$

$I(L) = \pi\int_0^p x^2\,dy = \pi\int_0^p e^{\frac{1}{2}y}dy = \pi\left[2e^{\frac{1}{2}y}\right]_0^p = \pi\left(2e^{\frac{1}{2}p} - 2\right)$

$I(L) = I(M)$ ofwel $I(L) = \frac{1}{2} \cdot I(\text{cilinder})$ geeft $\pi\left(2e^{\frac{1}{2}p} - 2\right) = \frac{1}{2}\pi p^2 \cdot p$

$2e^{\frac{1}{2}p} - 2 = \frac{1}{2}p^3$

Voer in $y_1 = 2e^{\frac{1}{2}x} - 2$ en $y_2 = \frac{1}{2}x^3$.
Intersect geeft $x = -1{,}223...$, $x = 0$ en $x = 1{,}801...$, dus $p \approx 1{,}80$.

16 Examentraining

16.1 Algemene vaardigheden

Bladzijde 147

1 $f_b(x) = \frac{x-b}{x^2-b^2} = \frac{x-b}{(x+b)(x-b)} = \frac{1}{x+b}$ met $x \neq b$

$\lim_{x \to b} \frac{1}{x+b} = \frac{1}{2b}$

Dus de perforatie is $\left(b, \frac{1}{2b}\right)$.

$\left(b, \frac{1}{2b}\right)$ op de lijn $y = 4x + 1$ geeft $4b + 1 = \frac{1}{2b}$

$$8b^2 + 2b = 1$$
$$8b^2 - 2b - 1 = 0$$
$$(2b+1)(4b-1) = 0$$
$$b = -\tfrac{1}{2} \vee b = \tfrac{1}{4}$$

Bladzijde 148

2 Stel x de lengte van de zijde van vierkant A.
Dan is de zijde van vierkant B gelijk aan $30 - x$ en de lengte van de zijde van vierkant C gelijk aan $20 - (30 - x) = x - 10$.
De oppervlakte van vierkant D is
$30 \cdot 20 - x^2 - (30-x)^2 - (x-10)^2 = 600 - x^2 - 900 + 60x - x^2 - x^2 + 20x - 100 = -3x^2 + 80x - 400$
Deze oppervlakte is maximaal voor $x = -\frac{b}{2a} = -\frac{80}{-6} = 13\tfrac{1}{3}$.
Dus de gevraagde lengte is $13\tfrac{1}{3}$.

3 **a** Stel $PQ = x$.
Dan is $AP + BQ = 2 - x$ en dit geeft $AP = 1 - \tfrac{1}{2}x$ en $AQ = 1 - \tfrac{1}{2}x + x = 1 + \tfrac{1}{2}x$.
De stelling van Pythagoras in $\triangle AQR$ geeft $AQ^2 + QR^2 = AR^2$

$$\left(1 + \tfrac{1}{2}x\right)^2 + x^2 = 2^2$$
$$1 + x + \tfrac{1}{4}x^2 + x^2 = 4$$
$$\tfrac{5}{4}x^2 + x - 3 = 0$$
$$D = 1^2 - 4 \cdot \tfrac{5}{4} \cdot -3 = 16$$
$$x = \frac{-1+4}{\frac{5}{2}} = \tfrac{6}{5} \vee x = \frac{-1-4}{\frac{5}{2}} = -2$$

vold. — vold. niet

Dus $PQ = \tfrac{6}{5}$.

b Teken MT loodrecht op AB met T op AB.
$AT = 1$, $AM = 2 - r$ en $MT = \tfrac{6}{5} + r$
De stelling van Pythagoras in $\triangle ATM$ geeft $AT^2 + TM^2 = AM^2$

$$1^2 + \left(\tfrac{6}{5} + r\right)^2 = (2-r)^2$$
$$1 + \tfrac{36}{25} + \tfrac{12}{5}r + r^2 = 4 - 4r + r^2$$
$$\tfrac{32}{5}r = \tfrac{39}{25}$$
$$r = \tfrac{39}{160}$$

Bladzijde 149

4 **a** Stel de straal van de cirkel is x.
Dan is $BP = BQ = x$, $AP = AR = 4 - x$ en $CQ = CR = 3 - x$.
De stelling van Pythagoras in $\triangle ABC$ geeft $AC^2 = AB^2 + BC^2$

$$AC^2 = 4^2 + 3^2$$
$$AC^2 = 25$$
$$AC = 5$$

$AC = 5$ geeft $AR + CR = 5$

$$4 - x + 3 - x = 5$$
$$-2x = -2$$
$$x = 1$$

Dus de straal van de ingeschreven cirkel van $\triangle ABC$ is 1.

b $\left.\begin{array}{l}\angle UAN = \angle PAM \\ \angle AUN = \angle APM\ (= 90°)\end{array}\right\} \triangle AUN \backsim \triangle APM$

Uit $\triangle AUN \backsim \triangle APM$ volgt $\dfrac{AU}{AP} = \dfrac{UN}{PM}$

$$\frac{AU}{4-1} = \frac{r}{1}$$
$$AU = 3r$$

c $NT = UP = AP - AU = 3 - 3r$

De stelling van Pythagoras in $\triangle NTM$ geeft $NT^2 + MT^2 = MN^2$

$$(3 - 3r)^2 + (1 - r)^2 = (r + 1)^2$$
$$9 - 18r + 9r^2 + 1 - 2r + r^2 = r^2 + 2r + 1$$
$$9r^2 - 22r + 9 = 0$$
$$D = (-22)^2 - 4 \cdot 9 \cdot 9 = 160$$
$$r = \frac{22 + \sqrt{160}}{18} \approx 1{,}92 \vee r = \frac{22 - \sqrt{160}}{18} \approx 0{,}52$$

vold. niet   vold.

Dus $r \approx 0{,}52$.

Bladzijde 150

5 **a** De stelling van Pythagoras in de getekende driehoek geeft

$$(5 - h)^2 + \left(\tfrac{1}{2}d\right)^2 = 5^2$$
$$(5 - h)^2 + \tfrac{1}{4}d^2 = 25$$
$$(5 - h)^2 = 25 - \tfrac{1}{4}d^2$$
$$(5 - h)^2 = \frac{100 - d^2}{4}$$
$$5 - h = \sqrt{\frac{100 - d^2}{4}} \vee 5 - h = -\sqrt{\frac{100 - d^2}{4}}$$
$$h = 5 - \frac{\sqrt{100 - d^2}}{2} \vee h = 5 + \frac{\sqrt{100 - d^2}}{2}$$
$$h = \frac{10 - \sqrt{100 - d^2}}{2} \vee h = \frac{10 + \sqrt{100 - d^2}}{2}$$

vold.   vold. niet

Dus $h = \dfrac{10 - \sqrt{100 - d^2}}{2}$.

b $340 = 0{,}102 \cdot \dfrac{29\,400}{10\pi h}$

$$10\pi h = \frac{0{,}102 \cdot 29\,400}{340} = 8{,}82$$
$$h = 0{,}280...$$

$\left.\begin{array}{l} h = 0{,}280... \\ h = \dfrac{10 - \sqrt{100 - d^2}}{2}\end{array}\right\} 10 - \sqrt{100 - d^2} = 2 \cdot 0{,}208..$

$$\sqrt{100 - d^2} = 9{,}438...$$
$$100 - d^2 = 89{,}085...$$
$$d^2 = 10{,}914...$$
$$d = 3{,}30...$$

Dus $d \approx 3{,}3$ mm.

6 $f_a(x) = \dfrac{4x^2 - 10x + 4}{2x - a} = \dfrac{(2x-1)(2x-4)}{2x - a}$

Er is dus een perforatie als $a = 1$ of als $a = 4$.

$f_4(x) = \dfrac{(2x-1)(2x-4)}{2x-4} = 2x - 1$ met $x \neq 2$.

De perforatie is (2, 3).

Bladzijde 151

7 **a** Uit $\triangle AMD \backsim \triangle AOB$ volgt $\dfrac{MD}{OB} = \dfrac{AM}{AO}$

$$\frac{r}{1} = \frac{a - 1 - r}{a}$$
$$ar = a - 1 - r$$
$$ar + r = a - 1$$
$$r(a+1) = a - 1$$
$$r = \frac{a-1}{a+1}$$

b $OCAB$ is een vierkant, dus $AB = OB = 1$ en $OA = \sqrt{2}$.

Dus $a = \sqrt{2}$. Dit geeft $r = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} \cdot \dfrac{\sqrt{2}-1}{\sqrt{2}-1} = \dfrac{2 - 2\sqrt{2} + 1}{2 - 1} = 3 - 2\sqrt{2}$.

Dus $p = 3$ en $q = -2$.

16.2 Differentiaal- en integraalrekening

Bladzijde 156

8 **a** De optie voor de integraal op de GR geeft $\pi \displaystyle\int_{0,0}^{55,3} (4{,}5 + 28{,}0 \cdot e^{-0{,}452x})^2 \, dx = 7994{,}1...$

Dus het volume is ongeveer 7994 cm.

b Stel $y = a(x - b)^2 + c$.

De top is C, dus $y = a(x - 87{,}5)^2 + 32{,}5$ door $D(155{,}0; 23{,}0)$ $\Big\}$ $a(155{,}0 - 87{,}5)^2 + 32{,}5 = 23{,}0$

$$4556{,}25a = -9{,}5$$
$$a = -0{,}00208...$$

Dus $y = -0{,}002(x - 87{,}5)^2 + 32{,}5$.

c 50 mL = 50 000 mm^3

$$\pi \int_{55,3}^{p} (g(x))^2 \, dx = \pi \int_{55,3}^{p} (-x^2 + 175x - 6600) \, dx = \left[-\tfrac{1}{3}x^3 + 87{,}5x^2 - 6600x\right]_{55,3}^{p}$$
$$= -\tfrac{1}{3}p^3 + 87{,}5p^2 - 6600p - \left(-\tfrac{1}{3} \cdot 55{,}3^3 + 87{,}5 \cdot 55{,}3^2 - 6600 \cdot 55{,}3\right)$$

Inhoud = 50 mL geeft $-\tfrac{1}{3}p^3 + 87{,}5p^2 - 6600p - \left(-\tfrac{1}{3} \cdot 55{,}3^3 + 87{,}5 \cdot 55{,}3^2 - 6600 \cdot 55{,}3\right) = 50\,000$

Voer in $y_1 = \tfrac{1}{3}x^3 + 87{,}5x^2 - 6600x - \left(-\tfrac{1}{3} \cdot 55{,}3^3 + 87{,}5 \cdot 55{,}3^2 - 6600 \cdot 55{,}3\right)$ en $y_2 = 50\,000$.

Intersect geeft $p \approx 81$, dus de x-coördinaat van P is ongeveer 81.

Bladzijde 157

9 **a** $V = \pi \displaystyle\int_0^h (r(x))^2 \, dx = \pi \int_0^h \tfrac{1}{100}(5 + 15x - 15x^2) \, dx = \pi\left[\tfrac{1}{100}\left(5x + 7\tfrac{1}{2}x^2 - 5x^3\right)\right]_0^h$

$$= \frac{\pi}{100}\left(5h + 7\tfrac{1}{2}h^2 - 5h^3\right) - 0 = \frac{\pi}{40}(2h + 3h^2 - 2h^3)$$

b $h = 1$ geeft $\dfrac{\pi}{40}(2 + 3 - 2) = \tfrac{3}{40}\pi$

$\tfrac{3}{4} \cdot \tfrac{3}{40}\pi = \tfrac{9}{160}\pi$

$\dfrac{\pi}{40}(2h + 3h^2 - 2h^3) = \tfrac{9}{160}\pi$ geeft $2h + 3h^2 - 2h^3 = \tfrac{9}{4}$

Voer in $y_1 = 2x + 3x^2 - 2x^3$ en $y_2 = \tfrac{9}{4}$.

Intersect geeft $x = 0{,}720...$ (vold.) en $x = 1{,}698...$ (vold. niet)

Dus de hoogte is ongeveer 72 cm.

10 $f(x) = \dfrac{x^2 - 4}{x^3 + 2x^2} = \dfrac{(x+2)(x-2)}{x^2(x+2)} = \dfrac{x-2}{x^2}$ met $x \neq -2$

$\lim\limits_{x \to -2} \dfrac{x-2}{x^2} = \dfrac{-2-2}{(-2)^2} = 1$

Dus de perforatie is $(-2, -1)$.

$f(x) = 0$ geeft $x = 2$

$f(x) = \dfrac{x-2}{x^2}$ geeft $f'(x) = \dfrac{x^2 \cdot 1 - (x-2) \cdot 2x}{x^4} = \dfrac{x - 2(x-2)}{x^3} = \dfrac{-x+4}{x^3}$

Stel de raaklijn door $(2, 0)$ is $y = ax + b$ met $a = f'(2) = \dfrac{-2+4}{2^3} = \frac{1}{4}$

$\left.\begin{array}{l} y = \frac{1}{4}x + b \\ \text{door } (2, 0) \end{array}\right\} \begin{array}{l} \frac{1}{4} \cdot 2 + b = 0 \\ b = -\frac{1}{2} \end{array}$

Dus de raaklijn is $y = \frac{1}{4}x - \frac{1}{2}$.

Substitutie van $(-2, -1)$ in deze vergelijking geeft $\frac{1}{4} \cdot -2 - \frac{1}{2} = -1$ ofwel $-1 = -1$ en dit klopt, dus de raaklijn gaat door de perforatie.

16

11 $f(x) = -x^3 + 3px^2$ geeft $f'(x) = -3x^2 + 6px$

$f'(x) = 0$ geeft $-3x^2 + 6px = 0$

$-3x(x - 2p) = 0$

$x = 0 \vee x = 2p$

$f(2p) = -(2p)^3 + 3p \cdot (2p)^2 = -8p^3 + 12p^3 = 4p^3$, dus $T(2p, 4p^3)$.

$f(x) = 0$ geeft $-x^3 + 3px^2 = 0$

$-x^2(x - 3p) = 0$

$x = 0 \vee x = 3p$

Dus $A(3p, 0)$.

De oppervlakte van rechthoek $OABC$ is $3p \cdot 4p^3 = 12p^4$.

De oppervlakte van het grijze gebied is

$\int_0^{3p} (-x^3 + 3px^2)\,dx = \left[-\frac{1}{4}x^4 + px^3\right]_0^{3p} = -\frac{1}{4} \cdot 81p^4 + p \cdot 27p^3 = 6\frac{3}{4}p^4$.

De verhouding van de twee oppervlakten is $12p^4 : 6\frac{3}{4}p^4 = 16 : 9$ en dit is onafhankelijk van p.

Bladzijde 159

12 **a** $\left.\begin{array}{l} \angle OPA = \angle Q'PQ \\ \angle POA = \angle PQ'Q\ (= 90°) \end{array}\right\} \triangle POA \backsim \triangle PQ'Q$

Uit $\triangle POA \backsim \triangle PQ'Q$ volgt $\dfrac{PO}{PQ'} = \dfrac{PA}{PQ}$

$\dfrac{p}{p+q} = \dfrac{\sqrt{p^2 + 35^2}}{280}$

$280p = p\sqrt{p^2 + 1225} + q\sqrt{p^2 + 1225}$

$q\sqrt{p^2 + 1225} = 280p - p\sqrt{p^2 + 1225}$

$q = \dfrac{280p}{\sqrt{p^2 + 1225}} - p$

b $q = \dfrac{280p}{\sqrt{p^2 + 1225}} - p$ geeft

$q' = \dfrac{\sqrt{p^2 + 1225} \cdot 280 - 280p \cdot \dfrac{1}{2\sqrt{p^2 + 1225}} \cdot 2p}{\left(\sqrt{p^2 + 1225}\right)^2} - 1$

$= \dfrac{280(p^2 + 1225) - 280p^2}{(p^2 + 1225)\left(\sqrt{p^2 + 1225}\right)} - 1 = \dfrac{280p^2 + 343\,000 - 280p^2}{(p^2 + 1225)\left(\sqrt{p^2 + 1225}\right)} - 1 = \dfrac{343\,000}{(p^2 + 1225)\left(\sqrt{p^2 + 1225}\right)} - 1$

c $q'(p) = 0$ geeft $\frac{343\,000}{(p^2+1225)\left(\sqrt{p^2+1225}\right)} = 1$

$(p^2 + 1225)^{1\frac{1}{2}} = 343\,000$
$p^2 + 1225 = 343\,000^{\frac{2}{3}}$
$p^2 + 1225 = 4900$
$p^2 = 3675$
$p = \sqrt{3675}$

$$\left.\begin{array}{l} q = \dfrac{280p}{\sqrt{p^2+1225}} - p \\ p = \sqrt{3675} \end{array}\right\} q = \frac{280\sqrt{3675}}{\sqrt{4900}} - \sqrt{3675} = 4\sqrt{3675} - \sqrt{3675} = 3\sqrt{3675}$$

Dus het maximum van q is $3\sqrt{3675}$.

16

13 **a** $f(x) = 0$ geeft $\frac{1}{6}\sqrt{87x - 3x^2 - 2x^3} = 0$
$87x - 3x^2 - 2x^3 = 0$
$-x(2x^2 + 3x - 87) = 0$
$x = 0 \vee 2x^2 + 3x - 87 = 0$
$x = 0 \vee D = 3^2 - 4 \cdot 2 \cdot -87 = 705$
$x = 0 \vee x = \frac{-3+\sqrt{705}}{4} = 5{,}88... \vee x = \frac{-3-\sqrt{705}}{4} = -7{,}38...$

Dus de lengte van het ei is ongeveer 5,9 cm.

b $\pi\int_0^{5,9} \frac{1}{36}(87x - 3x^2 - 2x^3)\,dx = \frac{1}{36}\pi\left[43\frac{1}{2}x^2 - x^3 - \frac{1}{2}x^4\right]_0^{5,9} = \frac{1}{36}\pi\left(43\frac{1}{2} \cdot 5{,}9^2 - 5{,}9^3 - \frac{1}{2} \cdot 5{,}9^4\right) \approx 61$

De gevraagde inhoud is ongeveer 61 cm^3.

14 $f(x) = 2x^2 - px^4$ geeft $f'(x) = 4x - 4px^3$
$f'(x) = 0$ geeft $4x - 4px^3 = 0$
$4x(1 - px^2) = 0$
$x = 0 \vee x^2 = \frac{1}{p}$
$x = 0 \vee x = \sqrt{\frac{1}{p}} \vee x = -\sqrt{\frac{1}{p}}$

Uit de symmetrie volgt $OB = OA$, dus $OA = AB = OB$.
Volgens een eigenschap van de halve gelijkzijdige driehoek moet gelden
$y_A = \sqrt{3} \cdot x_A$
$2 \cdot \frac{1}{p} - p \cdot \frac{1}{p^2} = \sqrt{3} \cdot \sqrt{\frac{1}{p}}$
$\frac{2}{p} - \frac{1}{p} = \sqrt{3} \cdot \sqrt{\frac{1}{p}}$
$\frac{1}{p} = \sqrt{3} \cdot \sqrt{\frac{1}{p}}$
$\sqrt{\frac{1}{p}} = \sqrt{3}$
$\frac{1}{p} = 3$
$p = \frac{1}{3}$

Bladzijde 160

15 **a** $v = 75$ geeft $\frac{100p^3}{p^3 + 25\,000} = 75$

$100p^3 = 75p^3 + 1\,875\,000$

$25p^3 = 1\,875\,000$

$p^3 = 75\,000$

$p = \sqrt[3]{75\,000}$

$p = 42{,}1...$

Dus de partiële zuurstofdruk is ongeveer 42 mmHg.

b $v = \frac{100p^3}{p^3 + 25\,000}$ geeft

$$\frac{dv}{dp} = \frac{(p^3 + 25\,000) \cdot 300p^2 - 100p^3 \cdot 3p^2}{(p^3 + 25\,000)^2} = \frac{(p^3 + 25\,000) \cdot 300p^2 - 100p^3 \cdot 3p^2}{(p^3 + 25\,000)^2} = \frac{7\,500\,000p^2}{(p^3 + 25\,000)^2}$$

Voer in $y_1 = \frac{7\,500\,000x^2}{(x^3 + 25\,000)^2}$.

De optie maximum geeft $x = 23{,}2...$ en $y = 2{,}87...$

Dus de grafiek is het steilst voor p is ongeveer 23.

c $\frac{v}{100 - v} = 0{,}00004p^3$ geeft $v = 0{,}00004p^3(100 - v)$

$v = 0{,}004p^3 - 0{,}00004p^3v$

$v + 0{,}00004p^3v = 0{,}004p^3$

$v(1 + 0{,}00004p^3) = 0{,}004p^3$

$v = \frac{0{,}004p^3}{1 + 0{,}00004p^3}$

$v = \frac{100p^3}{25\,000 + p^3}$

$v = \frac{100p^3}{p^3 + 25\,000}$

Bladzijde 161

16 **a** $f(x) = (x^2 - 1)\left(x - 1\frac{1}{2}\right) = x^3 - 1\frac{1}{2}x^2 - x + 1\frac{1}{2}$ geeft $f'(x) = 3x^2 - 3x - 1$

$g(x) = -x + 1\frac{1}{2}$ geeft $g'(x) = -1$

$f'(0) = -1$ en $g'(0) = -1$, dus $f'(0) = g'(0)$, zodat de grafieken van f en g elkaar raken in A.

b $f(x) = 0$ geeft $(x^2 - 1)\left(x - 1\frac{1}{2}\right) = 0$

$x = -1 \vee x = 1 \vee x = 1\frac{1}{2}$

De oppervlakte van het linkerdeel is $\int_0^1 f(x)\,dx = \int_0^1 (x^2 - 1)\left(x - 1\frac{1}{2}\right) dx = \int_0^1 \left(x^3 - 1\frac{1}{2}x^2 - x + 1\frac{1}{2}\right) dx$

$= \left[\frac{1}{4}x^4 - \frac{1}{2}x^3 - \frac{1}{2}x^2 + 1\frac{1}{2}x\right]_0^1 = \frac{1}{4} - \frac{1}{2} - \frac{1}{2} + 1\frac{1}{2} = \frac{3}{4}$.

De oppervlakte van het rechterdeel is $O(\triangle OAB) - \frac{3}{4} = \frac{1}{2} \cdot 1\frac{1}{2} \cdot 1\frac{1}{2} - \frac{3}{4} = 1\frac{1}{2}$.

$1\frac{1}{2} = 2 \cdot \frac{3}{4}$, dus de oppervlakte van het linkerdeel is twee keer zo groot als de oppervlakte van het rechterdeel.

c $h(x) = \frac{g(x)}{f(x)} = \frac{-x + 1\frac{1}{2}}{(x^2 - 1)\left(x - 1\frac{1}{2}\right)} = \frac{-\left(x - 1\frac{1}{2}\right)}{(x^2 - 1)\left(x - 1\frac{1}{2}\right)} = \frac{-1}{x^2 - 1}$ mits $x \neq 1\frac{1}{2}$

$\lim_{x \to 1\frac{1}{2}} h(x) = \lim_{x \to 1\frac{1}{2}} \frac{-1}{x^2 - 1} = \frac{-1}{2\frac{1}{4} - 1} = -\frac{4}{5}$

Dus de perforatie is $\left(1\frac{1}{2}, -\frac{4}{5}\right)$.

horizontale asymptoot:

$\lim_{x \to \infty} h(x) = \lim_{x \to \infty} \frac{-1}{x^2 - 1} = 0$ en $\lim_{x \to -\infty} h(x) = 0$

Dus de lijn $y = 0$ is horizontale asymptoot.

verticale asymptoot:

$(x^2 - 1)\left(x - 1\frac{1}{2}\right) = 0 \wedge -x + 1\frac{1}{2} \neq 0$

$x^2 - 1 = 0$

$x = 1 \vee x = -1$

Dus de verticale asymptoten zijn de lijnen $x = -1$ en $x = 1$.

17 **a** $I = \pi \int_0^h \left(\sqrt{22x - x^2}\right)^2 \mathrm{d}x = \pi \int_0^h (22x - x^2)\,\mathrm{d}x = \pi\left[11x^2 - \frac{1}{3}x^3\right]_0^h = \pi\left(11h^2 - \frac{1}{3}h^3\right) - 0 = \pi h^2\left(11 - \frac{1}{3}h\right)$

b $I = 425$ geeft $\pi h^2\left(11 - \frac{1}{3}h\right) = 425$

Voer in $y_1 = \pi x^2\left(11 - \frac{1}{3}x\right)$ en $y_2 = 425$.
Intersect geeft $x = 3{,}72...$
Dus de bal ligt ongeveer 37 mm diep in het water.

Bladzijde 162

18 **a** $f''(x) = 12(x - p)(x + p) = 12(x^2 - p^2) = 12x^2 - 12p^2$ geeft $f'(x) = 4x^3 - 12p^2x + a$

$f'(x) = 4x^3 - 12p^2x + a$ geeft $f(x) = x^4 - 6p^2x^2 + ax + b$

b $f_1(x) = -8x$ geeft $x^4 - 6x^2 - 8x + 5 = -8x$

$x^4 - 6x^2 + 5 = 0$

$(x^2 - 1)(x^2 - 5) = 0$

$x = 1 \vee x = -1 \vee x = \sqrt{5} \vee x = -\sqrt{5}$

Dus de x-coördinaten van de andere punten zijn $\sqrt{5}$ en $-\sqrt{5}$.

c $O(V_2) = \int_{-1}^{1} (f_1(x) - -8x)\,\mathrm{d}x = \int_{-1}^{1} (x^4 - 6x^2 - 8x + 5 + 8x)\,\mathrm{d}x = \int_{-1}^{1} (x^4 - 6x^2 + 5)\mathrm{d}x$

$= \left[\frac{1}{5}x^5 - 2x^3 + 5x\right]_{-1}^{1} = \frac{1}{5} - 2 + 5 - \left(-\frac{1}{5} + 2 - 5\right) = 6\frac{2}{5}$

$6\frac{2}{5} = 2 \cdot 3\frac{1}{5}$, dus de gezamenlijke oppervlakte van V_1 en V_3 is gelijk aan de oppervlakte van V_2.

19 **a** Voor de functie f geldt $y = 4 - \dfrac{4}{x + 1}$, dus voor f^{inv} geldt $x = 4 - \dfrac{4}{y + 1}$

$x - 4 = \dfrac{-4}{y + 1}$

$y + 1 = \dfrac{-4}{x - 4}$

$y = \dfrac{-4}{x - 4} - 1 = \dfrac{-4 - (x - 4)}{x - 4} = \dfrac{-x}{x - 4} = \dfrac{x}{4 - x}$

Dus f en g zijn elkaars inverse.

b $\int_0^3 (f(x) - x)\,\mathrm{d}x = \int_0^3 \left(4 - \dfrac{4}{x + 1} - x\right)\mathrm{d}x = \left[4x - 4\ln|x + 1| - \frac{1}{2}x^2\right]_0^3$

$= 12 - 4\ln(4) - 4\frac{1}{2} - 0 = 7\frac{1}{2} - 4\ln(4)$

De gevraagde oppervlakte is $2\left(7\frac{1}{2} - 4\ln(4)\right) = 15 - 8\ln(4)$.

Bladzijde 163

20 **a** $\int_0^a (f(x) - g(x))\,\mathrm{d}x = \int_a^4 g(x)\,\mathrm{d}x$

$\int_0^a \left(\sqrt{x} - \frac{1}{2}\sqrt{x}\right)\mathrm{d}x = \int_a^4 \frac{1}{2}\sqrt{x}\,\mathrm{d}x$

$\int_0^a \left(\frac{1}{2}x^{\frac{1}{2}}\right)\mathrm{d}x = \int_a^4 \frac{1}{2}x^{\frac{1}{2}}\,\mathrm{d}x$

$\left[\frac{1}{3}x^{1\frac{1}{2}}\right]_0^a = \left[\frac{1}{3}x^{1\frac{1}{2}}\right]_a^4$

$\left[\frac{1}{3}x\sqrt{x}\right]_0^a = \left[\frac{1}{3}x\sqrt{x}\right]_a^4$

$\frac{1}{3}a\sqrt{a} - 0 = \frac{1}{3} \cdot 4\sqrt{4} - \frac{1}{3}a\sqrt{a}$

$\frac{2}{3}a\sqrt{a} = \frac{8}{3}$

$a^{1\frac{1}{2}} = 4$

$a = 4^{\frac{2}{3}} = \sqrt[3]{16}$

b Stel $P\left(p, \sqrt{p}\right)$.

Dit geeft $M\left(\dfrac{2+p}{2}, \dfrac{0+\sqrt{p}}{2}\right)$ ofwel $M\left(1+\tfrac{1}{2}p, \tfrac{1}{2}\sqrt{p}\right)$.

$h\left(1+\tfrac{1}{2}p\right) = \sqrt{\tfrac{1}{2}\left(1+\tfrac{1}{2}p\right)-\tfrac{1}{2}} = \sqrt{\tfrac{1}{4}p} = \tfrac{1}{2}\sqrt{p}$, dus M ligt op de grafiek van h.

21 **a** $f(x) = \sqrt{25-x^2}$ geeft $f'(x) = \dfrac{1}{2\sqrt{25-x^2}} \cdot 2x = \dfrac{x}{\sqrt{25-x^2}}$

$$\sqrt{1+(f'(x))^2} = \sqrt{1+\frac{x^2}{25-x^2}} = \sqrt{\frac{25-x^2+x^2}{25-x^2}} = \sqrt{\frac{25}{25-x^2}} = \frac{\sqrt{25}}{\sqrt{25-x^2}} = \frac{5}{\sqrt{25-x^2}}$$

b $A = 2\pi \displaystyle\int_{5-h}^{5} f(x) \cdot \sqrt{1+(f'(x))^2}\,dx = 2\pi \int_{5-h}^{5} \sqrt{25-x^2} \cdot \frac{5}{\sqrt{25-x^2}}\,dx = 2\pi \int_{5-h}^{5} 5\,dx = 2\pi[5x]_{5-h}^{5}$

$= [10\pi x]_{5-h}^{5} = 50\pi - 10\pi(5-h) = 50\pi - 50\pi + 10\pi h = 10\pi h$

Bladzijde 164

22 **a** $A(p) = \displaystyle\int_{-9}^{p} \sqrt{x+9}\,dx = \int_{-9}^{p} (x+9)^{\frac{1}{2}}\,dx = \left[\tfrac{2}{3}(x+9)^{1\frac{1}{2}}\right]_{-9}^{p} = \tfrac{2}{3}(p+9)^{1\frac{1}{2}} - \tfrac{2}{3}(-9+9)^{1\frac{1}{2}} = \tfrac{2}{3}(p+9)^{1\frac{1}{2}}$

b $\displaystyle\int_{-9}^{0} \sqrt{x+9}\,dx = \left[\tfrac{2}{3}(x+9)^{1\frac{1}{2}}\right]_{-9}^{0} = \tfrac{2}{3}(0+9)^{1\frac{1}{2}} - \tfrac{2}{3}(-9+9)^{1\frac{1}{2}} = \tfrac{2}{3} \cdot 9\sqrt{9} = 18$

$A(p) = \tfrac{1}{8} \cdot 18$

$\tfrac{2}{3}(p+9)^{1\frac{1}{2}} = \tfrac{9}{4}$

$(p+9)^{1\frac{1}{2}} = \tfrac{27}{8}$

$p+9 = \left(\tfrac{27}{8}\right)^{\frac{2}{3}}$

$p = -9 + \tfrac{9}{4} = -6\tfrac{3}{4}$

c $y = \sqrt{x+9}$

$\downarrow$ spiegelen in de y-as

$y = \sqrt{-x+9}$

$\downarrow$ translatie $(-9, 0)$

$y = \sqrt{-(x+9)+9}$ ofwel $y = \sqrt{-x}$

Dus $g(x) = \sqrt{-x}$.

$f(x) = g(x)$ geeft $\sqrt{x+9} = \sqrt{-x}$

kwadrateren geeft

$x + 9 = -x$

$2x = -9$

$x = -4\tfrac{1}{2}$

vold.

$$\pi \int_{-4\frac{1}{2}}^{0} ((f(x))^2 - (g(x))^2)\,dx = \pi \int_{-4\frac{1}{2}}^{0} (x+9--x)\,dx = \pi \int_{-4\frac{1}{2}}^{0} (2x+9)\,dx = \pi\left[x^2+9x\right]_{-4\frac{1}{2}}^{0}$$

$$= \pi(0+0) - \pi\left(\left(-4\tfrac{1}{2}\right)^2 + 9 \cdot -4\tfrac{1}{2}\right) = -\pi\left(\tfrac{81}{4} - \tfrac{81}{2}\right) = \tfrac{81}{4}\pi$$

Dus de gevraagde inhoud is $20\tfrac{1}{4}\pi$.

Bladzijde 170

23 **a** $f_a(x) = (1 + ax) \cdot e^{ax}$ geeft $f'(x) = a \cdot e^{ax} + (1 + ax) \cdot a \cdot e^{ax} = (2a + a^2x) \cdot e^{ax}$

$f'(x) = 0$ geeft $(2a + a^2x) \cdot e^{ax} = 0$

$$2a + a^2x = 0$$
$$a^2x = -2a$$
$$x = -\frac{2}{a}$$

$x = -\frac{1}{a}$ geeft $f_a(x) = \left(1 + a \cdot -\frac{2}{a}\right) \cdot e^{a \cdot -\frac{2}{a}} = -e^{-2}$

Alle toppen liggen op de lijn $y = -e^{-2}$, dus alle punten P_a liggen op één lijn.

b $O(\triangle AOB) = \frac{1}{2} \cdot \frac{1}{a} \cdot 1 = \frac{1}{2a}$

De oppervlakte van het gebied onder de grafiek van f_a is

$$\int_{-\frac{1}{a}}^{0} (1 + ax) \cdot e^{ax}\, dx = \left[x \cdot e^{ax}\right]_{-\frac{1}{a}}^{0} = 0 \cdot e^0 - -\frac{1}{a} \cdot e^{-1} = \frac{1}{a} \cdot \frac{1}{e}$$

De oppervlakte van het gebied boven de grafiek van f_a is $\frac{1}{2a} - \frac{1}{ea} = \frac{1}{a}\left(\frac{1}{2} - \frac{1}{e}\right)$

De verhouding van de oppervlakten is $\frac{1}{a}\left(\frac{1}{2} - \frac{1}{e}\right) : \left(\frac{1}{a} \cdot \frac{1}{e}\right) = \left(\frac{1}{2} - \frac{1}{e}\right) : \frac{1}{e}$, dus onafhankelijk van a.

Bladzijde 171

24 **a** $25 \cdot e^{-k \cdot t_{99}} = 0{,}01 \cdot 25$

$e^{-k \cdot t_{99}} = 0{,}01$

$-k \cdot t_{99} = \ln(0{,}01)$

$-k \cdot t_{99} = -\ln(100)$

$t_{99} = \frac{\ln(100)}{k}$

b $a(t) = 25(e^{-0{,}1t} - e^{-0{,}4t})$ geeft $a'(t) = 25(-0{,}1\, e^{-0{,}1t} - -0{,}4\, e^{-0{,}4t}) = -2{,}5\, e^{-0{,}1t} + 10\, e^{-0{,}4t}$

Voer in $y_1 = -2{,}5\, e^{-0{,}1x} + 10\, e^{-0{,}4x}$.

De optie nulpunt geeft $x = 4{,}62...$

Dus $t_{max} \approx 4{,}6$.

c Voer in $y_2 = 25(e^{-0{,}1x} - e^{-0{,}4x})$.

De optie maximum geeft $x = 4{,}62...$ en $y = 11{,}81...$

Dus $a_{max} = 11{,}81...$ en $\frac{1}{2}a_{max} = 5{,}90...$

Voer in $y_3 = 5{,}90...$

Intersect met y_2 en y_3 geeft $x = 1{,}00...$ en $x = 14{,}29...$

De FWHM is $14{,}29... - 1{,}00... \approx 13$ uur.

Bladzijde 172

25 **a** $P = 1$ ofwel $\log(P) = 0$ geeft $4{,}146 - \frac{1144}{T - 53{,}15} = 0$

$$\frac{1144}{T - 53{,}15} = 4{,}146$$
$$T - 53{,}15 = \frac{1144}{4{,}146}$$
$$T = 53{,}15 + \frac{1144}{4{,}146}$$
$$T = 329{,}0...$$

Dus het kookpunt is ongeveer 329 kelvin.

b Als T toeneemt, dan neemt $T - 53{,}15$ toe en neemt de breuk $\frac{1144}{T - 53{,}15}$ af.

Er wordt dan minder afgetrokken van 4,146, dus $\log(P)$ is stijgend en daarmee is ook P stijgend.

c $\log(P) = 4{,}146 - \dfrac{1144}{T - 53{,}15}$ geeft $P = e^{4{,}146 - \frac{1144}{T - 53{,}15}}$

Voer in $y_1 = e^{4{,}146 - \frac{1144}{x - 53{,}15}}$.

De optie $\dfrac{dy}{dx}$ geeft $\left[\dfrac{dy}{dx}\right]_{x = 293} = 0{,}01065...$

Dus de gevraagde waarde van $\dfrac{dP}{dT}$ is ongeveer 0,011 bar/kelvin.

Bladzijde 173

d $\left.\begin{array}{l} \log(P) = 4{,}146 - \dfrac{1144}{T - 53{,}15} \\ P = \dfrac{p}{750} \\ T = t + 273{,}15 \end{array}\right\}$ $\log\left(\dfrac{p}{750}\right) = 4{,}146 - \dfrac{1144}{t + 273{,}15 - 53{,}15}$

$\log(p) - \log(750) = 4{,}146 - \dfrac{1144}{t + 220}$

$\log(p) = \log(750) + 4{,}146 - \dfrac{1144}{t + 220}$

$\log(p) = 7{,}021 - \dfrac{1144}{t + 220}$

Dus $a \approx 7{,}02$ en $b \approx 220$.

26 **a** $y = e^x$

↓ verm. y-as, $\frac{1}{2}$

$y = e^{2x}$

$g(x) = e^{2x} = (e^2)^x$

Dus $a = e^2$.

b $y = e^x$

↓ translatie (0, −1)

$y = e^x - 1$

↓ spiegelen in $y = x$

$x = e^y - 1$

Uit $x = e^y - 1$ volgt $e^y = x + 1$

$y = \ln(x + 1)$

$y = \ln(x + 1)$

↓ translatie (0, 1)

$y = \ln(x + 1) + 1$

Dus $h(x) = \ln(x + 1) + 1$.

Bladzijde 174

27 $f(x) = \dfrac{1}{p}$ geeft $x = p$, dus de oppervlakte van het grijze deel boven de grafiek is

$$\int_p^{2p}\left(\frac{1}{p} - f(x)\right)dx = \int_p^{2p}\left(\frac{1}{p} - \frac{1}{x}\right)dx = \left[\frac{x}{p} - \ln|x|\right]_p^{2p} = \frac{2p}{p} - \ln(2p) - \left(\frac{p}{p} - \ln(p)\right)$$

$$= 2 - \ln(2) - \ln(p) - 1 + \ln(p) = 1 - \ln(2)$$

en dit is onafhankelijk van p.

De oppervlakte van de rechthoek is $2p \cdot \dfrac{1}{p} = 2$, dus de oppervlakte van het deel van de rechthoek onder de grafiek is $2 - (1 - \ln(2)) = 1 + \ln(2)$ en dit is ook onafhankelijk van p.

28 **a** $y = \dfrac{1 + \ln(x)}{x}$

↓ verm. x-as, e

$y = \dfrac{e(1 + \ln(x))}{x}$

↓ verm. y-as, $\frac{1}{e}$

$y = \dfrac{e(1 + \ln(ex))}{ex} = \dfrac{1 + \ln(e) + \ln(x)}{x} = \dfrac{2 + \ln(x)}{x}$

Dus $c = 2$.

b $O(W) = \int_1^e (g_3(x) - f(x))\,dx = \int_1^e \left(\frac{3 + \ln(x)}{x} - \frac{1 + \ln(x)}{x}\right)dx = \int_1^e \frac{2}{x}\,dx = \left[2\ln|x|\right]_1^e = 2\ln(e) - 2\ln(1) = 2$

29 $y = 2\ln(x)$

$\downarrow$ translatie $(-a, 0)$

$y = 2\ln(x + a)$

Dus $g(x) = 2\ln(x + a)$.

$g(x) = 0$ geeft $2\ln(x + a) = 0$

$x + a = 1$

$x = 1 - a$

$g(0) = 2\ln(a)$

De grafiek van g snijdt de x-as in $P(1 - a, 0)$ en gaat door $Q(0, 2\ln(a))$.

P en Q zijn elkaars spiegelbeeld in de lijn $y = -x$ als $1 - a = -2\ln(a)$.

Voer in $y_1 = 1 - x$ en $y_2 = -2\ln(x)$.

Intersect geeft $x = 1 \vee x = 3{,}512...$

Dus $a \approx 3{,}51$.

Bladzijde 175

30 **a** Voor de halveringstijd t geldt $e^{-\lambda t} = \frac{1}{2}$

$-\lambda t = \ln(\frac{1}{2})$

$t = -\frac{\ln(\frac{1}{2})}{\lambda} = -\frac{\ln(\frac{1}{2})}{1{,}42 \cdot 10^{-11}} = 4{,}88... \cdot 10^{10}$

Dus de halveringstijd is ongeveer 49 miljard jaar.

b $a(t) = a(0) \cdot e^{-\lambda t}$ geeft $a(0) = a(t) \cdot e^{\lambda t}$

$\left.\begin{array}{l} a(t) + b(t) = a(0) + b(0) \\ a(0) = a(t) \cdot e^{\lambda t} \end{array}\right\}$ $a(t) + b(t) = a(t) \cdot e^{\lambda t} + b(0)$

$b(t) + a(t) - a(t) \cdot e^{\lambda t} = b(0)$

$b(t) + (1 - e^{\lambda t})a(t) = b(0)$

c Voor M_1 geldt $0{,}739 + (1 - e^{\lambda t}) \cdot 0{,}60 = b(0)$

Voor M_2 geldt $0{,}713 + (1 - e^{\lambda t}) \cdot 0{,}20 = b(0)$

Omdat beide meteorieten dezelfde waarde van $b(0)$ hebben, geldt

$0{,}739 + (1 - e^{1{,}42 \cdot 10^{-11} \cdot t}) \cdot 0{,}60 = 0{,}713 + (1 - e^{1{,}42 \cdot 10^{-11} \cdot t}) \cdot 0{,}20$.

Voer in $y_1 = 0{,}739 + (1 - e^{1{,}42 \cdot 10^{-11} \cdot x}) \cdot 0{,}60$ en $y_2 = 0{,}713 + (1 - e^{1{,}42 \cdot 10^{-11} \cdot x}) \cdot 0{,}20$.

Intersect geeft $x = 4{,}43... \cdot 10^9$.

Dus de leeftijd is ongeveer 4 miljard jaar.

Bladzijde 176

31 **a** $f(x) = \frac{1}{2a} \cdot (e^{ax} + e^{-ax} - 2)$ geeft $f'(x) = \frac{1}{2a} \cdot (a e^{ax} - a e^{-ax}) = \frac{1}{2}(e^{ax} - e^{-ax})$

$1 + (f'(x))^2 = 1 + \frac{1}{4}(e^{ax} - e^{-ax})^2 = 1 + \frac{1}{4}(e^{2ax} - 2e^0 + e^{-2ax}) = 1 + \frac{1}{4}e^{2ax} - \frac{1}{2} + \frac{1}{4}e^{-2ax} =$

$\frac{1}{4}e^{2ax} + \frac{1}{2} + \frac{1}{4}e^{-2ax} = \left(\frac{1}{2}e^{ax} + \frac{1}{2}e^{-ax}\right)^2$

Hiermee is het gevraagde bewezen.

b $a = \frac{1}{140}$ geeft $f(x) = 70\left(e^{\frac{1}{140}x} + e^{-\frac{1}{140}x} - 2\right)$ en $1 + (f'(x))^2 = \left(\frac{1}{2}e^{\frac{1}{140}x} + \frac{1}{2}e^{-\frac{1}{140}x}\right)^2$

$\int_0^{96} \left(\frac{1}{2}e^{\frac{1}{140} \cdot x} + \frac{1}{2}e^{-\frac{1}{140} \cdot x}\right)dx = \left[70e^{\frac{1}{140} \cdot x} - 70e^{-\frac{1}{140} \cdot x}\right]_0^{96} = 70e^{\frac{96}{140}} - 70e^{-\frac{96}{140}} - (70 - 70) = 103{,}7...$

Dus de lengte van de ankerketting is 103,7... meter.

$f(96) = 34{,}2...$ dus de waterdiepte is 34,2... meter

103,7... : 3 = 34,5... > 34,2..., dus er is voldaan aan de vuistregel.

Bladzijde 177

32 **a** $A(p) = \int_{-p}^{p} f(x)\,\mathrm{d}x = \left[\frac{-1}{\mathrm{e}^x + 1}\right]_{-p}^{p} = \frac{-1}{\mathrm{e}^p + 1} + \frac{1}{\mathrm{e}^{-p} + 1} = \frac{-1}{\mathrm{e}^p + 1} + \frac{\mathrm{e}^p}{1 + \mathrm{e}^p} = \frac{\mathrm{e}^p - 1}{\mathrm{e}^p + 1} = \frac{\mathrm{e}^p + 1 - 2}{\mathrm{e}^p + 1} = 1 - \frac{2}{\mathrm{e}^p + 1}$

b $L = \lim_{p \to \infty}\left(1 - \frac{2}{\mathrm{e}^p + 1}\right) = 1 - \lim_{\mathrm{e}^p \to \infty} \frac{2}{\mathrm{e}^p + 1} = 1 - 0 = 1$

$A(p) = \frac{1}{2}L$ geeft $1 - \frac{2}{\mathrm{e}^p + 1} = \frac{1}{2}$

$\frac{2}{\mathrm{e}^p + 1} = \frac{1}{2}$

$\mathrm{e}^p + 1 = 4$

$\mathrm{e}^p = 3$

$p = \ln(3)$

33 **a** $y = \ln\left(\frac{2x - 1}{x + 2}\right)$ geeft voor de inverse functie $x = \ln\left(\frac{2y - 1}{y + 2}\right)$

$\frac{2y - 1}{y + 2} = \mathrm{e}^x$

$\mathrm{e}^x \cdot y + 2\mathrm{e}^x = 2y - 1$

$\mathrm{e}^x \cdot y - 2y = -2\mathrm{e}^x - 1$

$y(\mathrm{e}^x - 2) = -2\mathrm{e}^x - 1$

$y = \frac{-2\mathrm{e}^x - 1}{\mathrm{e}^x - 2} = \frac{1 + 2\mathrm{e}^x}{2 - \mathrm{e}^x}$

Dus $g(x) = \frac{1 + 2\mathrm{e}^x}{2 - \mathrm{e}^x}$ is de inverse van f.

b $h(x) = 0$ geeft $\ln\left(\frac{2x - 1}{x + 2}\right) = 0$

$\frac{2x - 1}{x + 2} = 1$

$2x - 1 = x + 2$

$x = 3$

Dus $h(x) = \begin{cases} \ln\left(\frac{2x - 1}{x + 2}\right) & \text{voor } x \geq 3 \\ -\ln\left(\frac{2x - 1}{x + 2}\right) & \text{voor } \frac{1}{2} < x < 3 \end{cases}$

$\lim_{x \to \infty} h(x) = \lim_{x \to \infty} \ln\left(\frac{2x - 1}{x + 2}\right) = \lim_{x \to \infty} \ln\left(\frac{2 - \frac{1}{x}}{1 + \frac{2}{x}}\right) = \ln\left(\frac{2 - 0}{1 + 0}\right) = \ln(2)$

$f(x) = \ln(2)$ geeft $-\ln\left(\frac{2x - 1}{x + 2}\right) = \ln(2)$

$\ln\left(\frac{2x - 1}{x + 2}\right) = -\ln(2)$

$\ln\left(\frac{2x - 1}{x + 2}\right) = \ln\left(\frac{1}{2}\right)$

$\frac{2x - 1}{x + 2} = \frac{1}{2}$

$4x - 2 = x + 2$

$3x = 4$

$x = 1\frac{1}{3}$

Dus de x-coördinaat van A is $1\frac{1}{3}$.

Bladzijde 178

34 **a** $a^7 = \frac{1}{5}$ geeft $a = \left(\frac{1}{5}\right)^{\frac{1}{7}} \approx 0{,}79$

b $n = 1$ geeft $2 \cdot \sqrt{\frac{h_1}{4{,}9}} = 1{,}11$

$$\sqrt{\frac{h_1}{4{,}9}} = 0{,}555$$

$$\frac{h_1}{4{,}9} = 0{,}308...$$

$$h_1 = 1{,}509...$$

$n = 4$ geeft $2 \cdot \sqrt{\frac{h_4}{4{,}9}} = 0{,}68$

$$\sqrt{\frac{h_1}{4{,}9}} = 0{,}34$$

$$\frac{h_1}{4{,}9} = 0{,}1156$$

$$h_1 = 0{,}566...$$

$$a^3 = \frac{h_4}{h_1} = \frac{0{,}566...}{1{,}509...} = 0{,}375...$$

$a = 0{,}721...$

$h_1 = h_0 \cdot a^1$ geeft $1{,}509... = h_0 \cdot 0{,}721...$

$$h_0 = \frac{1{,}509...}{0{,}721...} = 2{,}092...$$

Dus $h_0 \approx 2{,}0$ m.

16.4 Goniometrie

Bladzijde 183

35 $\begin{cases} x = 4 + 2\sin(t) \\ y = 2\cos(t) \end{cases}$ met $0 \le t \le \pi$ is de rechterhelft van de cirkel met middelpunt (4, 0) en straal 2.

$\begin{cases} x = 4 + 4\sin(t) \\ y = 2\cos(t) \end{cases}$ met $0 \le t \le 2\pi$ is het beeld van de linker helft van de cirkel met middelpunt (4, 0) en straal 2 bij de vermenigvuldiging ten opzichte van de lijn $x = 4$ met factor 2.
Dus de lengte van het ei is $2 + 4 = 6$ cm en de breedte is 4 cm.

Bladzijde 184

36 **a** $O(V) = \int_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} (f(x) - g(x))\,\mathrm{d}x = \int_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} \left(\sin(x) - \sin\left(x + \frac{1}{3}\pi\right)\right)\mathrm{d}x = \left[-\cos(x) + \cos\left(x + \frac{1}{3}\pi\right)\right]_{\frac{1}{3}\pi}^{\frac{4}{3}\pi}$

$$= -\cos\left(\tfrac{4}{3}\pi\right) + \cos\left(\tfrac{5}{3}\pi\right) - \left(-\cos\left(\tfrac{1}{3}\pi\right) + \cos\left(\tfrac{2}{3}\pi\right)\right) = \tfrac{1}{2} + \tfrac{1}{2} - \left(-\tfrac{1}{2} + -\tfrac{1}{2}\right) = 2$$

b $h(x) = 0$ geeft $\frac{1}{2}\left(\sin(x) + \sin\left(x + \frac{1}{3}\pi\right)\right) = 0$

$\sin(x) = -\sin\left(x + \frac{1}{3}\pi\right)$

$\sin(x) = \sin\left(x + 1\frac{1}{3}\pi\right)$

$x = x + 1\frac{1}{3}\pi \vee x = \pi - \left(x + 1\frac{1}{3}\pi\right)$

geen opl. $\quad x = \pi - x - 1\frac{1}{3}\pi$

$2x = -\frac{1}{3}\pi$

$x = -\frac{1}{6}\pi$

Dus $b = -\frac{1}{6}\pi$.

De periode is 2π, dus de $x_{top} = \frac{1}{3}\pi$.

$h\left(\frac{1}{3}\pi\right) = \frac{1}{2}\left(\sin\left(\frac{1}{3}\pi\right) + \sin\left(\frac{2}{3}\pi\right)\right) = \frac{1}{2}\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3}\right) = \frac{1}{2}\sqrt{3}$

Dus $a = \frac{1}{2}\sqrt{3}$.

37 **a** $d(O, P) = \sqrt{(x(t))^2 + (y(t))^2} = \sqrt{\left(\frac{1}{2}\sin(t)\right)^2 + \left(\sin\left(t + \frac{1}{3}\pi\right)\right)^2}$

Voer in $y_1 = \sqrt{\left(\frac{1}{2}\sin(x)\right)^2 + \left(\sin\left(x + \frac{1}{3}\pi\right)\right)^2}$.

De optie maximum geeft $y = 1{,}037...$

Dus de maximale afstand van P tot O is ongeveer 1,04.

b $\begin{cases} x(t) = \frac{1}{2}\sin(t) \\ y(t) = \sin\left(t + \frac{1}{3}\pi\right) \end{cases}$ geeft $\begin{cases} x'(t) = \frac{1}{2}\cos(t) \\ y'(t) = \cos\left(t + \frac{1}{3}\pi\right) \end{cases}$

De snelheid op $t = 0$ is $v(0) = \sqrt{(x'(0))^2 + (y'(0))^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{2}}$.

c Substitutie van $\begin{cases} x(t) = \frac{1}{2}\sin(t) \\ y(t) = \sin\left(t + \frac{1}{3}\pi\right) \end{cases}$ in $y = 2x$ geeft $\sin\left(t + \frac{1}{3}\pi\right) = \sin(t)$

$t + \frac{1}{3}\pi = t + k \cdot 2\pi \vee t + \frac{1}{3}\pi = \pi - t + k \cdot 2\pi$

geen opl. $\quad 2t = \frac{2}{3}\pi + k \cdot 2\pi$

$t = \frac{1}{3}\pi + k \cdot \pi$

t op $[0, 2\pi]$ geeft $t = \frac{1}{3}\pi \vee t = 1\frac{1}{3}\pi$

$t = \frac{1}{3}\pi$ geeft $A\left(\frac{1}{4}\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$ en $t = 1\frac{1}{3}\pi$ geeft $B\left(-\frac{1}{4}\sqrt{3}, -\frac{1}{2}\sqrt{3}\right)$.

16

Bladzijde 185

38 **a** $O(OETS) = OE^2 = (\sin(\alpha) + \cos(\alpha))^2$

$\alpha = \frac{1}{6}\pi$ geeft $O(OETS) = \left(\frac{1}{2} + \frac{1}{2}\sqrt{3}\right)^2 = \frac{1}{4} + \frac{1}{2}\sqrt{3} + \frac{3}{4} = 1 + \frac{1}{2}\sqrt{3}$

b $\overrightarrow{GC} = \vec{c} - \vec{g} = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) + \cos(\alpha) \end{pmatrix} - \begin{pmatrix} \sin(\alpha) + \cos(\alpha) + 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sin(\alpha) - 1 \\ \sin(\alpha) + \cos(\alpha) - 1 \end{pmatrix}$

$CG\colon \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sin(\alpha) + \cos(\alpha) + 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -\sin(\alpha) - 1 \\ \sin(\alpha) + \cos(\alpha) - 1 \end{pmatrix}$

CG snijden met de y-as ($x = 0$) geeft $\sin(\alpha) + \cos(\alpha) + 1 + \lambda(-\sin(\alpha) - 1) = 0$

$\lambda(\sin(\alpha) + 1) = \sin(\alpha) + \cos(\alpha) + 1$

$\lambda = \dfrac{\sin(\alpha) + \cos(\alpha) + 1}{\sin(\alpha) + 1}$

$\lambda = \dfrac{\sin(\alpha) + \cos(\alpha) + 1}{\sin(\alpha) + 1}$ geeft

$y = 1 + \dfrac{\sin(\alpha) + \cos(\alpha) + 1}{\sin(\alpha) + 1}(\sin(\alpha) + \cos(\alpha) - 1) = 1 + \dfrac{(\sin(\alpha) + \cos(\alpha) + 1)(\sin(\alpha) + \cos(\alpha) - 1)}{\sin(\alpha) + 1}$

Dus $OP = 1 + \dfrac{(\sin(\alpha) + \cos(\alpha) + 1)(\sin(\alpha) + \cos(\alpha) - 1)}{\sin(\alpha) + 1}$.

c $OP = 1 + \dfrac{(\sin(\alpha) + \cos(\alpha) + 1)(\sin(\alpha) + \cos(\alpha) - 1)}{\sin(\alpha) + 1} = 1 + \dfrac{(\sin(\alpha) + \cos(\alpha))^2 - 1}{\sin(\alpha) + 1}$

$= 1 + \dfrac{\sin^2(\alpha) + 2\sin(\alpha)\cos(\alpha) + \cos^2(\alpha) - 1}{\sin(\alpha) + 1} = 1 + \dfrac{2\sin(\alpha)\cos(\alpha) + \sin^2(\alpha) + \cos^2(\alpha) - 1}{\sin(\alpha) + 1}$

$= 1 + \dfrac{\sin(2\alpha) + 1 - 1}{\sin(\alpha) + 1} = 1 + \dfrac{\sin(2\alpha)}{\sin(\alpha) + 1}$

d $OP = 1 + \dfrac{\sin(2\alpha)}{\sin(\alpha) + 1}$ geeft $\dfrac{d(OP)}{d\alpha} = \dfrac{(\sin(\alpha) + 1) \cdot 2\cos(2\alpha) - \sin(2\alpha) \cdot \cos(\alpha)}{(\sin(\alpha) + 1)^2}$

Voer in $y_1 = \dfrac{(\sin(x) + 1) \cdot 2\cos(2x) - \sin(2x) \cdot \cos(x)}{(\sin(x) + 1)^2}$.

De optie nulpunt geeft $x = 0{,}666...$

Dus de hoogte van P is maximaal voor $\alpha \approx 0{,}67$.

Bladzijde 186

39 **a** $\begin{cases} x(t) = 2\cos(t) - \cos(2t) \\ y(t) = 2\sin(t) - \sin(2t) \end{cases}$ geeft $\begin{cases} x'(t) = -2\sin(t) + 2\sin(2t) \\ y'(t) = 2\cos(t) - 2\cos(2t) \end{cases}$

Dit geeft $v = \sqrt{(-2\sin(t) + 2\sin(2t))^2 + (2\cos(t) - 2\cos(2t))^2} =$

$\sqrt{4\sin^2(t) - 8\sin(t)\sin(2t) + 4\sin^2(2t) + 4\cos^2(t) - 8\cos(t)\cos(2t) + 4\cos^2(2t)} =$

$\sqrt{4\sin^2(t) + 4\cos^2(t) + 4\sin^2(2t) + 4\cos^2(2t) - 8\sin(t)\sin(2t) - 8\cos(t)\cos(2t)} =$

$\sqrt{4 + 4 - 8(\cos(t)\cos(2t) + \sin(t)\sin(2t))} = \sqrt{8 - 8\cos(t - 2t)} = \sqrt{8 - 8\cos(-t)} = \sqrt{8 - 8\cos(t)}$.

De maximale snelheid van P is $\sqrt{8 - 8 \cdot -1} = \sqrt{16} = 4$.

b $x = 1$ geeft $2\cos(t) - \cos(2t) = 1$

$$2\cos(t) - (2\cos^2(t) - 1) = 1$$
$$2\cos(t) - 2\cos^2(t) + 1 = 1$$
$$2\cos(t)(1 - \cos(t)) = 0$$
$$\cos(t) = 0 \vee \cos(t) = 1$$
$$t = \tfrac{1}{2}\pi + k \cdot \pi \vee t = 0 + k \cdot 2\pi$$

t op $[0, 2\pi]$ geeft $t = 0 \vee t = \tfrac{1}{2}\pi \vee t = 1\tfrac{1}{2}\pi \vee t = 2\pi$

$t = \tfrac{1}{2}\pi$ geeft $y = 2\sin\left(\tfrac{1}{2}\pi\right) - \sin(\pi) = 2 - 0 = 2$

Dus $a = 2$.

40 **a** De lijn $x = \pi$ is verticale asymptoot geeft $1 - 2\cos(a\pi) = 0 \wedge \sin(a\pi) \neq 0$

$$\cos(a\pi) = \tfrac{1}{2} \wedge a\pi \neq k \cdot \pi$$
$$\left(a\pi = \tfrac{1}{3}\pi + k \cdot 2\pi \vee a\pi = -\tfrac{1}{3}\pi + k \cdot 2\pi\right) \wedge a \neq k$$
$$\left(a = \tfrac{1}{3} + k \cdot 2 \vee a = -\tfrac{1}{3} + k \cdot 2\right) \wedge a \neq k$$
$$a = \tfrac{1}{3} + k \cdot 2 \vee a = -\tfrac{1}{3} + k \cdot 2$$

16

b De grafiek van f_2 is puntsymmetrisch in $\left(\tfrac{1}{2}\pi, 0\right)$ als voor elke p geldt

$$\frac{f_2\left(\tfrac{1}{2}\pi - p\right) + f_2\left(\tfrac{1}{2}\pi + p\right)}{2} = 0$$
$$f_2\left(\tfrac{1}{2}\pi - p\right) = -f_2\left(\tfrac{1}{2}\pi + p\right)$$
$$\frac{\sin\left(2\left(\tfrac{1}{2}\pi - p\right)\right)}{1 - 2\cos\left(2\left(\tfrac{1}{2}\pi - p\right)\right)} = -\frac{\sin\left(2\left(\tfrac{1}{2}\pi - p\right)\right)}{1 - 2\cos\left(2\left(\tfrac{1}{2}\pi - p\right)\right)}$$
$$\frac{\sin(\pi - 2p)}{1 - \cos(\pi - 2p)} = -\frac{\sin(\pi + 2p)}{1 - \cos(\pi + 2p)}$$
$$\frac{\sin(2p)}{1 + \cos(2p)} = -\frac{-\sin(2p)}{1 + \cos(2p)}$$
$$\frac{\sin(2p)}{1 + \cos(2p)} = \frac{\sin(2p)}{1 + \cos(2p)}$$

Dit geldt voor elke p, dus de grafiek van f_2 is puntsymmetrisch in $\left(\tfrac{1}{2}\pi, 0\right)$.

Bladzijde 187

41 **a** $h_2\left(\tfrac{5}{3}\right) = 1 + 2\sin\left(\tfrac{1}{5}\pi \cdot \tfrac{5}{3} - \tfrac{1}{5}\pi\right) = 1 + 2\sin\left(\tfrac{2}{15}\pi\right)$

$h_3\left(\tfrac{5}{3}\right) = 1 + 2\sin\left(-\tfrac{3}{10}\pi \cdot \tfrac{25}{9} + \tfrac{6}{5}\pi \cdot \tfrac{5}{3} - \tfrac{31}{30}\pi\right) = 1 + 2\sin\left(\tfrac{2}{15}\pi\right)$

Dus de hoogtes zijn gelijk.

b $h_1(t) = 1 + 2\sin\left(\tfrac{3}{10}\pi \cdot t^2 - \tfrac{1}{6}\pi\right)$ geeft $h_1'(t) = 2\cos\left(\tfrac{3}{10}\pi \cdot t^2 - \tfrac{1}{6}\pi\right) \cdot \tfrac{3}{5}\pi t$

$h_1'\left(\tfrac{1}{3}\right) = 2\cos\left(\tfrac{3}{10}\pi \cdot \left(\tfrac{1}{3}\right)^2 - \tfrac{1}{6}\pi\right) \cdot \tfrac{3}{5}\pi \cdot \tfrac{1}{3} = 2\cos\left(\tfrac{1}{30}\pi - \tfrac{1}{6}\pi\right) \cdot \tfrac{1}{5}\pi = \tfrac{2}{5}\pi\cos\left(-\tfrac{2}{15}\pi\right) = \tfrac{2}{5}\pi\cos\left(\tfrac{2}{15}\pi\right)$

Dus de hellingen van h_1 en h_2 zijn gelijk op $t = \tfrac{1}{3}$.

c $\dfrac{h_2(1 - a) + h_2(1 + a)}{2} = 1$ geeft $h_2(1 - a) + h_2(1 + a) = 2$

$$1 + 2\sin\left(\tfrac{1}{5}\pi(1 - a) - \tfrac{1}{5}\pi\right) + 1 + 2\sin\left(\tfrac{1}{5}\pi(1 + a) - \tfrac{1}{5}\pi\right) = 2$$
$$2\sin\left(\tfrac{1}{5}\pi(1 - a) - \tfrac{1}{5}\pi\right) + 2\sin\left(\tfrac{1}{5}\pi(1 + a) - \tfrac{1}{5}\pi\right) = 0$$
$$\sin\left(-\tfrac{1}{5}\pi a\right) + \sin\left(\tfrac{1}{5}\pi a\right) = 0$$
$$-\sin\left(\tfrac{1}{5}\pi a\right) + \sin\left(\tfrac{1}{5}\pi a\right) = 0$$
$$0 = 0$$

Dit klopt voor elke a.

Bladzijde 188

42 **a** $A'B' = |x_A - x_B| = |\cos(3t) - \cos(t)|$

Voer in $y_1 = |\cos(3x) - \cos(x)|$.

De optie maximum geeft voor verschillende x-waarden telkens $y \approx 1{,}54$.

Dus de maximale lengte van $A'B'$ is ongeveer 1,54.

b $$a = \text{rc}_{AB} = \frac{\sin(3t) - \sin(t)}{\cos(3t) - \cos(t)} = \frac{2\sin\left(\dfrac{3t - t}{2}\right) \cdot \cos\left(\dfrac{3t + t}{2}\right)}{-2\sin\left(\dfrac{3t + t}{2}\right) \cdot \sin\left(\dfrac{3t - t}{2}\right)} = -\frac{\sin(t)\cos(2t)}{\sin(2t)\sin(t)} = -\frac{\cos(2t)}{\sin(2t)}$$

c $AB \,//\, l$ geeft $rc_{AB} = rc_l$

$$-\frac{\cos(2t)}{\sin(2t)} = -1$$
$$\sin(2t) = \cos(2t)$$
$$\frac{\sin(2t)}{\cos(2t)} = 1$$
$$\tan(2t) = 1$$
$$2t = \tfrac{1}{4}\pi + k \cdot \pi$$
$$t = \tfrac{1}{8}\pi + k \cdot \tfrac{1}{2}\pi$$

t op $\langle 0, 2\pi \rangle$ geeft $t = \frac{1}{8}\pi \vee t = \frac{5}{8}\pi \vee t = 1\frac{1}{8}\pi \vee t = 1\frac{5}{8}\pi$.

Bladzijde 189

43 **a** $y_B = 0$ geeft $2\cos(2t) = 0$

$$2t = \tfrac{1}{2}\pi + k \cdot \pi$$
$$t = \tfrac{1}{4}\pi + k \cdot \tfrac{1}{2}\pi$$

t op $[0, 2\pi]$ geeft $t = \frac{1}{4}\pi \vee t = \frac{3}{4}\pi \vee t = 1\frac{1}{4}\pi \vee t = 1\frac{3}{4}\pi$.

$t = \frac{1}{4}\pi$ geeft $A\left(\sin\left(\frac{1}{4}\pi\right), \cos\left(\frac{1}{4}\pi\right)\right) = A\left(\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\right)$, dus A op de lijn $y = x$.

$t = \frac{3}{4}\pi$ geeft $A\left(\sin\left(\frac{3}{4}\pi\right), \cos\left(\frac{3}{4}\pi\right)\right) = A\left(\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2}\right)$, dus A op de lijn $y = -x$.

$t = 1\frac{1}{4}\pi$ geeft $A\left(\sin\left(1\frac{1}{4}\pi\right), \cos\left(1\frac{1}{4}\pi\right)\right) = A\left(-\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2}\right)$, dus A op de lijn $y = x$.

$t = 1\frac{3}{4}\pi$ geeft $A\left(\sin\left(1\frac{3}{4}\pi\right), \cos\left(1\frac{3}{4}\pi\right)\right) = A\left(-\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\right)$, dus A op de lijn $y = -x$.

Dus als B op de x-as is, is A op de lijn $y = x$ of op de lijn $y = -x$.

b AB horizontaal geeft $y_A = y_B$

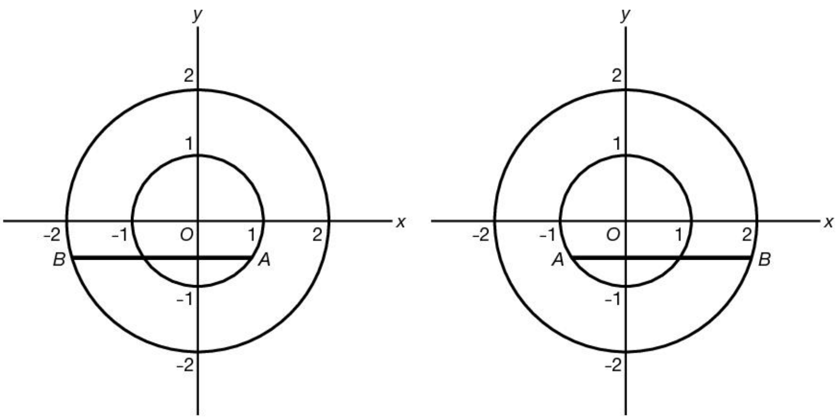
$$\cos(t) = 2\cos(2t)$$

Voer in $y_1 = \cos(x)$ en $y_2 = 2\cos(2x)$.

Intersect geeft $x = 0{,}567...$, $x = 2{,}205...$, $x = 4{,}077...$ en $x = 5{,}715...$

Bij de punten onder de x-as horen de t-waarden 2,205... en 4,077...

$t = 2{,}205...$ geeft de punten $A(0{,}8; -0{,}6)$ en $B(-1{,}9; -0{,}6)$, zie de linker figuur hieronder of $t = 4{,}077...$ geeft de punten $A(-0{,}8; -0{,}6)$ en $B(1{,}9; -0{,}6)$ zie de rechter figuur hieronder.



Bladzijde 190

44 **a** $x_P = -4\cos(\alpha)$ en $y_P = 2 + 4\sin(\alpha)$.

b $y(t) = -5t^2 + 2 + 20t \cdot \cos(\alpha) \cdot \sqrt{\sin(\alpha)} + 4\sin(\alpha)$ geeft $y'(t) = -10t + 20\cos(\alpha) \cdot \sqrt{\sin(\alpha)}$

$y'(t) = 0$ geeft $-10t + 20\cos(\alpha) \cdot \sqrt{\sin(\alpha)} = 0$

$$10t = 20\cos(\alpha) \cdot \sqrt{\sin(\alpha)}$$
$$10t = 20\cos(\alpha) \cdot \sqrt{\sin(\alpha)}$$
$$t = 2\cos(\alpha) \cdot \sqrt{\sin(\alpha)}$$

$t = 2\cos(\alpha) \cdot \sqrt{\sin(\alpha)}$ geeft

$$y_{\text{top}} = -5\left(2\cos(\alpha) \cdot \sqrt{\sin(\alpha)}\right)^2 + 2 + 20 \cdot 2\cos(\alpha) \cdot \sqrt{\sin(\alpha)} \cdot \cos(\alpha) \cdot \sqrt{\sin(\alpha)} + 4\sin(\alpha)$$
$$= -20\cos^2(\alpha)\sin(\alpha) + 2 + 40\cos^2(\alpha)\sin(\alpha) + 4\sin(\alpha) = 20\cos^2(\alpha)\sin(\alpha) + 2 + 4\sin(\alpha)$$
$$= 20(1 - \sin^2(\alpha))\sin(\alpha) + 2 + 4\sin(\alpha) = 20\sin(\alpha) - 20\sin^3(\alpha) + 2 + 4\sin(\alpha)$$
$$= 2 + 24\sin(\alpha) - 20\sin^3(\alpha)$$

Bladzijde 191

c Voer in $y_1 = 2 + 24\sin(x) - 20\sin^3(x)$.
De optie maximum geeft $x = 0{,}684...$
$\alpha = 0{,}684...$ geeft $y(t) = -5t^2 + 2 + 20t \cdot \cos(0{,}684..) \cdot \sqrt{\sin(0{,}684)} + 4\sin(0{,}684)$
$= -5t^2 + 2 + 12{,}32... \cdot t + 2{,}529... \approx -5t^2 + 12{,}3t + 4{,}5$

d $y(t) = 6$ geeft $-5t^2 + 12{,}3t + 4{,}5 = 6$
Voer in $y_1 = -5x^2 + 12{,}3x + 4{,}5$ en $y_2 = 6$.
Intersect geeft $x = 0{,}128...$ en $x = 2{,}331...$
Dus $t = 0{,}128...$ of $t = 2{,}331...$
$t = 0{,}128...$ geeft $x = -1{,}80...$ voldoet niet.
$t = 2{,}331...$ geeft $x = 20{,}44...$
De afstand van 24 meter van de muur moet dus minstens 4 meter minder worden. Dus de katapult moet minstens 4 meter in de richting van de muur worden geschoven.

16

45 **a** Voer in $y_1 = 20\cos(x)\left(\sin(x) + \sqrt{\sin^2(x) + 0{,}185}\right)$.
De optie maximum geeft $x = 0{,}743...$ en $y = 21{,}77...$
Dus de optimale stoothoek is ongeveer 0,74 rad.

b Voor $0 < \alpha < \frac{1}{2}\pi$ is $\sin(\alpha) > 0$.
$h = 0$ geeft
$r = 20\cos(\alpha)\left(\sin(\alpha) + \sqrt{\sin^2(\alpha)}\right) = 20\cos(\alpha)(\sin(\alpha) + \sin(\alpha)) = 40\cos(\alpha)\sin(\alpha) = 20\sin(2\alpha)$
r is maximaal als $\sin(2\alpha)$ maximaal is. Dus als $2\alpha = \frac{1}{2}\pi$ ofwel $\alpha = \frac{1}{4}\pi$.

16.5 Meetkunde

Bladzijde 197

46 **a** Stel $S(s, 0)$.
Dan is k: $\frac{x}{s} + \frac{y}{4} = 1$ ofwel $4x + sy = 4s$.
k raakt aan de cirkel als $d(M, k) = 2$

$$\frac{|4 \cdot 2 + s \cdot 0 - 4s|}{\sqrt{4^2 + s^2}} = 2$$
$$|8 - 4s| = 2\sqrt{16 + s^2}$$
$$64 - 64s + 16s^2 = 4(16 + s^2)$$
$$64 - 64s + 16s^2 = 64 + 4s^2$$
$$12s^2 - 64s = 0$$
$$4s(3s - 16) = 0$$
$$s = 0 \lor s = 5\tfrac{1}{3}$$
vold. niet   vold.

Dus $x_S = 5\frac{1}{3}$.

b Stel $A(a, pa)$.
A op de gegeven cirkel: $(x - 2)^2 + y^2 = 4$ geeft $(a - 2)^2 + p^2a^2 = 4$.
$OA = 3$ geeft dat A op de cirkel $x^2 + y^2 = 9$ ligt, dus $a^2 + p^2a^2 = 9$ ofwel $p^2a^2 = 9 - a^2$.
$p^2a^2 = 9 - a^2$ substitueren in $(a - 2)^2 + p^2a^2 = 4$ geeft $(a - 2)^2 + 9 - a^2 = 4$
$$a^2 - 4a + 4 + 9 - a^2 = 4$$
$$-4a = -9$$
$$a = 2\tfrac{1}{4}$$
$a = 2\frac{1}{4}$ substitueren is $p^2a^2 = 9 - a^2$ geeft $\frac{81}{16}p^2 = 9 - \frac{81}{16}$
$$p^2 = \tfrac{16}{9} - 1$$
$$p^2 = \tfrac{7}{9}$$
$$p = \tfrac{1}{3}\sqrt{7} \lor p = -\tfrac{1}{3}\sqrt{7}$$
vold.   vold. niet

Dus $p = \frac{1}{3}\sqrt{7}$.

47 $\begin{cases} x(t) = t^2 - 1 \\ y(t) = t(t^2 - 1) = t^3 - t \end{cases}$ geeft de snelheidsvector $\vec{v}(t) = \begin{pmatrix} 2t \\ 3t^2 - 1 \end{pmatrix}$

$\vec{v}(1) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ en $\vec{v}(-1) = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$\vec{v}(t) \triangleq \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ geeft $2t = 3t^2 - 1$

$3t^2 - 2t - 1 = 0$

$(3t + 1)(t - 1) = 0$

$t = -\frac{1}{3} \vee t = 1$

$\vec{v}(t) \triangleq \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ geeft $2t = -3t^2 + 1$

$3t^2 + 2t - 1 = 0$

$(3t - 1)(t + 1) = 0$

$t = \frac{1}{3} \vee t = -1$

Dus de opeenvolgende tijdstippen zijn $t = -1$, $t = -\frac{1}{3}$, $t = \frac{1}{3}$ en $t = 1$.
De tussenliggende tijden zijn alle gelijk aan $\frac{2}{3}$, dus even lang.

16

Bladzijde 198

48 **a** $DM = 4$ en $CM = 2$

De stelling van Pythagoras in $\triangle CMD$ geeft $CD^2 = MD^2 - MC^2 = 4^2 - 2^2 = 16 - 4 = 12$

Dus $CD = \sqrt{12} = 2\sqrt{3}$.

Trek $KS // PQ$, S op LQ.

De stelling van Pythagoras in $\triangle KLS$ geeft $KS^2 + LS^2 = KL^2$

$KS^2 + (3 - 1)^2 = (3 + 1)^2$

$KS^2 + 4 = 16$

$KS^2 = 12$

$\left.\begin{array}{l} KS = \sqrt{12} = 2\sqrt{3} \\ KS = PQ \end{array}\right\} PQ = 2\sqrt{3}$

Dus $CD = PQ$.

b $KT = 1 + r$, $KM = 4 - 1 = 3$ en $MT = 4 - r$

De cosinusregel geeft $KT^2 = KM^2 + MT^2 - 2 \cdot KM \cdot KT \cdot \cos(\alpha)$

$(1 + r)^2 = 3^2 + (4 - r)^2 - 2 \cdot 3 \cdot (4 - r) \cdot \cos(\alpha)$

$1 + 2r + r^2 = 9 + 16 - 8r + r^2 - (24 - 6r)\cos(\alpha)$

$(24 - 6r)\cos(\alpha) = 24 - 10r$

$\cos(\alpha) = \dfrac{24 - 10r}{24 - 6r} = \dfrac{12 - 5r}{12 - 3r}$

c $\dfrac{12 - 5r}{12 - 3r} = \dfrac{7r - 4}{4 - r}$ geeft $\dfrac{12 - 5r}{12 - 3r} = \dfrac{21r - 12}{12 - 3r}$

$12 - 5r = 21r - 12$

$-26r = -24$

$r = \frac{12}{13}$

Bladzijde 199

49 **a** $\cos(\angle(l, m)) = \dfrac{\left|\begin{pmatrix} 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ -3 \end{pmatrix}\right|} = \dfrac{1 + 6}{\sqrt{5} \cdot \sqrt{10}} = \dfrac{7}{\sqrt{50}}$

$\angle(l, m) \approx 8°$

b $\left.\begin{array}{l} x = t \wedge y = 2 - 2t \\ x^2 + (y - 1)^2 = 1 \end{array}\right\} t^2 + (1 - 2t)^2 = 1$

$t^2 + 1 - 4t + 4t^2 - 1 = 0$

$5t^2 - 4t = 0$

$t(5t - 4) = 0$

$t = 1 \vee t = \frac{4}{5}$

$t = \frac{4}{5}$ geeft $x = \frac{4}{5} \wedge y = 2 - 2 \cdot \frac{4}{5} = \frac{2}{5}$

Dus $B\left(\frac{4}{5}, \frac{2}{5}\right)$.

c $\vec{n}_l = \vec{r}_{AB} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, dus $l\colon x - 2y = c$
door $\left(\dfrac{1 + \frac{4}{5}}{2}, \dfrac{0 + \frac{2}{5}}{2}\right) = \left(\frac{9}{10}, \frac{2}{10}\right)$ $\Big\}$ $c = \frac{9}{10} - 2 \cdot \frac{2}{10} = \frac{1}{2}$

De middelloodlijn van AB is $l\colon x - 2y = \frac{1}{2}$.
De middelloodlijn van AD is $k\colon x = \frac{5}{6}$.
l en k snijden geeft $\frac{5}{6} - 2y = \frac{1}{2}$

$$-2y = -\tfrac{1}{3}$$
$$y = \tfrac{1}{6}$$

Dus het middelpunt van de cirkel door A, B en D is $M\left(\frac{5}{6}, \frac{1}{6}\right)$.

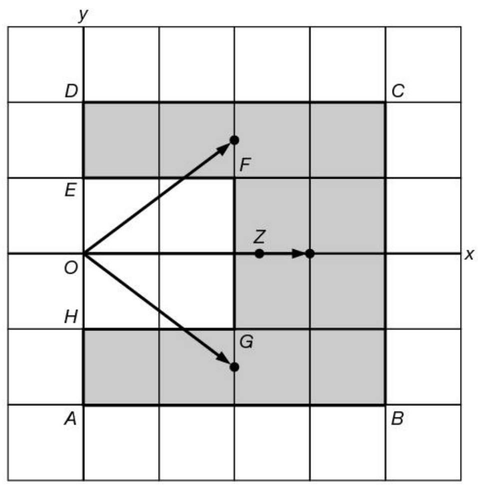
De straal van deze cirkel is $\sqrt{\left(1 - \frac{5}{6}\right)^2 + \left(0 - \frac{1}{6}\right)^2} = \sqrt{\frac{1}{36} + \frac{1}{36}} = \sqrt{\frac{1}{18}}$.

$d(M, C) = \sqrt{\left(\frac{3}{5} - \frac{5}{6}\right)^2 + \left(\frac{1}{5} - \frac{1}{6}\right)^2} = \sqrt{\frac{49}{900} + \frac{1}{900}} = \sqrt{\frac{50}{900}} = \sqrt{\frac{1}{18}}$

Dus C ligt ook op deze cirkel, zodat de punten A, B, C en D op één cirkel liggen.

16

50 De lijnen EF en HG verdelen de figuur in drie delen, elk met oppervlakte 4.
De zwaartepunten van de drie delen zijn de snijpunten van de diagonalen.
Breng een assenstelsel aan met de oorsprong in het midden van EH.
Dan zijn de vectoren naar de zwaartepunten van de drie delen $\begin{pmatrix} 2 \\ 1\frac{1}{2} \end{pmatrix}$, $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ en $\begin{pmatrix} 2 \\ -1\frac{1}{2} \end{pmatrix}$.



Dit geeft $\vec{z} = \frac{1}{12}\left(4 \cdot \begin{pmatrix} 2 \\ 1\frac{1}{2} \end{pmatrix} + 4 \cdot \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 4 \cdot \begin{pmatrix} 2 \\ -1\frac{1}{2} \end{pmatrix}\right) = \frac{1}{12}\left(\begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} 12 \\ 0 \end{pmatrix} + \begin{pmatrix} 8 \\ -6 \end{pmatrix}\right) = \frac{1}{12} \cdot \begin{pmatrix} 28 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\frac{1}{3} \\ 0 \end{pmatrix}$

Bladzijde 200

51 **a** $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AB}_L = \begin{pmatrix} p \\ q \end{pmatrix} + (\vec{b} - \vec{a})_L = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 2 - p \\ 0 - q \end{pmatrix}_L = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} q \\ 2 - p \end{pmatrix} = \begin{pmatrix} p + q \\ 2 - p + q \end{pmatrix}$

b $\overrightarrow{MA} = \vec{a} - \vec{m} = \vec{a} - \frac{1}{2} \cdot \overrightarrow{OB} = \begin{pmatrix} p \\ q \end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} p - 1 \\ q \end{pmatrix}$

$\overrightarrow{ED} = \vec{d} - \vec{e} = \begin{pmatrix} p + q \\ 2 - p + q \end{pmatrix} - \begin{pmatrix} p - q \\ p + q \end{pmatrix} = \begin{pmatrix} 2q \\ 2 - 2p \end{pmatrix}$

$\overrightarrow{MA} \cdot \overrightarrow{ED} = \begin{pmatrix} p - 1 \\ q \end{pmatrix} \cdot \begin{pmatrix} 2q \\ 2 - 2p \end{pmatrix} = 2pq - 2q + 2q - 2pq = 0$

Dus de lijn MA staat loodrecht op de lijn ED.

52 **a** $l\colon ax + y = b$
↓ spiegelen in $y = x$
$m\colon ay + x = b$ ofwel $m\colon x + ay = b$

$$\cos(\angle(l, m)) = \frac{|\vec{n}_l \cdot \vec{n}_m|}{|\vec{n}_l| \cdot |\vec{n}_m|} = \frac{\left|\begin{pmatrix} a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ a \end{pmatrix}\right|}{\left|\begin{pmatrix} a \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ a \end{pmatrix}\right|} = \frac{|a + a|}{\sqrt{a^2 + 1} \cdot \sqrt{a^2 + 1}} = \frac{|2a|}{a^2 + 1} = \frac{2a}{a^2 + 1}$$

b $\alpha = 30°$ geeft $\frac{2a}{a^2+1} = \cos(30°)$

$$\frac{2a}{a^2+1} = \tfrac{1}{2}\sqrt{3}$$
$$\tfrac{1}{2}\sqrt{3} \cdot a^2 + \tfrac{1}{2}\sqrt{3} = 2a$$
$$\tfrac{1}{2}\sqrt{3} \cdot a^2 - 2a + \tfrac{1}{2}\sqrt{3} = 0$$
$$D = (-2)^2 - 4 \cdot \tfrac{1}{2}\sqrt{3} \cdot \tfrac{1}{2}\sqrt{3} = 4 - 3 = 1$$
$$a = \frac{2+1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \vee a = \frac{2-1}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \tfrac{1}{3}\sqrt{3}$$

$x^2 = \tfrac{1}{2}y$ ofwel $y = 2x^2$ geeft $\frac{dy}{dx} = 4x$
De richtingscoëfficiënt van $ax + y = b$ is $-a$.
$4x = -\sqrt{3}$ geeft $x = -\tfrac{1}{4}\sqrt{3} \vee 4x = -\tfrac{1}{3}\sqrt{3}$ geeft $x = -\tfrac{1}{12}\sqrt{3}$
Dus $x_P = -\tfrac{1}{4}\sqrt{3} \vee x_P = -\tfrac{1}{12}\sqrt{3}$.

16

Bladzijde 201

53 **a** Stel de straal is r en het middelpunt is $M(0, m)$.
c raakt c_1 geeft $m = r + 3$
c raakt c_2 geeft $m^2 = (r + 12)^2 - 15^2$

$$(r+3)^2 = (r+12)^2 - 15^2$$
$$r^2 + 6r + 9 = r^2 + 24r + 144 - 225$$
$$-18r = -90$$
$$r = 5$$

$m = r + 3 = 8$, dus c_3: $x^2 + (y-8)^2 = 25$.

b Een van de gemeenschappelijke raaklijnen heeft vergelijking $x = 3$.
Stel de andere gemeenschappelijk raaklijnen zijn l: $y = ax + b$ ofwel l: $ax - y + b = 0$.
Voor raken aan c_1 en c_2 geldt $d((0, 0), l) = 3 \wedge d((15, 0), l) = 12$

$$\frac{|b|}{\sqrt{a^2+1}} = 3 \wedge \frac{|15a+b|}{\sqrt{a^2+1}} = 12$$
$$|b| = 3\sqrt{a^2+1} \wedge |15a+b| = 12\sqrt{a^2+1}$$

$|b| = 3\sqrt{a^2+1}$ geeft $4 \cdot |b| = |15a + b|$

$$4b = 15a + b \vee 4b = -15a - b$$
$$3b = 15a \vee 5b = -15a$$
$$b = 5a \vee b = -3a$$

$|b| = 3\sqrt{a^2+1}$
$b = 5a$

$$|5a| = 3\sqrt{a^2+1}$$
$$25a^2 = 9(a^2+1)$$
$$25a^2 = 9a^2 + 9$$
$$16a^2 = 9$$
$$a^2 = \tfrac{9}{16}$$
$$a = \tfrac{3}{4} \vee a = -\tfrac{3}{4}$$

$a = \tfrac{3}{4}$ geeft $b = 5 \cdot \tfrac{3}{4} = 3\tfrac{3}{4}$, dus l_1: $y = \tfrac{3}{4}x + 3\tfrac{3}{4}$.
$a = -\tfrac{3}{4}$ geeft $b = 5 \cdot -\tfrac{3}{4} = -3\tfrac{3}{4}$, dus l_2: $y = -\tfrac{3}{4}x - 3\tfrac{3}{4}$.
Er zijn twee gemeenschappelijke raaklijnen gevonden, dus $b = -3a$ hoeft niet meer bekeken te worden.
De gemeenschappelijke raaklijnen zijn l_1: $y = \tfrac{3}{4}x + 3\tfrac{3}{4}$ en l_2: $y = -\tfrac{3}{4}x - 3\tfrac{3}{4}$.

54 **a** $$\overrightarrow{OS} = \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AP} + \tfrac{1}{2}\overrightarrow{AP}_R = \begin{pmatrix}2\\0\end{pmatrix} + \tfrac{1}{2}(\vec{p} - \vec{a}) + \tfrac{1}{2}(\vec{p} - \vec{a})_R = \begin{pmatrix}2\\0\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}2\cos(t) - 2\\2\sin(t)\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}2\sin(t)\\-2\cos(t) + 2\end{pmatrix}$$
$$= \begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}\cos(t) - 1\\\sin(t)\end{pmatrix} + \begin{pmatrix}\sin(t)\\-\cos(t) + 1\end{pmatrix} = \begin{pmatrix}2 + \cos(t) - 1 + \sin(t)\\0 + \sin(t) - \cos(t) + 1\end{pmatrix} = \begin{pmatrix}1 + \cos(t) + \sin(t)\\1 - \cos(t) + \sin(t)\end{pmatrix}$$

b $$d(M, S) = \sqrt{(1 + \cos(t) + \sin(t) - 1)^2 + (1 - \cos(t) + \sin(t) - 1)^2}$$
$$= \sqrt{(\cos(t) + \sin(t))^2 + (-\cos(t) + \sin(t))^2}$$
$$= \sqrt{\cos^2(t) + 2\cos(t)\sin(t) + \sin^2(t) + \cos^2(t) - 2\cos(t)\sin(t) + \sin^2(t)}$$
$$= \sqrt{\cos^2(t) + \sin^2(t) + 2\cos(t)\sin(t) + \cos^2(t) + \sin^2(t) - 2\cos(t)\sin(t)} = \sqrt{1+1} = \sqrt{2}$$

Dus de afstand van S tot M is constant.

Bladzijde 202

55 $\begin{cases} x(t) = 8{,}4t \\ y(t) = h + 11{,}2t - 4{,}9t^2 \end{cases}$ geeft $\begin{cases} x'(t) = 8{,}4 \\ y'(t) = 11{,}2 - 9{,}8t \end{cases}$

$v(0) = \sqrt{(x'(0))^2 + (y'(0))^2} = \sqrt{8{,}4^2 + 11{,}2^2} = \sqrt{196} = 14$

Dus de snelheid op $t = 0$ is 14 m/s.

56 **a** $f(x) = 3\sqrt{x} - x$ geeft $f'(x) = 3 \cdot \dfrac{1}{2\sqrt{x}} - 1 = \dfrac{3}{2\sqrt{x}} - 1$

$f'(x) = 0$ geeft $\dfrac{3}{2\sqrt{x}} - 1 = 0$

$\dfrac{3}{2\sqrt{x}} = 1$

$2\sqrt{x} = 3$

$\sqrt{x} = 1\frac{1}{2}$

$x = 2\frac{1}{4}$

$f\left(2\frac{1}{4}\right) = 3\sqrt{2\frac{1}{4}} - 2\frac{1}{4} = 3 \cdot 1\frac{1}{2} - 2\frac{1}{4} = 2\frac{1}{4}$

Dus $T\left(2\frac{1}{4}, 2\frac{1}{4}\right)$.

b $\overrightarrow{TO} = \vec{0} - \vec{t} = \begin{pmatrix} -2\frac{1}{4} \\ -2\frac{1}{4} \end{pmatrix}$ en $\overrightarrow{TA} = \vec{a} - \vec{t} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} - \begin{pmatrix} 2\frac{1}{4} \\ 2\frac{1}{4} \end{pmatrix} = \begin{pmatrix} 6\frac{3}{4} \\ -2\frac{1}{4} \end{pmatrix}$

$$\cos(\angle OTA) = \frac{\begin{pmatrix} -2\frac{1}{4} \\ -2\frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 6\frac{3}{4} \\ -2\frac{1}{4} \end{pmatrix}}{\left|\begin{pmatrix} -2\frac{1}{4} \\ -2\frac{1}{4} \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 6\frac{3}{4} \\ -2\frac{1}{4} \end{pmatrix}\right|} = \frac{\begin{pmatrix} -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ -1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ -1 \end{pmatrix}\right|} = \frac{-3 + 1}{\sqrt{2} \cdot \sqrt{10}} = \frac{-2}{2\sqrt{5}} = \frac{-1}{\sqrt{5}}$$

Dit geeft $\angle OTA = 116{,}5...^\circ$

Dus $\angle OTA \approx 117^\circ$.

Bladzijde 203

57 $\overrightarrow{OP} = \begin{pmatrix} p \\ p^2 \end{pmatrix}$ geeft middelloodlijn: $px + p^2y = c$

het midden van OP is $\left(\frac{1}{2}p, \frac{1}{2}p^2\right)$

$\Big\}$ $c = p \cdot \frac{1}{2}p + p^2 \cdot \frac{1}{2}p^2$

Dus middelloodlijn: $px + p^2y = \frac{1}{2}p^2 + \frac{1}{2}p^4$.

Snijden met de y-as geeft $p \cdot 0 + p^2y = \frac{1}{2}p^2 + \frac{1}{2}p^4$

$y = \frac{1}{2} + \frac{1}{2}p^2$

$\lim\limits_{p \downarrow 0} y = \lim\limits_{p \downarrow 0}\left(\frac{1}{2} + \frac{1}{2}p^2\right) = \frac{1}{2}$

Dus de y-coördinaat nadert tot $\frac{1}{2}$.

58 $\overrightarrow{OA} = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$

$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 2\sin(2t) \\ 2\cos(2t) \end{pmatrix} - \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} = \begin{pmatrix} 2\sin(2t) - \sin(t) \\ 2\cos(2t) - \cos(t) \end{pmatrix}$

$OA \perp AB$ geeft $\overrightarrow{OA} \cdot \overrightarrow{AB} = 0$

$\begin{pmatrix} 2\sin(2t) - \sin(t) \\ 2\cos(2t) - \cos(t) \end{pmatrix} \cdot \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} = 0$

$2\sin(2t)\sin(t) - \sin^2(t) + 2\cos(2t)\cos(t) - \cos^2(t) = 0$

$2\sin(2t)\sin(t) + 2\cos(2t)\cos(t) = \sin^2(t) + \cos^2(t)$

$2\sin(2t)\sin(t) + 2\cos(2t)\cos(t) = 1$

$\sin(2t)\sin(t) + \cos(2t)\cos(t) = \frac{1}{2}$

$2\sin(t)\cos(t)\sin(t) + (1 - 2\sin^2(t))\cos(t) = \frac{1}{2}$

$2\sin^2(t)\cos(t) + \cos(t) - 2\sin^2(t)\cos(t) = \frac{1}{2}$

$\cos(t) = \frac{1}{2}$

$t = \frac{1}{3}\pi$

59 **a** $f_5(x) = \dfrac{4x^2 - 10x + 4}{2x - 5} = \dfrac{2x(2x-5) + 10x - 10x + 4}{2x - 5} = 2x + \dfrac{4}{2x - 5}$

$\lim\limits_{x \to \infty} \dfrac{4}{2x - 5} = 0$, dus de scheve asymptoot is de lijn $y = 2x$.

$$\cos(\beta) = \frac{\begin{pmatrix}1\\2\end{pmatrix} \cdot \begin{pmatrix}0\\1\end{pmatrix}}{\left|\begin{pmatrix}1\\2\end{pmatrix}\right| \cdot \left|\begin{pmatrix}0\\1\end{pmatrix}\right|} = \frac{2}{\sqrt{5} \cdot 1} = \frac{2}{\sqrt{5}}$$

Dit geeft $\beta \approx 26{,}6°$.

b $f_a(x) = \dfrac{4x^2 - 10x + 4}{2x - a}$ geeft

$$f_a'(x) = \frac{(2x-a) \cdot (8x - 10) - (4x^2 - 10x + 4) \cdot 2}{(2x-a)^2} = \frac{16x^2 - 20x - 8ax + 10a - 8x^2 + 20x - 8}{(2x-a)^2}$$
$$= \frac{8x^2 - 8ax + 10a - 8}{(2x-a)^2}$$

$f_a'(0) = 0$ geeft $\dfrac{10a - 8}{(0-a)^2} = 0$

$10a - 8 = 0$

$a = \frac{4}{5}$

$$f_{\frac{4}{5}}'(x) = \frac{8x^2 - \frac{32}{5}x}{\left(2x - \frac{4}{5}\right)^2} = \frac{8x\left(x - \frac{4}{5}\right)}{\left(2x - \frac{4}{5}\right)^2}$$

De andere top heeft x-coördinaat $\frac{4}{5}$ en is dus de rechter top.
Dus de linker top ligt op de y-as voor $a = \frac{4}{5}$.

Bladzijde 204

60 **a** $\overrightarrow{OC} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = \begin{pmatrix}42\\0\end{pmatrix} + \frac{2}{3}\begin{pmatrix}21 - 42\\21\sqrt{3} - 0\end{pmatrix} = \begin{pmatrix}42\\0\end{pmatrix} + \begin{pmatrix}-14\\14\sqrt{3}\end{pmatrix} = \begin{pmatrix}28\\14\sqrt{3}\end{pmatrix} \triangleq \begin{pmatrix}2\\\sqrt{3}\end{pmatrix}$

$\overrightarrow{OD} = \frac{1}{3}\overrightarrow{OB} = \frac{1}{3}\begin{pmatrix}21\\21\sqrt{3}\end{pmatrix} = \begin{pmatrix}7\\7\sqrt{3}\end{pmatrix}$ en $\overrightarrow{AD} = \begin{pmatrix}7\\7\sqrt{3}\end{pmatrix} - \begin{pmatrix}42\\0\end{pmatrix} = \begin{pmatrix}-35\\7\sqrt{3}\end{pmatrix} \triangleq \begin{pmatrix}-5\\\sqrt{3}\end{pmatrix}$

OC: $\begin{pmatrix}x\\y\end{pmatrix} = \lambda\begin{pmatrix}2\\\sqrt{3}\end{pmatrix}$ en AD: $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}42\\0\end{pmatrix} + \mu\begin{pmatrix}-5\\\sqrt{3}\end{pmatrix}$ snijden geeft

$$\begin{cases} 2\lambda = 42 - 5\mu \\ \lambda\sqrt{3} = \mu\sqrt{3} \end{cases}$$

$\lambda\sqrt{3} = \mu\sqrt{3}$ geeft $\lambda = \mu$
$2\lambda = 42 - 5\mu$ } $2\lambda = 42 - 5\lambda$

$7\lambda = 42$

$\lambda = 6$

$\lambda = 6$ geeft $E\left(12, 6\sqrt{3}\right)$
Dus $x_E = 12$.

b $\overrightarrow{EB} = \vec{b} - \vec{e} = \begin{pmatrix}21\\21\sqrt{3}\end{pmatrix} - \begin{pmatrix}12\\6\sqrt{3}\end{pmatrix} = \begin{pmatrix}9\\15\sqrt{3}\end{pmatrix}$

$\overrightarrow{EA} \triangleq \overrightarrow{AD} \triangleq \begin{pmatrix}-5\\\sqrt{3}\end{pmatrix}$

$\overrightarrow{EB} \cdot \overrightarrow{EA} = \begin{pmatrix}9\\15\sqrt{3}\end{pmatrix} \cdot \begin{pmatrix}-5\\\sqrt{3}\end{pmatrix} = -45 + 45 = 0$

Dus $\angle(EB, EA) = 90°$ ofwel $\angle AEB = 90°$.

16

61 **a** $y = \frac{1}{4}$ geeft $\sin(t) \cdot \cos(t) = \frac{1}{4}$

$2\sin(t) \cdot \cos(t) = \frac{1}{2}$

$\sin(2t) = \frac{1}{2}$

$2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \frac{5}{6}\pi + k \cdot 2\pi$

$t = \frac{1}{12}\pi + k \cdot \pi \vee t = \frac{5}{12}\pi + k \cdot \pi$

$0 \leq t < 2\pi$ geeft $t = \frac{1}{12}\pi \vee t = \frac{5}{12}\pi \vee t = 1\frac{1}{12}\pi \vee t = 1\frac{5}{12}\pi$

b $\cos(t) = 0 \wedge \sin(t)\cos(t) = 0$

$\cos(t) = 0$

$t = \frac{1}{2}\pi + k \cdot \pi$

$0 \leq t < 2\pi$ geeft $t = \frac{1}{2}\pi \vee t = 1\frac{1}{2}\pi$

$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos(t) \\ \sin(t) \cdot \cos(t) \end{pmatrix}$ geeft

$\vec{v}(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} -\sin(t) \\ \cos(t) \cdot \cos(t) + \sin(t) \cdot -\sin(t) \end{pmatrix} = \begin{pmatrix} -\sin(t) \\ \cos^2(t) - \sin^2(t) \end{pmatrix} = \begin{pmatrix} -\sin(t) \\ \cos(2t) \end{pmatrix}$

$\vec{v}\left(\frac{1}{2}\pi\right) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ en $\vec{v}\left(1\frac{1}{2}\pi\right) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$, dus de hoek tussen de richtingen is 90°.

Bladzijde 205

62 **a** Stel $P(0, p)$.

$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 0 \\ p \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ p-3 \end{pmatrix}$

$\overrightarrow{BP} = \vec{p} - \vec{b} = \begin{pmatrix} 0 \\ p \end{pmatrix} - \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \begin{pmatrix} -6 \\ p-7 \end{pmatrix}$

$AP \perp BP$ geeft $\begin{pmatrix} 2 \\ p-3 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ p-7 \end{pmatrix} = 0$

$-12 + p^2 - 10p + 21 = 0$

$p^2 - 10p + 9 = 0$

$(p-1)(p-9) = 0$

$p = 1 \vee p = 9$

$p = 1$ geeft $Q(0, 1)$ en $p = 9$ geeft $P(0, 9)$.

b Stel AB: $y = ax + b$ met $a = \text{rc}_{AB} = \dfrac{7-3}{6--2} = \dfrac{4}{8} = \frac{1}{2}$.

$\left.\begin{matrix} y = \frac{1}{2}x + b \\ \text{door } A(-2, 3) \end{matrix}\right\} \frac{1}{2} \cdot -2 + b = 3$

$-1 + b = 3$

$b = 4$

Dus AB: $y = \frac{1}{2}x + 4$ ofwel $\frac{1}{2}x - y + 4 = 0$.

$d(M, AB) = \dfrac{\left|-1\frac{1}{2} - 0 + 4\right|}{\sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2}} = \dfrac{\frac{5}{2}}{\sqrt{\frac{5}{4}}} = \frac{5}{2} \cdot \dfrac{2}{\sqrt{5}} = \sqrt{5}$

$r^2 = \left(\sqrt{5}\right)^2 + \left(\frac{1}{2}RS\right)^2 = 5 + \left(3\sqrt{5}\right)^2 = 5 + 45 = 50$

Dus $r = \sqrt{50} = 5\sqrt{2}$.

63 **a** P op de parabool geeft $y_P = \frac{1}{8}p^2 + 2$.

Dus $P\left(p, \frac{1}{8}p^2 + 2\right)$.

$FP = \sqrt{(p-0)^2 + \left(\frac{1}{8}p^2 + 2 - 4\right)^2} = \sqrt{(p-0)^2 + \left(\frac{1}{8}p^2 - 2\right)^2} = \sqrt{p^2 + \frac{1}{64}p^4 - \frac{1}{2}p^2 + 4}$

$= \sqrt{\frac{1}{64}p^4 + \frac{1}{2}p^2 + 4} = \sqrt{\left(\frac{1}{8}p^2 + 2\right)^2} = \frac{1}{8}p^2 + 2$

b $P\left(p, \frac{1}{8}p^2 + 2\right)$, dus het midden van PP' is $\left(p, \frac{1}{16}p^2 + 1\right)$.
Hieruit volgt m: $y = \frac{1}{16}p^2 + 1$.
$d(F, m) = 4 - \left(\frac{1}{16}p^2 + 1\right) = 3 - \frac{1}{16}p^2$
$d(F, P) = d(F, m)$ geeft $\frac{1}{8}p^2 + 2 = 3 - \frac{1}{16}p^2$

$$\frac{3}{16}p^2 = 1$$
$$p^2 = \frac{16}{3}$$
$$p = \sqrt{\frac{16}{3}} = \frac{4}{\sqrt{3}} = \frac{4}{3}\sqrt{3}$$

16.6 Eindexamens 2016

2016, tijdvak 1

Bladzijde 206

1 $f(x) = \frac{1}{2}e^{\frac{1}{2}x} + 2\,e^{-\frac{1}{2}x} + 1\frac{1}{2}$ geeft $f'(x) = \frac{1}{2} \cdot \frac{1}{2}e^{\frac{1}{2}x} - \frac{1}{2} \cdot 2e^{-\frac{1}{2}x} = \frac{1}{4}e^{\frac{1}{2}x} - e^{-\frac{1}{2}x}$

$f'(x) = 0$ geeft $\frac{1}{4}e^{\frac{1}{2}x} - e^{-\frac{1}{2}x} = 0$

$$e^{\frac{1}{2}x} = 4\,e^{-\frac{1}{2}x}$$
$$e^x = 4$$
$$x = \ln(4)$$

Dus $x_T = \ln(4)$.

Bladzijde 207

2 $f(\ln(4)) = \frac{1}{2} \cdot 2 + 2 \cdot \frac{1}{2} + 1\frac{1}{2} = 3\frac{1}{2}$, dus $T\left(\ln(4), 3\frac{1}{2}\right)$.
$f(0) = \frac{1}{2} \cdot 1 + 2 \cdot 1 + 1\frac{1}{2} = 4$, dus $A(0, 4)$.

$\left.\begin{array}{l} y = a(x-b) + c \\ \text{top } \left(\ln(4), 3\frac{1}{2}\right) \end{array}\right\} y = a(x - \ln(4)) + 3\frac{1}{2}$

$\left.\begin{array}{l} y = a(x - \ln(4)) + 3\frac{1}{2} \\ \text{door } A(0, 4) \end{array}\right\} a(0 - \ln(4)) + 3\frac{1}{2} = 0$

$$-a\ln(4) = -3\frac{1}{2}$$
$$a = \frac{3\frac{1}{2}}{\ln(4)}$$

$y = \dfrac{7}{2\ln(4)}(x - \ln(4)) + 3\frac{1}{2}$

Los op $\left| f(x) - \dfrac{7}{2\ln(4)}(x - \ln(4)) + 3\frac{1}{2} \right| = 1$

Voer in $y_1 = \left| f(x) - \dfrac{7}{2\ln(4)}(x - \ln(4)) + 3\frac{1}{2} \right|$ en $y_2 = 1$.

Intersect geeft $x \approx 5{,}1$.

Bladzijde 208

3 $AB = 1 + 4 = 5$

$\cos(\alpha) = \dfrac{AE}{AC} = \dfrac{AE}{1}$, dus $AE = \cos(\alpha)$.

$\sin(\alpha) = \dfrac{CE}{AC} = \dfrac{CE}{1}$, dus $CE = \sin(\alpha)$.

De stelling van Pythagoras in $\triangle CED$ geeft $ED^2 + CE^2 = CD^2$

$$ED^2 + \sin^2(\alpha) = 4^2$$
$$ED^2 = 16 - \sin^2(\alpha)$$
$$ED = \sqrt{16 - \sin^2(\alpha)}$$

$s = BD = AB - AE - ED = 5 - \cos(\alpha) - \sqrt{16 - \sin^2(\alpha)}$

4 Het maximale verschil tussen z en s is $|z - s| = \left|1 - \cos(\alpha) + \frac{1}{8}\sin^2(\alpha) - \left(5 - \cos(\alpha) - \sqrt{16 - \sin^2(\alpha)}\right)\right| =$
$\left|1 - \cos(\alpha) + \frac{1}{8}\sin^2(\alpha) - 5 + \cos(\alpha) + \sqrt{16 - \sin^2(\alpha)}\right| = \left|\frac{1}{8}\sin^2(\alpha) - 4 + \sqrt{16 - \sin^2(\alpha)}\right|$.

Voer in $y_1 = \left|\frac{1}{8}\sin^2(x) - 4 + \sqrt{16 - \sin^2(x)}\right|$.
De optie maximum geeft $x = 1{,}57...$ en $y = 0{,}0020...$
Dus het maximale verschil is 0,002.

5 $z = 1 - \cos(\alpha) + \frac{1}{8}\sin^2(\alpha)$ geeft $\frac{dz}{d\alpha} = \sin(\alpha) + 2 \cdot \frac{1}{8}\sin(\alpha) \cdot \cos(\alpha) = \sin(\alpha) + \frac{1}{4}\sin(\alpha) \cdot \cos(\alpha)$

Voer in $y_1 = \sin(x) + \frac{1}{4}\sin(x) \cdot \cos(x)$.
De optie maximum geeft $x = 1{,}344...$ en $y = 1{,}029...$
Dus de maximale snelheid van de zuiger is 1,03.

Bladzijde 209

6 $y = ax$ substitueren in $(x-1)^2 + y^2 = 1$ geeft $(x-1)^2 + (ax)^2 = 1$

$$x^2 - 2x + 1 + a^2x^2 = 1$$
$$(a^2+1)x^2 = 2x$$
$$x = 0 \vee (a^2+1)x = 2$$
$$x = 0 \vee x = \frac{2}{a^2+1}$$

Dus $S\left(\frac{2}{a^2+1}, \frac{2a}{a^2+1}\right)$.

$$\overrightarrow{OP} = \overrightarrow{OS} + \overrightarrow{OS_R} = \begin{pmatrix} \frac{2}{a^2+1} \\ \frac{2a}{a^2+1} \end{pmatrix} + \begin{pmatrix} \frac{2a}{a^2+1} \\ \frac{-2}{a^2+1} \end{pmatrix} = \begin{pmatrix} \frac{2a+2}{a^2+1} \\ \frac{2a-2}{a^2+1} \end{pmatrix}$$

Dus $x_P = \frac{2a+2}{a^2+1}$ en $y_P = \frac{2a-2}{a^2+1}$.

7 $a = 1$ geeft $P(2, 0)$
$a = 0$ geeft $P(2, -2)$
$a = -1$ geeft $P(0, -2)$
De cirkel door de punten (0, 0), (2, 0), (2, −2) en (0, −2) heeft middelpunt (1, −1) en straal $\sqrt{2}$.
Dus een vergelijking van deze cirkel is $(x-1)^2 + (y+1)^2 = 2$.

8 x_P is maximaal in een snijpunt van de cirkel met de horizontale lijn door het middelpunt. Dat is de lijn $y = -1$.

$y = -1$ geeft $\frac{2a-2}{a^2+1} = -1$

$$2a - 2 = -a^2 - 1$$
$$a^2 + 2a - 1 = 0$$
$$(a+1)^2 - 1 - 1 = 0$$
$$(a+1)^2 = 2$$
$$a + 1 = \sqrt{2} \vee a + 1 = -\sqrt{2}$$
$$a = -1 + \sqrt{2} \vee a = -1 - \sqrt{2}$$

$a = -1 + \sqrt{2}$ geeft $(1 + \sqrt{2}, -1)$ en $a = -1 - \sqrt{2}$ geeft $(1 - \sqrt{2}, -1)$.
Dus x_P is maximaal voor $a = -1 + \sqrt{2}$.

Bladzijde 210

9 $x = 0$ geeft $\sin(2t) + \sin(t) = 0$

$$\sin(2t) = -\sin(t)$$
$$\sin(2t) = \sin(-t)$$
$$2t = -t + k \cdot 2\pi \vee 2t = \pi - -t + k \cdot 2\pi$$
$$3t = k \cdot 2\pi \vee t = \pi + k \cdot 2\pi$$
$$t = k \cdot \tfrac{2}{3}\pi \vee t = \pi + k \cdot 2\pi$$

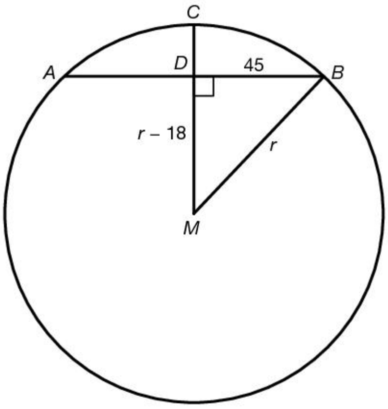
$0 < t < \pi$ geeft $t = \frac{2}{3}\pi$

$\frac{dx}{dt} = 2 \cdot \cos(2t) + \cos(t)$ en $\frac{dy}{dt} = -\sin(t)$ geeft $\vec{v} = \begin{pmatrix} 2\cos(2t) + \cos(t) \\ -\sin(t) \end{pmatrix}$

$t = \frac{2}{3}\pi$ geeft $\vec{v} = \begin{pmatrix} 2 \cdot -\frac{1}{2} - \frac{1}{2} \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2} \\ -\frac{1}{2}\sqrt{3} \end{pmatrix}$

Dus in B is de snelheid $\sqrt{(-1\frac{1}{2})^2 + (-\frac{1}{2}\sqrt{3})^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$.

10



Stel de straal van de cirkel is r.
$CD = 18$ geeft $MD = r - 18$.
De stelling van Pythagoras in $\triangle MBD$ geeft $MB^2 = MD^2 + BD^2$

$$r^2 = (r-18)^2 + 45^2$$
$$r^2 = r^2 - 36r + 324 + 2025$$
$$36r = 2349$$
$$r = 65\tfrac{1}{4}$$

Dus de straal van de cirkel is 65 cm.

Bladzijde 211

11 $a = 1$ geeft $b = 1 + 16 = 17$

$$\text{inhoud} = \pi \cdot \int_1^{17} \left(16^2 - \left(\frac{16}{\sqrt{x}}\right)^2\right) \mathrm{d}x = \pi \cdot \int_1^{17} \left(256 - \frac{256}{x}\right) \mathrm{d}x =$$

$$\pi\left[256x - 256\ln(x)\right]_1^{17} = \pi(256 \cdot 17 - 256\ln(17)) - \pi \cdot (256 \cdot 1 - 256\ln(0)) = \pi(4096 - 256\ln(17))$$

12 $b = a + AB = a + AD = a + \dfrac{16}{\sqrt{a}} = a + 16a^{-\frac{1}{2}}$

$$\frac{\mathrm{d}b}{\mathrm{d}a} = 1 - \tfrac{1}{2} \cdot 16a^{-1\frac{1}{2}} = 1 - 8a^{-1\frac{1}{2}}$$

$\dfrac{\mathrm{d}b}{\mathrm{d}a} = 0$ geeft $1 - 8a^{-1\frac{1}{2}} = 0$

$$1 = 8a^{-1\frac{1}{2}}$$
$$a^{-1\frac{1}{2}} = \tfrac{1}{8}$$
$$a^{-1\frac{1}{2}} = 2^{-3}$$
$$a^{-1\frac{1}{2}} = (2^2)^{-1\frac{1}{2}}$$
$$a = 2^2 = 4$$

$a = 4$ geeft $b = 4 + \dfrac{16}{\sqrt{4}} = 4 + \dfrac{16}{2} = 4 + 8 = 12$

Dus de minimale waarde van b is 12.

Bladzijde 212

13 $y = \dfrac{1}{c(x-1)} + 1$

↓ spiegelen in $y = x$

$$x = \frac{1}{c(y-1)} + 1$$
$$x - 1 = \frac{1}{c(y-1)}$$
$$c(x-1)(y-1) = 1$$
$$y - 1 = \frac{1}{c(x-1)}$$
$$y = \frac{1}{c(x-1)} + 1$$

Dus f_c is de inverse van zichzelf.

14 $\dfrac{f_c(1+p)+f_c(1-p)}{2} = \dfrac{\dfrac{1}{c(1+p-1)}+1+\dfrac{1}{c(1-p-1)}+1}{2} = \dfrac{\dfrac{1}{cp}+1+\dfrac{1}{-cp}+1}{2} = \dfrac{2}{2} = 1$

Dus de grafiek van f_c is puntsymmetrisch ten opzichte van S.

15 Oplossen van de vergelijking $f_c(x) = x$ geeft $\dfrac{1}{c(x-1)} + 1 = x$

$$\frac{1}{c(x-1)} = x - 1$$
$$c(x-1)^2 = 1$$
$$(x-1)^2 = \frac{1}{c}$$
$$x - 1 = \sqrt{\frac{1}{c}} \vee x - 1 = -\sqrt{\frac{1}{c}}$$
$$x = 1 + \sqrt{\frac{1}{c}} \vee x = 1 - \sqrt{\frac{1}{c}}$$

Dus $x_A = 1 - \sqrt{\dfrac{1}{c}}$.

A ligt op de lijn $y = x$, dus $y_A = 1 - \sqrt{\dfrac{1}{c}}$.

$\lim\limits_{c \to \infty} x_A = \lim\limits_{c \to \infty} y_A = \lim\limits_{c \to \infty} 1 - \sqrt{\dfrac{1}{c}} = 1 - \sqrt{0} = 1$, dus het limietpunt van A is $S(1, 1)$.

Bladzijde 213

16 $\alpha + \beta + 2 \cdot 90° = 360°$ (volle hoek), dus $\beta = 180° - \alpha$.
De oppervlakte van de lichtgrijze delen is $p^2 + q^2$.
De cosinusregel in $r^2 = p^2 + q^2 - 2pq\cos(\alpha)$
De cosinusregel in $s^2 = p^2 + q^2 - 2pq\cos(\beta) = p^2 + q^2 - 2pq\cos(180° - \alpha) = p^2 + q^2 + 2pq\cos(\alpha)$
Hieruit volgt $\frac{1}{2}r^2 + \frac{1}{2}s^2 = \frac{1}{2}(p^2 + q^2 - 2pq\cos(\alpha)) + \frac{1}{2}(p^2 + q^2 + 2pq\cos(\alpha)) = p^2 + q^2$.
Dus de oppervlakte van de donkergrijze delen is gelijk aan de oppervlakte van de lichtgrijze delen.

2016, tijdvak 2

Bladzijde 214

1 $y = (x+1)^3 - 1$
$\downarrow$ spiegelen in $y = x$
$x = (y+1)^3 - 1$
$(y+1)^3 = x + 1$
$y + 1 = \sqrt[3]{x+1}$
$y = \sqrt[3]{x+1} - 1$
Dus g is de inverse van f.

2 De grafieken van f en $g = f^{\text{inv}}$ snijden elkaar op de lijn $y = x$.
Dus los op $f(x) = x$.
Dit geeft $(x+1)^3 - 1 = x$
$(x+1)^3 = x + 1$
$x + 1 = 0 \vee (x+1)^2 = 1$
$x = -1 \vee x + 1 = 1 \vee x + 1 = -1$
$x = -1 \vee x = 0 \vee x = -2$
De gemeenschappelijke punten zijn $(-1, -1)$, $(0, 0)$ en $(-2, -2)$.

Bladzijde 215

3 $E = \frac{I_{spot}}{4\pi r^2} \cdot \cos(\alpha) = \frac{I_{spot}}{4\pi r^2} \cdot \frac{x}{r} = \frac{I_{spot}}{4\pi} \cdot \frac{x}{(x^2 + d^2) \cdot \sqrt{x^2 + d^2}} = \frac{I_{spot}}{4\pi} \cdot \frac{x}{(x^2 + d^2)^{\frac{3}{2}}}$

4 $d = 10$ geeft $E = \frac{I_{spot}}{4\pi^2} \cdot \frac{x}{(x^2 + 100)^{\frac{3}{2}}}$

$\frac{dE}{dx} = \frac{I_{spot}}{4\pi^2} \cdot \frac{(x^2 + 100)^{\frac{3}{2}} \cdot 1 - x \cdot \frac{3}{2}(x^2 + 100)^{\frac{1}{2}} \cdot 2x}{(x^2 + 100)^3} = \frac{I_{spot}}{4\pi^2} \cdot \frac{(x^2 + 100)^{\frac{3}{2}} - 3x^2(x^2 + 100)^{\frac{1}{2}}}{(x^2 + 100)^3}$

$= \frac{I_{spot}}{4\pi^2} \cdot \frac{x^2 + 100 - 3x^2}{(x^2 + 100)^{2\frac{1}{2}}} = \frac{I_{spot}}{4\pi^2} \cdot \frac{100 - 2x^2}{(x^2 + 100)^{2\frac{1}{2}}}$

$\frac{dE}{dx} = 0$ geeft $\frac{I_{spot}}{4\pi^2} \cdot \frac{100 - 2x^2}{(x^2 + 100)^{2\frac{1}{2}}} = 0$

$100 - 2x^2 = 0$

$x^2 = 50$

$x = \sqrt{50} = 7{,}071...$

Dus E is maximaal voor $x \approx 7{,}1$ mm.

5 De horizontale afstand van de rechter spot tot P is $40 - d$.

De totale verlichtingssterkte in P is $E_{\text{linker spot}} + E_{\text{rechter spot}} = \frac{500}{4\pi} \cdot \frac{25}{(25^2 + d^2)^{\frac{3}{2}}} + \frac{500}{4\pi} \cdot \frac{25}{(25^2 + (40 - d)^2)^{\frac{3}{2}}}$.

Voer in $y_1 = \frac{500}{4\pi} \cdot \frac{25}{(25^2 + x^2)^{\frac{3}{2}}} + \frac{500}{4\pi} \cdot \frac{25}{(25^2 + (40 - x)^2)^{\frac{3}{2}}}$.

De optie maximum geeft $x = 1{,}92...$ en $y = 0{,}0736...$ en $x = 38{,}07...$ en $y = 0{,}0736...$

De optie minimum geeft $x = 20$ en $y = 0{,}0606...$

80% van 0,0736... is 0,0589...

De minimale waarde van de verlichtingssterkte is dus minstens 80% van de hoogste waarde.

Het werkoppervlak wordt dus voldoende gelijkmatig verlicht.

Bladzijde 216

6 De stelling van Pythagoras in $\triangle ADM$ geeft $AD^2 + (DN - 1)^2 = 1^2$ ofwel $AD^2 = 1 - (DN - 1)^2$

De stelling van Pythagoras in $\triangle ADN$ geeft $AD^2 + DN^2 = r^2$ ofwel $AD^2 = r^2 - DN^2$

Hieruit volgt $1 - (DN - 1)^2 = r^2 - DN^2$

$1 - DN^2 + 2DN - 1 = r^2 - DN^2$

$2DN = r^2$

$DN = \frac{1}{2}r^2$

7 $\left.\begin{array}{l} CN = r \\ MN = 1 \end{array}\right\} CM = r - 1$

$\left.\begin{array}{l} CD = DM \\ CM = r - 1 \end{array}\right\} DN = r - \frac{1}{2}(r - 1) = \frac{1}{2}r + \frac{1}{2}$

Ook geldt $DN = \frac{1}{2}r^2$, dus $\frac{1}{2}r^2 = \frac{1}{2}r + \frac{1}{2}$

$r^2 = r + 1$

$r^2 - r - 1 = 0$

$D = (-1)^2 - 4 \cdot 1 \cdot -1 = 5$

$r = \frac{1 - \sqrt{5}}{2} \vee r = \frac{1 + \sqrt{5}}{2}$

vold. niet   vold.

Dus $r = \frac{1}{2} + \frac{1}{2}\sqrt{5}$.

Bladzijde 217

8 Er is een perforatie als $1 + \cos(2x) = 0 \wedge \cos(x) = 0$

$$\cos(2x) = -1 \wedge x = \tfrac{1}{2}\pi + k \cdot \pi$$
$$2x = \pi + k \cdot 2\pi \wedge x = \tfrac{1}{2}\pi + k \cdot \pi$$
$$x = \tfrac{1}{2}\pi + k \cdot \pi \wedge x = \tfrac{1}{2}\pi + k \cdot \pi$$
$$x = \tfrac{1}{2}\pi + k \cdot \pi$$

$$\lim_{x \to \frac{1}{2}\pi} \left(\frac{1 + \cos(2x)}{\cos(x)} + 1 \right) = \lim_{x \to \frac{1}{2}\pi} \left(\frac{1 + 2\cos^2(x) - 1}{\cos(x)} + 1 \right) = \lim_{x \to \frac{1}{2}\pi} \left(\frac{2\cos^2(x)}{\cos(x)} + 1 \right) =$$

$$\lim_{x \to \frac{1}{2}\pi} (2\cos(x) + 1) = 2 \cdot 0 + 1 = 1$$

$$\lim_{x \to 1\frac{1}{2}\pi} \left(\frac{1 + \cos(2x)}{\cos(x)} + 1 \right) = \lim_{x \to 1\frac{1}{2}\pi} (2\cos(x) + 1) = 2 \cdot 0 + 1 = 1$$

Dus de perforaties zijn $\left(\tfrac{1}{2}\pi, 1\right)$ en $\left(1\tfrac{1}{2}\pi, 1\right)$.

9 $AP = f(p) - 1 = \ln(p^2 + 1) - 1$ en

$$BP = 1 - g(p) = 1 - \ln\left(\frac{e^2}{p^2 + 1}\right) = 1 - \ln(e^2) + \ln(p^2 + 1) = 1 - 2 + \ln(p^2 + 1) = \ln(p^2 + 1) - 1$$

Dus $AP = BP$.

Bladzijde 218

10 $y = \ln(x^2 + 1)$

$x^2 + 1 = e^y$

$x^2 = e^y - 1$

$$\text{inhoud} = 2 \cdot \pi \cdot \int_0^1 (e^y - 1)dy = 2\pi\left[e^y - y\right]_0^1 = 2\pi(e - 1 - (1 - 0)) = 2\pi(e - 2)$$

11 $y = \ln(x^2 + 1)$

↓ translatie (2, 0)

$y = \ln((x - 2)^2 + 1) = \ln(x^2 - 4x + 5)$

Voor het snijpunt geldt $\ln(x^2 + 1) = \ln(x^2 - 4x + 5)$

$$x^2 + 1 = x^2 - 4x + 5$$
$$4x = 4$$
$$x = 1$$

$f(x) = \ln(x^2 + 1)$ geeft $f'(x) = \dfrac{1}{x^2 + 1} \cdot 2x = \dfrac{2x}{x^2 + 1}$

$$f'(1) = \frac{2}{1 + 1} = 1$$

De afgeleide van de verschoven grafiek voor $x = 1$ is $f'(-1) = \dfrac{-2}{1 + 1} = -1$

$1 \cdot -1 = -1$, dus de grafieken snijden elkaar loodrecht.

Bladzijde 219

12 Voer in $y_1 = 125\cos\left(\dfrac{2\pi}{745}x\right)$ en $y_2 = 40$. Intersect geeft $x = 147{,}62...$ en $x = 596{,}37...$

De droogligtijd is $596{,}37... - 147{,}62... = 449{,}74... \approx 450$ minuten.

13 De grafiek van h is symmetrisch in de lijn $x = \frac{745}{2}$.

$$\text{Dus} \begin{cases} t_2 + t_1 = 745 \\ t_2 - t_1 = D \end{cases} \; -$$

$$2t_1 = 745 - D$$
$$t_1 = \tfrac{745}{2} - \tfrac{1}{2}D$$

$$z = h(t_1) = 125\cos\left(\frac{2\pi}{745}\left(\tfrac{745}{2} - \tfrac{1}{2}D\right)\right) = 125\cos\left(\pi - \frac{\pi}{745}D\right)$$

Bladzijde 220

14 $z = 125\cos\left(\pi - \frac{\pi}{745}D\right)$ geeft $\frac{dz}{dD} = -125\sin\left(\pi - \frac{\pi}{745}D\right) \cdot -\frac{\pi}{745} = \frac{125\pi}{745}\sin\left(\pi - \frac{\pi}{745}D\right)$

$\left[\frac{dz}{dD}\right]_{D=372{,}5} = \frac{125\pi}{745}\sin\left(\pi - \frac{\pi}{745}\cdot 372{,}5\right) = \frac{125\pi}{745}\sin\left(\tfrac{1}{2}\pi\right) = \frac{125\pi}{745}$

Dus de helling bij figuur 3 is $\frac{745}{125\pi} \approx 1{,}9$.

$D = 8 \cdot 10^{-5}z^3 + 1{,}7z + 372{,}5$ geeft $\frac{dD}{dz} = 2{,}4 \cdot 10^{-4}z^2 + 1{,}7$

$\left[\frac{dD}{dz}\right]_{z=0} = 1{,}7$

Dus de helling bij figuur 4 is 1,7.

Bladzijde 221

15 $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$, dus een normaalvector van de middelloodlijn van AB is $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

$\left.\begin{array}{l} 3x - y = c \\ M_{AB} \text{ is } (3, 1) \end{array}\right\} c = 3 \cdot 3 - 1 = 8$

$3x - y = 8$ snijden met k geeft $3(2 + 4t) - (1 + 5t) = 8$

$6 + 12t - 1 - 5t = 8$

$7t = 3$

$t = \frac{3}{7}$

$t = \frac{3}{7}$ geeft $P\left(2 + \frac{3}{7} \cdot 4,\ 1 + \frac{3}{7} \cdot 5\right) = P\left(3\frac{5}{7},\ 3\frac{1}{7}\right)$

16 $P(2 + 4t,\ 1 + 5t)$

$d(P, y\text{-as}) = 2 + 4t$

$rc_m = \frac{0 - 2}{6 - 0} = -\frac{1}{3}$ en m door (0, 2), dus m: $y = -\frac{1}{3}x + 2$ ofwel $x + 3y = 6$.

$d(P, m) = \frac{|2 + 4t + 3(1 + 5t) - 6|}{\sqrt{1^2 + 3^2}} = \frac{|19t - 1|}{\sqrt{10}}$

Voer in $y_1 = 2 + 4x$ en $y_2 = \frac{|19x - 1|}{\sqrt{10}}$.

Intersect geeft $x = -0{,}168...$ en $x = 1{,}153...$

Invullen in $2 + 4t$ geeft 1,327... en 6,613...

Dus de stralen zijn 1,33 en 6,61.

Gemengde opgaven

13 Limieten en asymptoten

Bladzijde 222

1 **a** $\lim\limits_{x \to 3} \dfrac{x^3 - 9x}{x^2 - 9} = \lim\limits_{x \to 3} \dfrac{x(x^2 - 9)}{x^2 - 9} = \lim\limits_{x \to 3} x = 3$

b $\lim\limits_{x \to \infty} \dfrac{2a - x^2}{(x + a)(x - a)} = \lim\limits_{x \to \infty} \dfrac{2a - x^2}{x^2 - a^2} = \lim\limits_{x \to \infty} \dfrac{\frac{2a}{x^2} - 1}{1 - \frac{a^2}{x^2}} = \dfrac{0 - 1}{1 - 0} = -1$

c $\lim\limits_{x \to \infty} \dfrac{\ln(\sqrt{x})}{3 + \ln(x^4)} = \lim\limits_{x \to \infty} \dfrac{\frac{1}{2}\ln(x)}{3 + 4\ln(x)} = \lim\limits_{x \to \infty} \dfrac{\frac{1}{2}}{\frac{3}{\ln(x)} + 4} = \dfrac{\frac{1}{2}}{0 + 4} = \frac{1}{8}$

d $\lim\limits_{x \to 3} \dfrac{e^{2x} - e^6}{e^3 - e^x} = \lim\limits_{x \to 3} \dfrac{(e^x - e^3)(e^x + e^3)}{-(e^x - e^3)} = \lim\limits_{x \to 3} -(e^x + e^3) = -2e^3$

e $\lim\limits_{x \to \infty} \dfrac{e^{2x+2}}{e^{2x} + 2} = \lim\limits_{x \to \infty} \dfrac{\frac{e^{2x+2}}{e^{2x}}}{1 + \frac{2}{e^{2x}}} = \lim\limits_{x \to \infty} \dfrac{e^2}{1 + \frac{2}{e^{2x}}} = \dfrac{e^2}{1 + 0} = e^2$

f $\lim\limits_{x \downarrow 0} \dfrac{\ln\left(\frac{1}{x}\right)}{2 + \ln(x^3)} = \lim\limits_{x \downarrow 0} \dfrac{\ln(x^{-1})}{2 + 3\ln(x)} = \lim\limits_{x \downarrow 0} \dfrac{-\ln(x)}{2 + 3\ln(x)} = \lim\limits_{x \downarrow 0} \dfrac{-1}{\frac{2}{\ln(x)} + 3} = \dfrac{-1}{0 + 3} = -\frac{1}{3}$

2 **a** De grafiek van $f(x) = \dfrac{ax + 4}{2x - 3}$ heeft een perforatie als $\begin{cases} ax + 4 = 0 \\ 2x - 3 = 0 \end{cases} \begin{vmatrix} 2 \\ a \end{vmatrix}$ geeft $\begin{cases} 2ax + 8 = 0 \\ 2ax - 3a = 0 \end{cases}$ −

$8 + 3a = 0$
$3a = -8$
$a = -2\frac{2}{3}$

b De grafieken van f en f^{inv} snijden elkaar op de lijn $y = x$, dus $f(4) = 4$

$\dfrac{4a + 4}{8 - 3} = 4$
$4a + 4 = 20$
$4a = 16$
$a = 4$

c $\lim\limits_{x \to \infty} \dfrac{ax + 4}{2x - 3} = \lim\limits_{x \to \infty} \dfrac{a + \frac{4}{x}}{2 - \frac{3}{x}} = \dfrac{a + 0}{2 - 0} = \frac{1}{2}a$, dus de horizontale asymptoot is de lijn $y = \frac{1}{2}a$.

$2x - 3 = 0$ geeft $x = 1\frac{1}{2}$, dus de horizontale asymptoot is de lijn $x = 1\frac{1}{2}$.

Het snijpunt $\left(1\frac{1}{2}, \frac{1}{2}a\right)$ ligt op de parabool $y = x^2$ als $\frac{1}{2}a = \left(1\frac{1}{2}\right)^2$

$\frac{1}{2}a = 2\frac{1}{4}$
$a = 4\frac{1}{2}$

3 **a** $f(x) = \dfrac{x^2 - x}{2 - |x-1|} = \begin{cases} \dfrac{x^2 - x}{2 - (x-1)} = \dfrac{x^2 - x}{3 - x} \text{ voor } x \geq 1 \\ \dfrac{x^2 - x}{2 - (-x+1)} = \dfrac{x^2 - x}{x+1} \text{ voor } x < 1 \end{cases}$

De verticale asymptoten zijn de lijnen $x = 3$ en $x = -1$.

$f(x) = \dfrac{x^2 - x}{3 - x} = \dfrac{-x(3-x) + 3x - x}{3 - x} = -x + \dfrac{2x}{3-x} = -x + \dfrac{-2(3-x) + 6}{3 - x} = -x - 2 + \dfrac{6}{3-x}$

$\lim\limits_{x \to \infty} \dfrac{6}{3-x} = 0$, dus de lijn $y = -x - 2$ is scheve asymptoot.

$f(x) = \dfrac{x^2 - x}{x+1} = \dfrac{x(x+1) - x - x}{x+1} = x - \dfrac{2x}{x+1} = x - \dfrac{2(x+1) - 2}{x+1} = x - 2 + \dfrac{2}{x+1}$

$\lim\limits_{x \to -\infty} \dfrac{2}{x+1} = 0$, dus de lijn $y = x - 2$ is scheve asymptoot.

b $f(x) = \dfrac{x^2 - x}{x+1}$ geeft $f'(x) = \dfrac{(x+1)(2x-1) - (x^2 - x) \cdot 1}{(x+1)^2} = \dfrac{2x^2 - x + 2x - 1 - x^2 + x}{(x+1)^2} = \dfrac{x^2 + 2x - 1}{(x+1)^2}$

$f(x) = 0$ geeft $x^2 + 2x - 1 = 0$

$(x+1)^2 - 1 - 1 = 0$

$(x+1)^2 = 2$

$x + 1 = \sqrt{2} \vee x + 1 = -\sqrt{2}$

$x = -1 + \sqrt{2} \vee x = -1 - \sqrt{2}$

vold. niet vold.

$y_A = f(-1 - \sqrt{2}) = \dfrac{(-1-\sqrt{2})^2 - (-1-\sqrt{2})}{-1 + \sqrt{2} + 1} = \dfrac{1 + 2\sqrt{2} + 2 + 1 + \sqrt{2}}{\sqrt{2}} = \dfrac{4 + 3\sqrt{2}}{\sqrt{2}} = 2\sqrt{2} + 3$

c Voer in $y_1 = \dfrac{x^2 - x}{2 - |x-1|}$.

De optie maximum geeft $x \approx -2{,}41$ en $y \approx -5{,}83$ en $x \approx 5{,}45$ en $y \approx -9{,}90$.

De optie minimum geeft $x \approx 0{,}41$ en $y \approx -0{,}17$.

$f(x) = p$ heeft precies twee oplossingen voor $-9{,}90 < p < -5{,}83$ en voor $p > -0{,}17$.

d Voor $x < 1$:

$f(x) = 2$ geeft $\dfrac{x^2 - x}{x+1} = 2$

$x^2 - x = 2x + 2$

$x^2 - 3x - 2 = 0$

$D = (-3)^2 - 4 \cdot 1 \cdot -2 = 17$

$x = \dfrac{3 + \sqrt{17}}{2} \vee x = \dfrac{3 - \sqrt{17}}{2}$

vold. niet vold.

Voor $x \geq 1$:

$f(x) = 2$ geeft $\dfrac{x^2 - x}{3 - x} = 2$

$x^2 - x = 6 - 2x$

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$

$x = 2 \vee x = -3$

vold. vold. niet

$f(x) \leq 2$ geeft $x < -1 \vee \dfrac{3 - \sqrt{17}}{2} \leq x \leq 2 \vee x > 3$

Bladzijde 223

4 **a** verticale asymptoten:

$\sin^2(x) - \frac{1}{2} = 0 \wedge \sin(x) \neq 0$

$\sin^2(x) = \frac{1}{2} \wedge \sin(x) \neq 0$

$\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$

$x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = 1\frac{1}{4}\pi + k \cdot 2\pi$

x op $[-\pi, \pi]$ geeft $x = -\frac{3}{4}\pi \vee x = -\frac{1}{4}\pi \vee x = \frac{1}{4}\pi \vee x = \frac{3}{4}\pi$

Dus de verticale asymptoten zijn de lijnen $x = -\frac{3}{4}\pi$, $x = -\frac{1}{4}\pi$, $x = \frac{1}{4}\pi$ en $x = \frac{3}{4}\pi$.

b $f(x) = \dfrac{\sin(x)}{\sin^2(x) - \frac{1}{2}}$ geeft $f'(x) = \dfrac{\left(\sin^2(x) - \frac{1}{2}\right) \cdot \cos(x) - \sin(x) \cdot 2\sin(x) \cdot \cos(x)}{\left(\sin^2(x) - \frac{1}{2}\right)^2} =$

$\dfrac{\sin^2(x)\cos(x) - \frac{1}{2}\cos(x) - 2\sin^2(x) \cdot \cos(x)}{\left(\sin^2(x) - \frac{1}{2}\right)^2} = \dfrac{-\sin^2(x)\cos(x) - \frac{1}{2}\cos(x)}{\left(\sin^2(x) - \frac{1}{2}\right)^2} = \dfrac{-\cos(x)\left(\sin^2(x) + \frac{1}{2}\right)}{\left(\sin^2(x) - \frac{1}{2}\right)^2}$

$f'(x) = 0$ geeft $-\cos(x)\left(\sin^2(x) + \frac{1}{2}\right) = 0$

$\cos(x) = 0 \vee \sin^2(x) = -\frac{1}{2}$

$\cos(x) = 0$

$x = -\frac{1}{2}\pi \vee x = \frac{1}{2}\pi$

$f\left(-\frac{1}{2}\pi\right) = \dfrac{-1}{(-1)^2 - \frac{1}{2}} = \dfrac{-1}{1 - \frac{1}{2}} = -2$ en $f\left(\frac{1}{2}\pi\right) = 2$

Dus $A\left(-\frac{1}{2}\pi, -2\right)$ en $B\left(\frac{1}{2}\pi, 2\right)$.

$\overrightarrow{OA} = \begin{pmatrix} -\frac{1}{2}\pi \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} \frac{1}{2}\pi \\ 2 \end{pmatrix} = \overrightarrow{OB}$, dus de lijn door A en B gaat door de oorsprong.

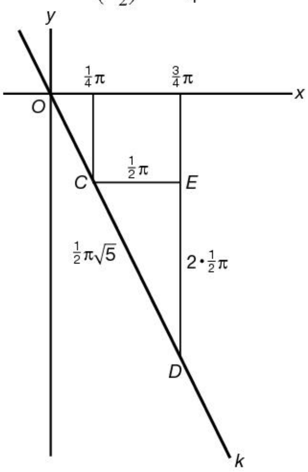
Alternatieve uitwerking

De grafiek van f is puntsymmetrisch in O want voor elke p geldt

$f(-p) = \dfrac{\sin(-p)}{\sin^2(-p) - \frac{1}{2}} = \dfrac{-\sin(p)}{(-\sin(p))^2 - \frac{1}{2}} = -\dfrac{\sin(p)}{\sin^2(p) - \frac{1}{2}} = -f(p)$.

De toppen zijn dus elkaars spiegelbeeld in O en liggen dus op één lijn die door de oorsprong gaat.

c $f'(0) = \dfrac{-\frac{1}{2}}{\left(-\frac{1}{2}\right)^2} = \dfrac{-\frac{1}{2}}{\frac{1}{4}} = -2$



$CE = \frac{3}{4}\pi - \frac{1}{4}\pi = \frac{1}{2}\pi$

$rc_k = -2$ geeft $ED = 2 \cdot \frac{1}{2}\pi$

De stelling van Pythagoras in $\triangle CDE$ geeft $CD = \frac{1}{2}\pi\sqrt{5}$.

5 **a** $\lim\limits_{x \to 2} f_{p,q}(x)$ bestaat als $\lim\limits_{x \uparrow 2}(x^2 + 4px) = \lim\limits_{x \downarrow 2}(2x + q)$

$4 + 8p = 4 + q$

$q = 8p$

$\lim\limits_{x \to 3} f_{p,q}(x)$ bestaat als $\lim\limits_{x \uparrow 3}(2x + q) = \lim\limits_{x \downarrow 3}(x^3 - 4x + p)$

$6 + q = 27 - 12 + p$

$q = 9 + p$

$q = 8p$ invullen in $q = 9 + p$ geeft $8p = 9 + p$

$7p = 9$

$p = 1\frac{2}{7}$

$q = 8 \cdot 1\frac{2}{7} = 10\frac{2}{7}$

b $A(3, 10)$ is een perforatie als $\lim\limits_{x \uparrow 3}(2x + q) = 10 \wedge \lim\limits_{x \downarrow 3}(x^3 - 4x + p) = 10$

$6 + q = 10 \wedge 27 - 12 + p = 10$

$q = 4 \wedge p = -5$

c $f_{p,q}$ heeft een extreme waarde voor $x = -1$ als $\dfrac{-4p}{2} = -1$

$-4p = -2$

$p = \frac{1}{2}$

$\lim\limits_{x \to 3} f_{p,q}(x)$ bestaat als $q = p + 9$, dus $q = \frac{1}{2} + 9 = 9\frac{1}{2}$

6 **a** $f_a(x) = \frac{x^3 + ax^2 + 4}{x^2 - 2x} = \frac{x(x^2 - 2x) + 2x^2 + ax^2 + 4}{x^2 - 2x} = x + \frac{(a+2)x^2 + 4}{x^2 - 2x}$

$= x + \frac{(a+2)(x^2 - 2x) + 2x(a+2) + 4}{x^2 - 2x} = x + a + 2 + \frac{2x(a+2) + 4}{x^2 - 2x}$

$$\lim_{x \to \infty} \frac{2x(a+2) + 4}{x^2 - 2x} = \lim_{x \to \infty} \frac{\frac{2(a+2)}{x} + \frac{4}{x^2}}{1 - \frac{2}{x}} = \frac{0 + 0}{1 - 0} = 0$$

Dus scheve asymptoot is de lijn $y = x + a + 2$.
$A(2, 3)$ op de scheve asymptoot geeft $2 + a + 2 = 3$
$a = -1$

b $f_a(x) = \frac{x^3 + ax^2 + 4}{x^2 - 2x}$ geeft $f_a'(x) = \frac{(x^2 - 2x)(3x^2 + 2ax) - (x^3 + ax^2 + 4)(2x - 2)}{(x^2 - 2x)^2}$

Top op de lijn $x = 1$ geeft $f'(1) = 0$

$\frac{(1-2)(3 + 2a) - (1 + a + 4)(2 - 2)}{(1-2)^2} = 0$

$-3 - 2a - 0 = 0$

$-2a = 3$

$a = -1\frac{1}{2}$

c De grafiek van $f_a(x) = x + a + 2 + \frac{2x(a+2) + 4}{x^2 - 2x}$ heeft een perforatie als zowel $2x(a + 2) + 4 = 0$ als $x^2 - x = 0$.
$x^2 - 2x = 0$ geeft $x(x - 2) = 0$
$x = 0 \vee x = 2$
$x = 0$ geeft $2x(a + 2) + 4 = 4$, dus voldoet niet.
$x = 2$ geeft $2 \cdot 2(a + 2) + 4 = 0$
$4a + 8 + 4 = 0$
$4a = -12$
$a = -3$
Dus voor $a = -3$ heeft de grafiek van f_a een perforatie (2, 1).
Echter $f_{-3}(x) = x - 1 + \frac{-2x + 4}{x^2 - 2x}$, dus de grafiek van f_{-3} is geen rechte lijn.
Dus voor geen enkele waarde van a is de grafiek een rechte lijn met perforatie.

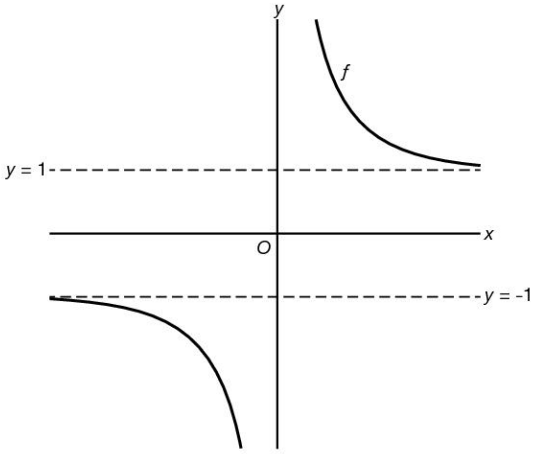
7 **a** $f(x) = \frac{e^x + 1}{e^x - 1}$

verticale asymptoot:
$e^x - 1 = 0 \wedge e^x + 1 \neq 0$
$e^x = 1$
$x = 0$
Dus de verticale asymptoot is de lijn $x = 0$.
horizontale asymptoot:

$$\lim_{x \to -\infty} \frac{e^x + 1}{e^x - 1} = \frac{0 + 1}{0 - 1} = -1$$

$$\lim_{x \to \infty} \frac{e^x + 1}{e^x - 1} = \frac{1 + \frac{1}{e^x}}{1 - \frac{1}{e^x}} = \frac{1 + 0}{1 - 0} = 1$$

Dus de horizontale asymptoten zijn de lijnen $y = -1$ en $y = 1$.



b $f(x) = \dfrac{|x^3 - 4| + x^2}{x^2 - 1} = \begin{cases} \dfrac{-x^3 + 4 + x^2}{x^2 - 1} = \dfrac{-x^3 + x^2 + 4}{x^2 - 1} \text{ voor } x^3 - 4 < 0 \text{ ofwel } x < \sqrt[3]{4} \\ \dfrac{x^3 - 4 + x^2}{x^2 - 1} = \dfrac{x^3 + x^2 - 4}{x^2 - 1} \text{ voor } x \geq \sqrt[3]{4} \end{cases}$

verticale asymptoten:
$x^2 - 1 = 0$
$x^2 = 1$
$x = -1 \vee x = 1$
Dus de verticale asymptoten zijn de lijnen $x = -1$ en $x = 1$.
scheve asymptoten:
$x < \sqrt[3]{4}$

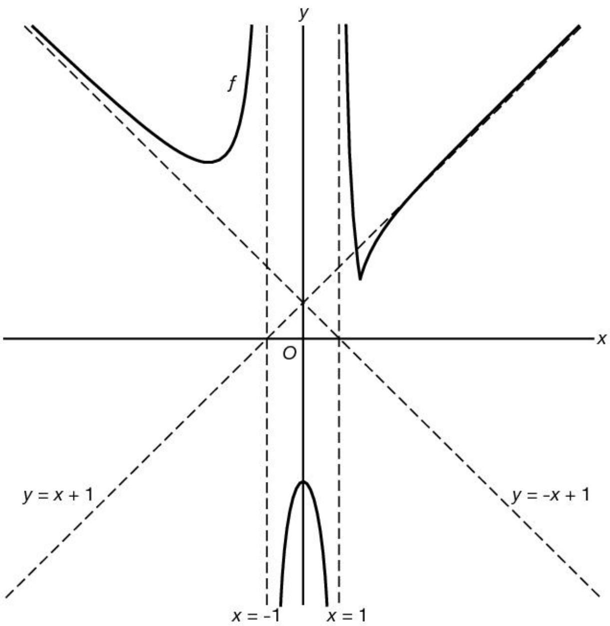
$$\frac{-x^3 + x^2 + 4}{x^2 - 1} = \frac{-x(x^2 - 1) - x + x^2 + 4}{x^2 - 1} = -x + \frac{x^2 - x + 4}{x^2 - 1} = -x + \frac{x^2 - 1 + 1 - x + 4}{x^2 - 1} = -x + 1 + \frac{-x + 5}{x^2 - 1}$$

$$\lim_{x \to -\infty} \frac{-x + 5}{x^2 - 1} = \lim_{x \to -\infty} \frac{\frac{-1}{x} + \frac{5}{x^2}}{1 - \frac{1}{x^2}} = \frac{0 + 0}{1 - 0} = 0$$, dus de lijn $y = -x + 1$ is scheve asymptoot.

$x \geq \sqrt[3]{4}$

$$\frac{x^3 + x^2 - 4}{x^2 - 1} = \frac{x(x^2 - 1) + x + x^2 + 4}{x^2 - 1} = x + \frac{x^2 + x + 4}{x^2 - 1} = x + \frac{x^2 - 1 + 1 + x + 4}{x^2 - 1} = x + 1 + \frac{x + 5}{x^2 - 1}$$

$$\lim_{x \to \infty} \frac{x + 5}{x^2 - 1} = \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{5}{x^2}}{1 - \frac{1}{x^2}} = \frac{0 + 0}{1 - 0} = 0$$, dus de lijn $y = x + 1$ is scheve asymptoot.



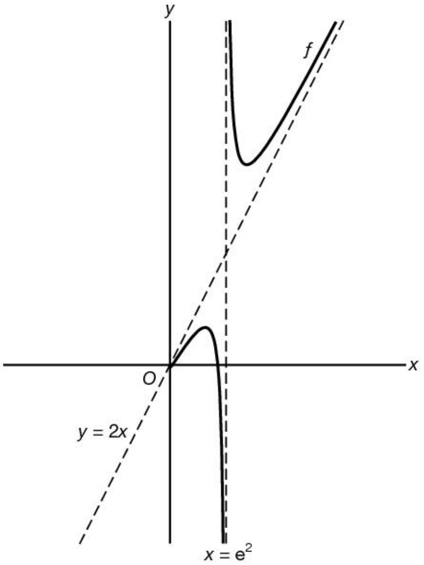
c $f(x) = 2x - 1 + \dfrac{\ln(x)}{\ln(x) - 2}$
verticale asymptoot:
$\ln(x) - 2 = 0$
$\ln(x) = 2$
$x = e^2$
Dus de verticale asymptoot is de lijn $x = e^2$.
scheve asymptoot:

$$\lim_{x \to \infty} \frac{\ln(x)}{\ln(x) - 2} = \lim_{\ln(x) \to \infty} \frac{1}{1 - \frac{2}{\ln(x)}} = \frac{1}{1 - 0} = 1$$

Dus de scheve asymptoot is de lijn $y = 2x$.

$$\lim_{x \downarrow 0}\left(2x - 1 + \frac{\ln(x)}{\ln(x) - 2}\right) = -1 + \lim_{\ln(x) \to -\infty} \frac{\ln(x)}{\ln(x) - 2} = -1 + \lim_{\ln(x) \to -\infty} \frac{1}{1 - \frac{2}{\ln(x)}} = -1 + \frac{1}{1 - 0} = -1 + 1 = 0$$

Dus de grafiek van f nadert de oorsprong als x naar 0 daalt.



Bladzijde 224

8 **a** verticale asymptoot:

$2\,e^{-x} - 4 = 0$

$2\,e^{-x} = 4$

$e^{-x} = 2$

$-x = \ln(2)$

$x = -\ln(2)$

Dus de verticale asymptoot is de lijn $x = -\ln(2)$

horizontale asymptoot:

$$\lim_{x \to -\infty} \frac{e^x + 1}{2\,e^{-x} - 4} = \lim_{x \to -\infty} \frac{e^{2x} + e^x}{2 - 4e^x} = \frac{0 + 0}{2 - 0} = 0$$

Dus de horizontale asymptoot is de lijn $y = 0$.

Voor x naar ∞ gaat de teller $e^x + 1$ naar oneindig en de noemer $2\,e^{-x} - 4$ naar -4, dus $f(x)$ gaat naar min oneindig. Er is geen sprake van een scheve asymptoot.

b $f(x) = -1$ geeft $\dfrac{e^x + 1}{2e^{-x} - 4} = -1$

$e^x + 1 = -2e^{-x} + 4$

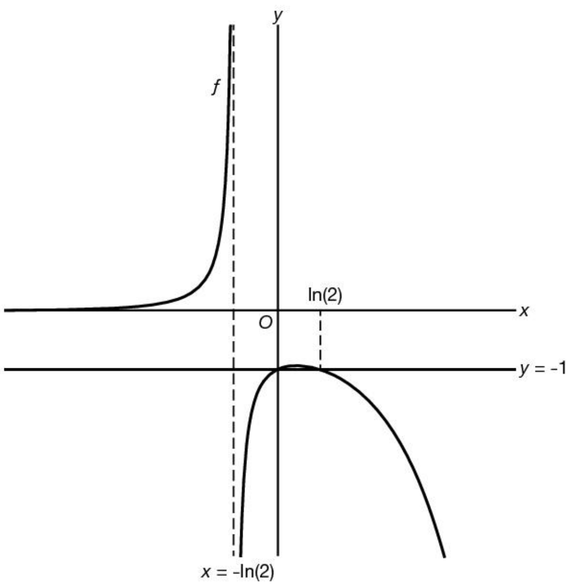
$e^x - 3 + 2e^{-x} = 0$

$e^{2x} - 3e^x + 2 = 0$

$(e^x - 1)(e^x - 2) = 0$

$e^x = 1 \vee e^x = 2$

$x = 0 \vee x = \ln(2)$



$f(x) \geq -1$ geeft $x < -\ln(2) \vee 0 < x < \ln(2)$

c $f(x)=\dfrac{e^x+1}{2e^{-x}-4}$ geeft $f'(x)=\dfrac{(2e^{-x}-4)\cdot e^x-(e^x+1)\cdot -2e^{-x}}{(2e^{-x}-4)^2}=\dfrac{2-4e^x+2-2e^{-x}}{(2e^{-x}-4)^2}=\dfrac{4-4e^x-2e^{-x}}{(2e^{-x}-4)^2}$

$f'(0)=\dfrac{4-4-2}{(2-4)^2}=\dfrac{-2}{4}=-\frac{1}{2}$

$f(0)=\dfrac{2}{-2}=-1$, dus $A(0, -1)$, dus $B(-2, 0)$.

$O(\triangle OAB)=\frac{1}{2}\cdot 2\cdot 1=1$

9 **a** verticale asymptoot:

$e^{\frac{1}{x}}-a=0$

$e^{\frac{1}{x}}=a$

$\dfrac{1}{x}=\ln(a)$

$x=\dfrac{1}{\ln(a)}$

Dus de verticale asymptoot is de lijn $x=\dfrac{1}{\ln(a)}$.

horizontale asymptoot:

$\lim\limits_{x\to -\infty}\dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}-a}=\lim\limits_{x\to \infty}\dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}-a}=\dfrac{2\cdot 1}{1-a}$

Dus de horizontale asymptoot is de lijn $y=\dfrac{2}{1-a}$.

Het snijpunt van de asymptoten in het punt $\left(\dfrac{1}{\ln(a)}, \dfrac{2}{1-a}\right)$.

Op de lijn $y=x-1$ geeft $\dfrac{2}{1-a}=\dfrac{1}{\ln(a)}-1$.

Voer in $y_1=\dfrac{2}{1-x}$ en $y_2=\dfrac{1}{\ln(x)}-1$.

Intersect geeft $x=5{,}700...$

Dus $a\approx 5{,}70$.

b $f_a(x)=\dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}-a}$ geeft $f_a'(x)=\dfrac{\left(e^{\frac{1}{x}}-a\right)\cdot 2e^{\frac{1}{x}}\cdot -\dfrac{1}{x^2}-2e^{\frac{1}{x}}\cdot e^{\frac{1}{x}}\cdot -\dfrac{1}{x^2}}{\left(e^{\frac{1}{x}}-a\right)^2}=\dfrac{2e^{\frac{2}{x}}-ae^{\frac{1}{x}}-2e^{\frac{2}{x}}}{-x^2\cdot\left(e^{\frac{1}{x}}-a\right)^2}=\dfrac{ae^{\frac{1}{x}}}{x^2\cdot\left(e^{\frac{1}{x}}-a\right)^2}$

$f_a'(1)=2$ geeft $\dfrac{a\mathrm{e}}{(\mathrm{e}-a)^2}=2$

$2(\mathrm{e}-a)^2=a\mathrm{e}$

$2(\mathrm{e}^2-2a\mathrm{e}+a^2)=a\mathrm{e}$

$2\mathrm{e}^2-4a\mathrm{e}+2a^2=a\mathrm{e}$

$2a^2-5a\mathrm{e}+2\mathrm{e}^2=0$

$D=(-5\mathrm{e})^2-4\cdot 2\cdot 2\mathrm{e}^2=25\mathrm{e}^2-16\mathrm{e}^2=9\mathrm{e}^2$

$a=\dfrac{5\mathrm{e}-3\mathrm{e}}{4}=\frac{1}{2}\mathrm{e}\vee a=\dfrac{5\mathrm{e}+3\mathrm{e}}{4}=2\mathrm{e}$

Dus voor $a=\frac{1}{2}\mathrm{e}\vee a=2\mathrm{e}$.

c x en y verwisselen in $y=\dfrac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}}-a}$ geeft $x=\dfrac{2e^{\frac{1}{y}}}{e^{\frac{1}{y}}-a}$

$xe^{\frac{1}{y}}-ax=2e^{\frac{1}{y}}$

$(x-2)e^{\frac{1}{y}}=ax$

$e^{\frac{1}{y}}=\dfrac{ax}{x-2}$

$\dfrac{1}{y}=\ln\left(\dfrac{ax}{x-2}\right)$

$y=\dfrac{1}{\ln\left(\dfrac{ax}{x-2}\right)}$

Dus $f_a^{\text{inv}}(x)=\dfrac{1}{\ln\left(\dfrac{ax}{x-2}\right)}$.

10 **a** verticale asymptoot:

$\ln(x^2) + 1 = 0$
$\ln(x^2) = -1$
$x^2 = \mathrm{e}^{-1}$
$x = \mathrm{e}^{-\frac{1}{2}} \vee x = -\mathrm{e}^{-\frac{1}{2}}$

Dus de verticale asymptoten zijn de lijnen $x = \mathrm{e}^{-\frac{1}{2}}$ en $x = -\mathrm{e}^{-\frac{1}{2}}$.

horizontale asymptoot:

$$\lim_{x \to \infty} \frac{\ln(x^2) - 1}{\ln(x^2) + 1} = \lim_{x \to -\infty} \frac{\ln(x^2) - 1}{\ln(x^2) + 1} = \lim_{x \to \infty} \frac{1 - \frac{1}{\ln(x^2)}}{1 + \frac{1}{\ln(x^2)}} = \frac{1 - 0}{1 + 0} = 1$$

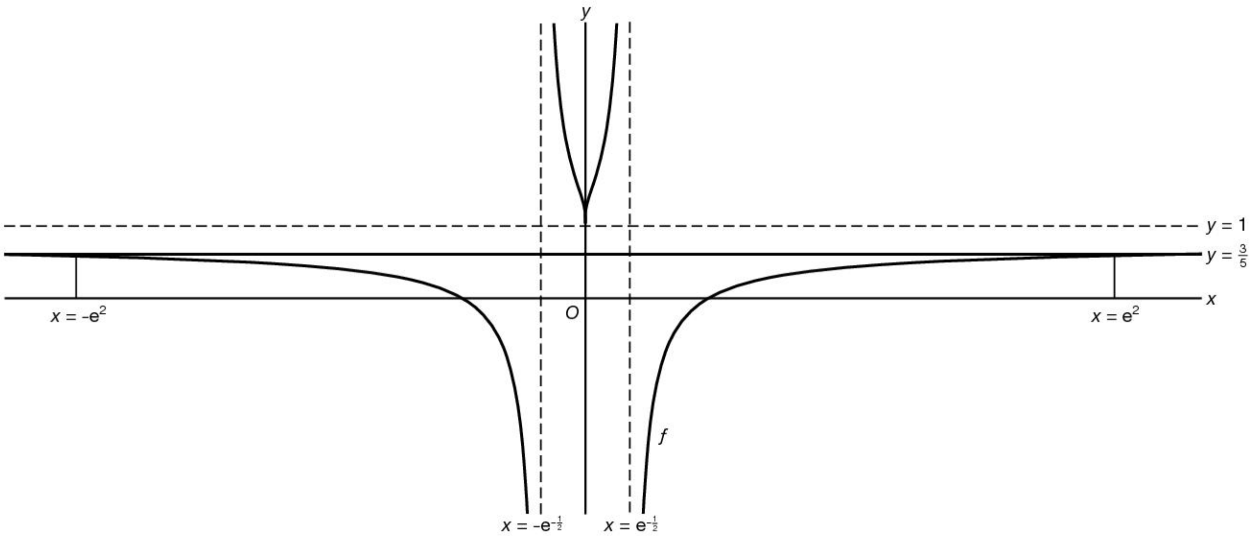
Dus de horizontale asymptoot is de lijn $y = 1$.

b $$\lim_{x \to 0} \frac{\ln(x^2) - 1}{\ln(x^2) + 1} = \lim_{x \to 0} \frac{1 - \frac{1}{\ln(x^2)}}{1 + \frac{1}{\ln(x^2)}} = \frac{1 - 0}{1 + 0} = 1$$

Dus de perforatie is het punt $(0, 1)$.

c $f(x) = \frac{3}{5}$ geeft $\frac{\ln(x^2) - 1}{\ln(x^2) + 1} = \frac{3}{5}$

$3\ln(x^2) + 3 = 5\ln(x^2) - 5$
$-2\ln(x^2) = -8$
$\ln(x^2) = 4$
$x^2 = \mathrm{e}^4$
$x = \mathrm{e}^2 \vee x = -\mathrm{e}^2$



$f(x) \geq \frac{3}{5}$ geeft $x \leq -\mathrm{e}^2 \vee -\mathrm{e}^{-\frac{1}{2}} < x < \mathrm{e}^{-\frac{1}{2}} \vee x \geq \mathrm{e}^2$

d $f(\mathrm{e}) = \frac{1}{3}$, dus $A\left(\mathrm{e}, \frac{1}{3}\right)$.

$$f(x) = \frac{\ln(x^2) - 1}{\ln(x^2) + 1} \text{ geeft } f'(x) = \frac{(\ln(x^2) + 1) \cdot \frac{1}{x^2} \cdot 2x - (\ln(x^2) - 1) \cdot \frac{1}{x^2} \cdot 2x}{(\ln(x^2) + 1)^2}$$

$$= \frac{2\ln(x^2) + 2 - 2\ln(x^2) + 2}{x(\ln(x^2) + 1)^2} = \frac{4}{x(\ln(x^2) + 1)^2}$$

Stel de raaklijn k: $y = ax + b$ met $a = f'(\mathrm{e}) = \dfrac{4}{\mathrm{e}(\ln(\mathrm{e}^2)+1)^2} = \dfrac{4}{\mathrm{e}(2+1)^2} = \dfrac{4}{9\mathrm{e}}$.

$$\left.\begin{array}{l} y = \dfrac{4}{9\mathrm{e}}x + b \\ A\left(\mathrm{e}, \frac{1}{3}\right) \end{array}\right\} \begin{array}{l} \dfrac{4}{9\mathrm{e}} \cdot \mathrm{e} + b = \frac{1}{3} \\ \frac{4}{9} + b = \frac{1}{3} \\ b = -\frac{1}{9} \end{array}$$

$y = \dfrac{4}{9\mathrm{e}}x - \frac{1}{9}$ snijden met de x-as geeft $\dfrac{4}{9\mathrm{e}}x - \frac{1}{9} = 0$

$$4x - \mathrm{e} = 0$$
$$4x = \mathrm{e}$$
$$x = \tfrac{1}{4}\mathrm{e}$$

Dus $B\left(\frac{1}{4}\mathrm{e}, 0\right)$.

14 Meetkunde toepassen

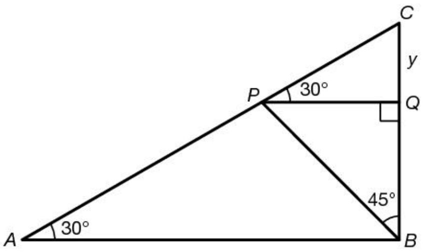
11 **a** Stel $BC = x$. Dan is $AB = x\sqrt{3}$ en $AC = 2x$.

omtrek = 6 geeft $x + x\sqrt{3} + 2x = 6$

$$x\left(3 + \sqrt{3}\right) = 6$$
$$x = \frac{6}{3+\sqrt{3}} = \frac{6}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{6\left(3-\sqrt{3}\right)}{9-3} = 3 - \sqrt{3}$$

Dus $BC = 3 - \sqrt{3}$ en $AB = x\sqrt{3} = \left(3 - \sqrt{3}\right) \cdot \sqrt{3} = 3\sqrt{3} - 3$.

b Teken PQ loodrecht op BC.



Stel $CQ = y$. Dan is $PQ = y\sqrt{3}$, $CP = 2y$ en $BQ = y\sqrt{3}$.

$BC = 3 - \sqrt{3}$ en $BC = y + y\sqrt{3}$ geeft

$y + y\sqrt{3} = 3 - \sqrt{3}$

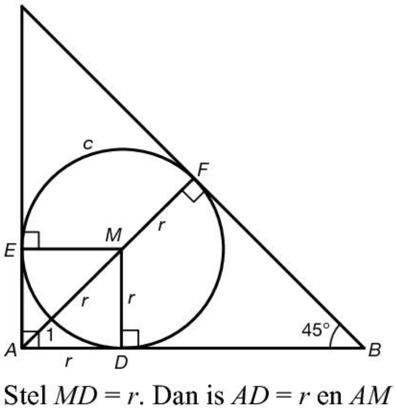
$y\left(1 + \sqrt{3}\right) = 3 - \sqrt{3}$

$y = \dfrac{3-\sqrt{3}}{1+\sqrt{3}} = \dfrac{3-\sqrt{3}}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{3 - 3\sqrt{3} - \sqrt{3} + 3}{1-3} = \dfrac{-4\sqrt{3}+6}{-2} = 2\sqrt{3} - 3$

Dus $CP = 4\sqrt{3} - 6$ en $AP = AC - CP = 2\left(3 - \sqrt{3}\right) - \left(4\sqrt{3} - 6\right) = 6 - 2\sqrt{3} - 4\sqrt{3} + 6 = 12 - 6\sqrt{3}$.

Bladzijde 225

12 Teken AM en teken MD loodrecht op AB.



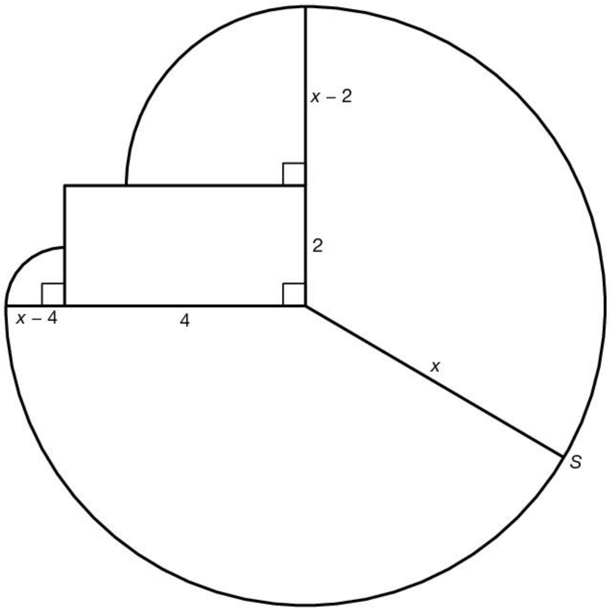
Stel $MD = r$. Dan is $AD = r$ en $AM = r + 1$.

$$\left.\begin{array}{l} AM = r\sqrt{2} \\ AM = r+1 \end{array}\right\} \begin{array}{l} r\sqrt{2} = r+1 \\ r\sqrt{2} - r = 1 \\ r\left(\sqrt{2}-1\right) = 1 \\ r = \dfrac{1}{\sqrt{2}-1} = \dfrac{1}{\sqrt{2}-1} \cdot \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = \dfrac{\sqrt{2}+1}{2-1} = \sqrt{2}+1 \end{array}$$

Dus $AF = 2r + 1 = 2\left(\sqrt{2}+1\right) + 1 = 2\sqrt{2} + 3$.

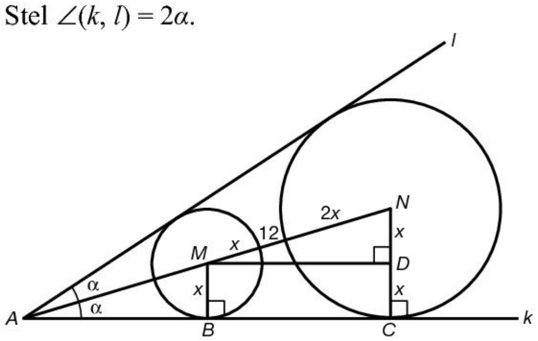
$BF = CF = AF = 2\sqrt{2} + 3$ en $AB = AC = \sqrt{2}\left(2\sqrt{2}+3\right) = 4 + 3\sqrt{2}$.
Dus de omtrek van $\triangle ABC$ is $AB + BC + AC = 4 + 3\sqrt{2} + 2\left(2\sqrt{2}+3\right) + 4 + 3\sqrt{2} = 14 + 10\sqrt{2}$.

13 Stel de lengte van de ketting x.
Neem eerst $4 < x < 6$.



Oppervlakte $= \frac{1}{4} \cdot \pi(x-4)^2 + \frac{3}{4} \cdot \pi x^2 + \frac{1}{4} \cdot \pi(x-2)^2$
Voer in $y_1 = \frac{1}{4} \cdot \pi(x-4)^2 + \frac{3}{4} \cdot \pi x^2 + \frac{1}{4} \cdot \pi(x-2)^2$ en $y_2 = 90$.
Intersect geeft $x = 5{,}712...$ en dit voldoet.
Dus de lengte van de ketting is 5,7 m.

14 Stel $\angle(k, l) = 2\alpha$.



$$\left.\begin{array}{l} \cos(2\alpha) = 1 - 2\sin^2(\alpha) \\ \cos(2\alpha) = \frac{41}{49} \end{array}\right\} \begin{array}{l} \frac{41}{49} = 1 - 2\sin^2(\alpha) \\ 2\sin^2(\alpha) = 1 - \frac{41}{49} = \frac{8}{49} \\ \sin^2(\alpha) = \frac{4}{49} \\ \sin(\alpha) = \frac{2}{7} \vee \sin(\alpha) = -\frac{2}{7} \\ \text{vold.} \qquad\qquad \text{vold. niet} \end{array}$$

$$\left.\begin{array}{l} \sin(\angle DMN) = \sin(\alpha) = \dfrac{DN}{MN} = \dfrac{x}{3x+12} \\ \sin(\alpha) = \frac{2}{7} \end{array}\right\} \begin{array}{l} \dfrac{x}{3x+12} = \frac{2}{7} \\ 7x = 6x + 24 \\ x = 24 \end{array}$$

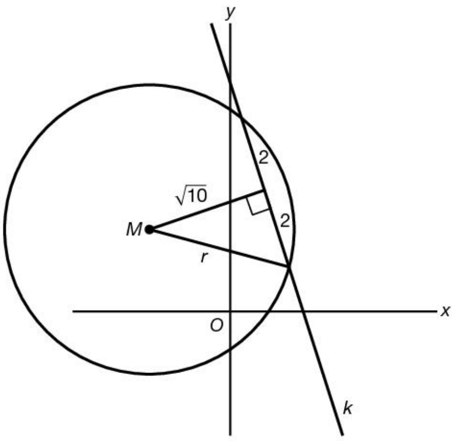
Dus $MN = 3 \cdot 24 + 12 = 84$ cm.

15 **a** $k\colon 3x + y - 6 = 0$

$$d(M, k) = \frac{|3 \cdot -2 + 2 - 6|}{\sqrt{3^2 + 1^2}} = \frac{10}{\sqrt{10}} = \sqrt{10}$$

Dus $c_1\colon (x + 2)^2 + (y - 2)^2 = 10$.

b $d(M, k) = \sqrt{10}$



$r^2 = (\sqrt{10})^2 + 2^2 = 10 + 4 = 14$

Dus $c_2\colon (x + 2)^2 + (y - 2)^2 = 14$.

c m loodrecht op k geeft $m\colon x - 3y = c$.

$d(M, m) = \sqrt{10}$

$$\frac{|-2 - 3 \cdot 2 - c|}{\sqrt{10}} = \sqrt{10}$$

$|-8 - c| = 10$

$-8 - c = 10 \vee -8 - c = -10$

$c = -18 \vee c = 2$

Dus $m_1\colon x - 3y = -18$ en $m_2\colon x - 3y = 2$.

d Stel $n\colon y = ax - 2$ ofwel $n\colon ax - y - 2 = 0$.

$d(M, n) = \sqrt{10}$ geeft $\dfrac{|-2a - 2 - 2|}{\sqrt{a^2 + 1}} = \sqrt{10}$

$|-2a - 4| = \sqrt{10a^2 + 10}$

$4a^2 + 16a + 16 = 10a^2 + 10$

$-6a^2 + 16a + 6 = 0$

$3a^2 - 8a - 3 = 0$

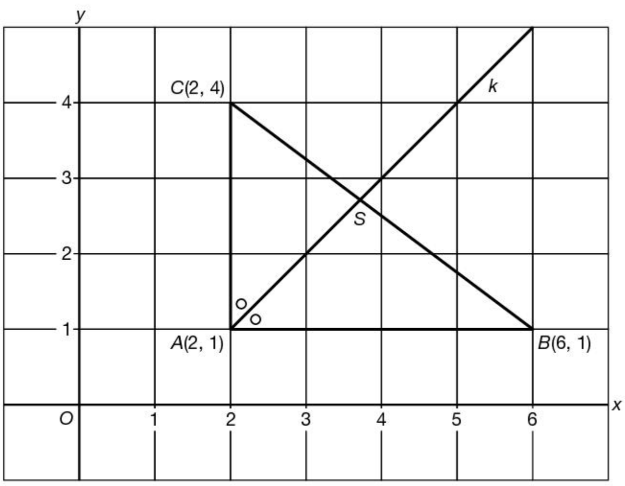
$D = (-8)^2 - 4 \cdot 3 \cdot -3 = 64 + 36 = 100$

$a = \dfrac{8 + 10}{6} = 3 \vee a = \dfrac{8 - 10}{6} = -\frac{1}{3}$

Dus $n_1\colon y = 3x - 2$ en $n_2\colon y = -\frac{1}{3}x - 2$.

Bladzijde 226

16 **a**



$k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\text{rc}_{BC} = \dfrac{4-1}{2-6} = \dfrac{3}{-4} = -\frac{3}{4}$

$\left.\begin{array}{l} BC: y = -\frac{3}{4}x + b \\ \text{door } B(6, 1) \end{array}\right\} -\frac{3}{4} \cdot 6 + b = 1$

$-4\frac{1}{2} + b = 1$

$b = 5\frac{1}{2}$

Dus $BC: y = -\frac{3}{4}x + 5\frac{1}{2}$.

Snijden van k en BC geeft $1 + \lambda = -\frac{3}{4}(2 + \lambda) + 5\frac{1}{2}$

$4 + 4\lambda = -3(2 + \lambda) + 22$

$4 + 4\lambda = -6 - 3\lambda + 22$

$7\lambda = 12$

$\lambda = 1\frac{5}{7}$

$\lambda = 1\frac{5}{7}$ geeft $x = 2 + 1\frac{5}{7} = 3\frac{5}{7}$ en $y = 1 + 1\frac{5}{7} = 2\frac{5}{7}$

Dus $S\left(3\frac{5}{7}, 2\frac{5}{7}\right)$.

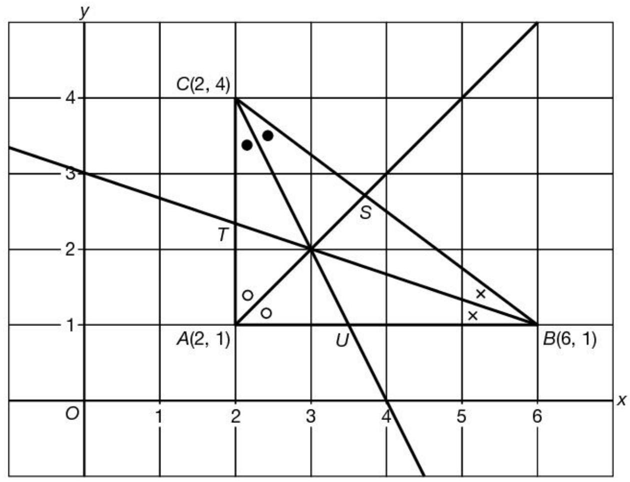
b $AB = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

$BS = \sqrt{\left(3\frac{5}{7} - 6\right)^2 + \left(2\frac{5}{7} - 1\right)^2} = \sqrt{\left(-\frac{16}{7}\right)^2 + \left(\frac{12}{7}\right)^2} = \sqrt{\frac{256}{49} + \frac{144}{49}} = \sqrt{\frac{400}{49}} = \frac{20}{7} = 2\frac{6}{7}$

$CS = 5 - 2\frac{6}{7} = \frac{15}{7}$

$\left.\begin{array}{l} \dfrac{BS}{CS} = \dfrac{\frac{20}{7}}{\frac{15}{7}} = \dfrac{20}{15} = \dfrac{4}{3} \\ \dfrac{AB}{AC} = \dfrac{4}{3} \end{array}\right\} \dfrac{BS}{CS} = \dfrac{AB}{AC}$

c



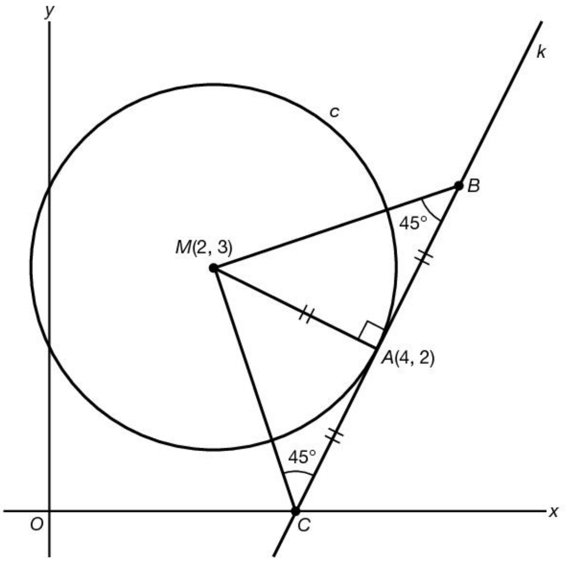
$p = 4$ en $q = 5$ geeft $\vec{t} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \frac{4}{9}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2\frac{1}{3} \end{pmatrix}$

Dus $T\left(2, 2\frac{1}{3}\right)$.

d $\vec{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \frac{3}{8}\begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 3\frac{1}{2} \\ 1 \end{pmatrix}$

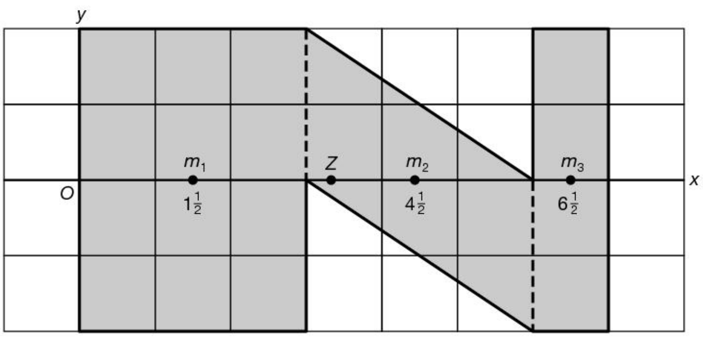
Dus $U\left(3\frac{1}{2}, 1\right)$.

17



$\overrightarrow{MA} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, dus $\overrightarrow{AB} = \overrightarrow{MA}_L = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ en $\overrightarrow{AC} = \overrightarrow{MA}_R = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$.
Dus $B(5, 4)$ en $C(3, 0)$.

18 Het zwaartepunt ligt op de x-as.



$m_1 = 3 \cdot 4 = 12$, $m_2 = 3 \cdot 2 = 6$ en $m_3 = 1 \cdot 4 = 4$

$z = \frac{1}{12 + 6 + 4}\left(12 \cdot 1\frac{1}{2} + 6 \cdot 4\frac{1}{2} + 4 \cdot 6\frac{1}{2}\right) = \frac{1}{22} \cdot 71 = 3\frac{5}{22}$

Dus het zwaartepunt ligt $3\frac{5}{22}$ rechts van O op de x-as.

Bladzijde 227

19 **a** $x(t) = \cos^2(t) - \cos(t)$ geeft $x'(t) = 2\cos(t) \cdot -\sin(t) + \sin(t) = -\sin(2t) + \sin(t)$
$y(t) = \sin(t)$ geeft $y'(t) = \cos(t)$
raaklijn horizontaal geeft $y'(t) = 0 \wedge x'(t) \neq 0$
$\cos(t) = 0 \wedge -\sin(2t) + \sin(t) \neq 0$
$t = \frac{1}{2}\pi + k \cdot \pi$
$-\pi \leq t \leq \pi$ geeft $t = -\frac{1}{2}\pi \vee t = \frac{1}{2}\pi$
$t = -\frac{1}{2}\pi$ geeft $(0, -1)$ en $t = \frac{1}{2}\pi$ geeft $(0, 1)$.
dus de raaklijn is horizontaal in de punten $(0, -1)$ en $(0, 1)$.
raaklijn verticaal geeft $x'(t) = 0 \wedge y'(t) \neq 0$
$-2\sin(t)\cos(t) + \sin(t) = 0 \wedge \cos(t) \neq 0$
$\sin(t)(-2\cos(t) + 1) = 0 \wedge \cos(t) \neq 0$
$\sin(t) = 0 \vee \cos(t) = \frac{1}{2}$
$t = k \cdot \pi \vee t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$
$-\pi \leq t \leq \pi$ geeft $t = -\pi \vee t = -\frac{1}{3}\pi \vee t = 0 \vee t = \frac{1}{3}\pi \vee t = \pi$
$t = -\pi$ en $t = \pi$ geeft $(2, 0)$
$t = -\frac{1}{3}\pi$ geeft $\left(-\frac{1}{4}, -\frac{1}{2}\sqrt{3}\right)$ en $t = \frac{1}{3}\pi$ geeft $\left(-\frac{1}{4}, \frac{1}{2}\sqrt{3}\right)$
$t = 0$ geeft $(0, 0)$
Dus de raaklijn is verticaal in de punten $(2, 0)$, $\left(-\frac{1}{4}, -\frac{1}{2}\sqrt{3}\right)$, $(0, 0)$ en $\left(-\frac{1}{4}, \frac{1}{2}\sqrt{3}\right)$.

b $\vec{r}(t) = \begin{pmatrix} \cos^2(t) - \cos(t) \\ \sin(t) \end{pmatrix}$

$\vec{v}\,(t) = \begin{pmatrix} -\sin(2t) + \sin(t) \\ \cos(t) \end{pmatrix}$

$\vec{v}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, dus de baansnelheid is 1.

c $x = 0$ geeft $\cos^2(t) - \cos(t) = 0$

$\cos(t)(\cos(t) - 1) = 0$

$\cos(t) = 0 \vee \cos(t) = 1$

$t = \frac{1}{2}\pi + k \cdot \pi \vee t = k \cdot 2\pi$

$-\pi \leq t \leq \pi$ geeft $t = \frac{1}{2}\pi \vee t = 0 \vee t = \frac{1}{2}\pi$

$t = \frac{1}{2}\pi$ geeft $(0, 1)$

$\vec{a}\,(t) = \begin{pmatrix} -2\cos(2t) + \cos(t) \\ -\sin(t) \end{pmatrix}$

$$a_b\left(\tfrac{1}{2}\pi\right) = \frac{\begin{pmatrix} -\sin(\pi) + \sin\left(\frac{1}{2}\pi\right) \\ \cos\left(\frac{1}{2}\pi\right) \end{pmatrix}\begin{pmatrix} -2\cos(\pi) + \cos\left(\frac{1}{2}\pi\right) \\ -\sin(t) \end{pmatrix}}{\left|\begin{pmatrix} -\sin(\pi) + \sin\left(\frac{1}{2}\pi\right) \\ \cos\left(\frac{1}{2}\pi\right) \end{pmatrix}\right|} = \frac{\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|} = \frac{1}{1} = 1$$

Dus de baanversnelling als P de positieve y-as passeert is 1.

d Voor elke p geldt $x(-p) = \cos^2(-p) - \cos(-p) = \cos^2(p) - \cos(p) = x(p)$ en $y(-p) = \sin(-p) = -\sin(p) = -y(p)$

Dus de baan van P is symmetrisch in de x-as.

e $x = \frac{3}{4}$ geeft $\cos^2(t) - \cos(t) = \frac{3}{4}$

$\cos^2(t) - \cos(t) - \frac{3}{4} = 0$

$\left(\cos(t) - 1\frac{1}{2}\right)\left(\cos(t) + \frac{1}{2}\right) = 0$

$\cos(t) = 1\frac{1}{2} \vee \cos(t) = -\frac{1}{2}$

$t = \frac{2}{3}\pi + k \cdot 2\pi \vee t = -\frac{2}{3}\pi + k \cdot 2\pi$

$-\pi \leq t \leq \pi$ geeft $t = \frac{2}{3}\pi \vee t = -\frac{2}{3}\pi$

$t = \frac{2}{3}\pi$ geeft $C\left(\frac{3}{4}, \frac{1}{2}\sqrt{3}\right)$

$t = -\frac{2}{3}\pi$ geeft $D\left(\frac{3}{4}, -\frac{1}{2}\sqrt{3}\right)$

f $x = \cos^2(t) - \cos(t)$ en $y = \sin(t)$ substitueren in $y = x^2$ geeft

$\sin(t) = (\cos^2(t) - \cos(t))^2$

Voer in $y_1 = \sin(x)$ en $y_2 = (\cos^2(x) - \cos(x))^2$.

Intersect geeft $x = 0$ en $y = 0$ en $x = 2{,}182...$ en $y = 0{,}818...$

Dus $E(2{,}18; 0{,}82)$.

20 **a** $x'(t) = 2t - 4$ en $y'(t) = \dfrac{1}{t^2 + \frac{1}{2}} \cdot 2t = \dfrac{2t}{t^2 + \frac{1}{2}}$

raaklijn horizontaal geeft $y'(t) = 0 \wedge x'(t) \neq 0$

$2t = 0 \wedge 2t - 4 \neq 0$

$t = 0$

$t = 0$ geeft het punt $\left(0, \ln\left(\frac{1}{2}\right)\right)$.

raaklijn verticaal geeft $x'(t) = 0 \wedge y'(t) \neq 0$

$2t - 4 = 0 \wedge 2t \neq 0$

$2t = 4$

$t = 2$

$t = 2$ geeft het punt $\left(-4, \ln\left(4\frac{1}{2}\right)\right)$.

b $x = 0$ geeft $t^2 - 4t = 0$

$t(t - 4) = 0$

$t = 0 \vee t = 4$

$\vec{r}\,(t) = \begin{pmatrix} t^2 - 4t \\ \ln\left(t^2 + \frac{1}{2}\right) \end{pmatrix}$

$\vec{v}\,(t) = \begin{pmatrix} 2t - 4 \\ \dfrac{2t}{t^2 + \frac{1}{2}} \end{pmatrix}$

$t = 4$ geeft $\vec{v}(4) = \begin{pmatrix} 2 \cdot 4 - 4 \\ \dfrac{2 \cdot 4}{4^2 + \frac{1}{2}} \end{pmatrix} = \begin{pmatrix} 4 \\ \dfrac{8}{16\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} 4 \\ \frac{16}{33} \end{pmatrix}$

$v(4) = \sqrt{4^2 + \left(\frac{16}{33}\right)^2} = \sqrt{16\frac{256}{1089}}$

Dus de baansnelheid waarmee P de positieve y-as passeert is $\sqrt{16\frac{256}{1089}}$.

c $y = 0$ geeft $\ln\left(t^2 + \frac{1}{2}\right) = 0$

$t^2 + \frac{1}{2} = 1$

$t^2 = \frac{1}{2}$

$t = \frac{1}{2}\sqrt{2} \vee t = -\frac{1}{2}\sqrt{2}$

$y'(t) = \dfrac{2t}{t^2 + \frac{1}{2}}$ geeft $y''(t) = \dfrac{\left(t^2 + \frac{1}{2}\right) \cdot 2 - 2t \cdot 2t}{\left(t^2 + \frac{1}{2}\right)^2} = \dfrac{2t^2 + 1 - 4t^2}{\left(t^2 + \frac{1}{2}\right)^2} = \dfrac{-2t^2 + 1}{\left(t^2 + \frac{1}{2}\right)^2}$

$\vec{a}(t) = \begin{pmatrix} 2 \\ \dfrac{-2t^2 + 1}{\left(t^2 + \frac{1}{2}\right)^2} \end{pmatrix}$

$t = -\frac{1}{2}\sqrt{2}$

$$a_b\left(-\tfrac{1}{2}\sqrt{2}\right) = \frac{\begin{pmatrix} 2 \cdot -\frac{1}{2}\sqrt{2} - 4 \\ \dfrac{2 \cdot -\frac{1}{2}\sqrt{2}}{\left(-\frac{1}{2}\sqrt{2}\right)^2 + \frac{1}{2}} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ \dfrac{-2 \cdot \left(-\frac{1}{2}\sqrt{2}\right)^2 + 1}{\left(\left(-\frac{1}{2}\sqrt{2}\right)^2 + \frac{1}{2}\right)^2} \end{pmatrix}}{\left|\begin{pmatrix} 2 \cdot -\frac{1}{2}\sqrt{2} - 4 \\ \dfrac{2 \cdot -\frac{1}{2}\sqrt{2}}{\left(-\frac{1}{2}\sqrt{2}\right)^2 + \frac{1}{2}} \end{pmatrix}\right|} = \frac{\begin{pmatrix} -\sqrt{2} - 4 \\ \dfrac{-\sqrt{2}}{\frac{1}{2} + \frac{1}{2}} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ \dfrac{-1 + 1}{1} \end{pmatrix}}{\left|\begin{pmatrix} -\sqrt{2} - 4 \\ \dfrac{-\sqrt{2}}{\frac{1}{2} + \frac{1}{2}} \end{pmatrix}\right|} = \frac{\begin{pmatrix} -\sqrt{2} - 4 \\ -\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} -\sqrt{2} - 4 \\ -\sqrt{2} \end{pmatrix}\right|}$$

$$= \frac{-2\sqrt{2} - 8}{\sqrt{\left(-\sqrt{2} - 4\right)^2 + \left(-\sqrt{2}\right)^2}} = \frac{-2\sqrt{2} - 8}{\sqrt{2 + 8\sqrt{2} + 16 + 2}} = \frac{-2\sqrt{2} - 8}{\sqrt{20 + 8\sqrt{2}}} = 1{,}935...$$

Dus de baanversnelling is 1,94.

d $x = q$ geeft $t^2 - 4t = q$

$(t - 2)^2 - 4 = q$

$(t - 2)^2 = q + 4$

$t - 2 = \sqrt{q + 4} \vee t - 2 = -\sqrt{q + 4}$

$t = 2 + \sqrt{q + 4} \vee t = 2 - \sqrt{q + 4}$

$AB = y\left(2 + \sqrt{q + 4}\right) - y\left(2 - \sqrt{q + 4}\right) = \ln\left(\left(2 + \sqrt{q + 4}\right)^2 + \frac{1}{2}\right) - \ln\left(\left(2 - \sqrt{q + 4}\right)^2 + \frac{1}{2}\right)$

Voer in $y_1 = \ln\left(\left(2 + \sqrt{x + 4}\right)^2 + \frac{1}{2}\right) - \ln\left(\left(2 - \sqrt{x + 4}\right)^2 + \frac{1}{2}\right)$ en $y_2 = 1$.

Intersect geeft $x = -3{,}69137...$

Dus $q \approx -3{,}6914$.

e k: $3x + 2y = 15$ geeft $\text{rc}_k = -\frac{3}{2}$

$\dfrac{y'(t)}{x'(t)} = -\frac{3}{2}$ geeft $y'(t) = -\frac{3}{2}x'(t)$

$\dfrac{2t}{t^2 + \frac{1}{2}} = -\frac{3}{2}(2t - 4)$

Voer in $y_1 = \dfrac{2x}{x^2 + \frac{1}{2}}$ en $y_2 = -\frac{3}{2}(2x - 4)$.

Intersect geeft $x = 1{,}66008...$

$t = 1{,}66008...$ geeft $x = -3{,}884...$ en $y = 1{,}180...$

Dus $C(-3{,}88; 1{,}18)$.

Bladzijde 228

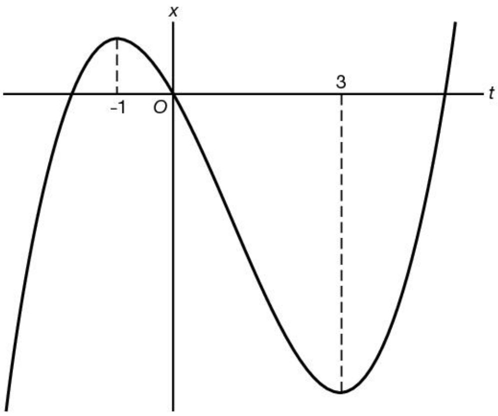
21 **a** $x'(t) = 3t^2 - 6t - 9 = 0$

$x'(t) = 0$ geeft $3t^2 - 6t - 9 = 0$

$t^2 - 2t - 3 = 0$

$(t + 1)(t - 3) = 0$

$t = -1 \vee t = 3$



max. is $x(-1) = 5$ en min. is $x(3) = -27$.
Dus $x = p$ snijdt de baan in drie punten voor $-27 < p < 5$.

b $y = 0$ geeft $t^2 - 1 = 0$
$t^2 = 1$
$t = 1 \vee t = -1$
$t = 1$ geeft $(-11, 0)$
$\vec{v}(t) = \begin{pmatrix} 3t^2 - 6t - 9 \\ 2t \end{pmatrix}$ en $\vec{a}(t) = \begin{pmatrix} 6t - 6 \\ 2 \end{pmatrix}$

$\vec{v}(1) = \begin{pmatrix} -12 \\ 2 \end{pmatrix}$, dus de baansnelheid is $\sqrt{(-12)^2 + 2^2} = \sqrt{148} \approx 12{,}17$.

$\vec{a}(1) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, dus de baanversnelling is $a_b(1) = \dfrac{\begin{pmatrix} -12 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix}}{\sqrt{148}} = \dfrac{4}{\sqrt{148}} \approx 0{,}33$.

c $y = 3$ geeft $t^2 - 1 = 3$
$t^2 = 4$
$t = 2 \vee t = -2$
$t = 2$ geeft het punt $A(-22, 3)$ en $t = -2$ geeft het punt $B(-2, 3)$.
$\vec{v}(2) = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$, dus $rc_k = -\frac{4}{9}$.

$\left.\begin{array}{l} k\colon y = -\frac{4}{9}x + b \\ \text{door } A(-22, 3) \end{array}\right\}$ $-\frac{4}{9} \cdot -22 + b = 3$
$\frac{88}{9} + b = 3$
$b = -6\frac{7}{9}$
Dus $k\colon y = -\frac{4}{9}x - 6\frac{7}{9}$.

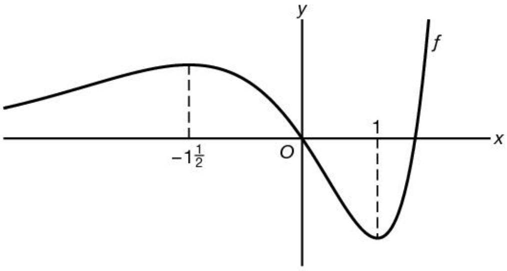
$\vec{v}(-2) = \begin{pmatrix} 15 \\ -4 \end{pmatrix}$, dus $rc_k = -\frac{4}{15}$.

$\left.\begin{array}{l} m\colon y = -\frac{4}{15}x + b \\ \text{door } A(-2, 3) \end{array}\right\}$ $-\frac{4}{15} \cdot -2 + b = 3$
$\frac{8}{15} + b = 3$
$b = 2\frac{7}{15}$
Dus $m\colon y = -\frac{4}{15}x + 2\frac{7}{15}$.

k en m snijden geeft $-\frac{4}{9}x - 6\frac{7}{9} = -\frac{4}{15}x + 2\frac{7}{15}$
$-20x - 305 = -12x + 111$
$-8x = 416$
$x = -52$
$x = -52$ geeft $y = 16\frac{1}{3}$
Dus $S\left(-52, 16\frac{1}{3}\right)$.

15 Afgeleiden en primitieven

22 **a** $f(x) = \left(x^2 - 1\frac{1}{2}x\right)e^x$ geeft $f'(x) = \left(2x - 1\frac{1}{2}\right)e^x + \left(x^2 - 1\frac{1}{2}\right)e^x = \left(x^2 + \frac{1}{2}x - 1\frac{1}{2}\right)e^x$
$f'(x) = 0$ geeft $x^2 + \frac{1}{2}x - 1\frac{1}{2} = 0$
$(x - 1)\left(x + 1\frac{1}{2}\right) = 0$
$x = 1 \vee x = -1\frac{1}{2}$



max. is $f\left(-1\frac{1}{2}\right) = \left(\left(-1\frac{1}{2}\right)^2 - 1\frac{1}{2} \cdot -1\frac{1}{2}\right)\mathrm{e}^{-1\frac{1}{2}} = 4\frac{1}{2}\mathrm{e}^{-1\frac{1}{2}}$

min. is $f(1) = \left(1 - 1\frac{1}{2}\right)\mathrm{e}^1 = -\frac{1}{2}\mathrm{e}$

b $f''(x) = \left(2x + \frac{1}{2}\right)\mathrm{e}^x + \left(x^2 + \frac{1}{2}x - 1\frac{1}{2}\right)\mathrm{e}^x = \left(x^2 + 2\frac{1}{2}x - 1\right)\mathrm{e}^x$

$f''(x) = 0$ geeft $x^2 + 2\frac{1}{2}x - 1 = 0$

$2x^2 + 5x - 2 = 0$

$D = 5^2 - 4 \cdot 2 \cdot -2 = 41$

$x = \dfrac{-5 - \sqrt{41}}{4} \vee x = \dfrac{-5 + \sqrt{41}}{4}$

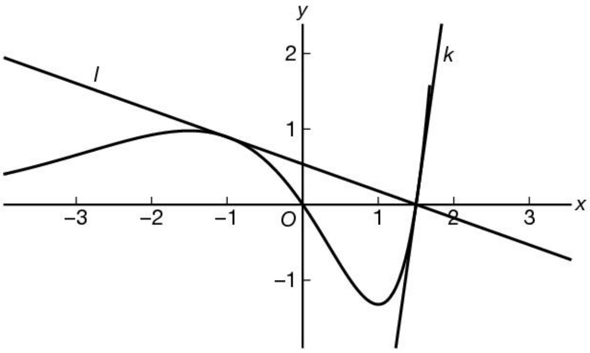
De x-coördinaten van de buigpunten zijn $-1\frac{1}{4} - \frac{1}{4}\sqrt{41}$ en $-1\frac{1}{4} + \frac{1}{4}\sqrt{41}$.

c $f'(0) = -1\frac{1}{2}\mathrm{e}^0 = -1\frac{1}{2} < 0$

$f''(0) = -\mathrm{e}^0 = -1 < 0$

Dus toenemend dalend.

d



$f\left(1\frac{1}{2}\right) = 0$, dus $\left(1\frac{1}{2}, 0\right)$ ligt op de grafiek van f.

$f'\left(1\frac{1}{2}\right) = \left(2\frac{1}{4} + \frac{3}{4} - 1\frac{1}{2}\right)\mathrm{e}^{1\frac{1}{2}} = 1\frac{1}{2}\mathrm{e}\sqrt{\mathrm{e}}$

k: $y = 1\frac{1}{2}\mathrm{e}\sqrt{\mathrm{e}} \cdot x + b$
door $\left(1\frac{1}{2}, 0\right)$ $\Big\}$ $2\frac{1}{4}\mathrm{e}\sqrt{\mathrm{e}} + b = 0$

$b = -2\frac{1}{4}\mathrm{e}\sqrt{\mathrm{e}}$

Dus k: $y = 1\frac{1}{2}\mathrm{e}\sqrt{\mathrm{e}} \cdot x - 2\frac{1}{4}\mathrm{e}\sqrt{\mathrm{e}}$ is de eerste gezochte lijn.

Raaklijn door $\left(1\frac{1}{2}, 0\right)$, dus de x-coördinaat van het raakpunt volgt uit $f'(x) = \dfrac{f(x)}{x - 1\frac{1}{2}}$.

$\left(x^2 + \frac{1}{2}x - 1\frac{1}{2}\right)\mathrm{e}^x = \dfrac{\left(x^2 - 1\frac{1}{2}x\right)\mathrm{e}^x}{x - 1\frac{1}{2}}$

$\left(x^2 + \frac{1}{2}x - 1\frac{1}{2}\right) = \dfrac{\left(x^2 - 1\frac{1}{2}x\right)}{x - 1\frac{1}{2}}$

$\left(x^2 + \frac{1}{2}x - 1\frac{1}{2}\right)\left(x - 1\frac{1}{2}\right) = x^2 - 1\frac{1}{2}x$

$\left(x^2 + \frac{1}{2}x - 1\frac{1}{2}\right)\left(x - 1\frac{1}{2}\right) = x\left(x - 1\frac{1}{2}\right)$

$x^2 + \frac{1}{2}x - 1\frac{1}{2} = x$

$x^2 - \frac{1}{2}x - 1\frac{1}{2} = 0$

$(x + 1)\left(x - 1\frac{1}{2}\right) = 0$

$x = -1 \vee x = 1\frac{1}{2}$
vold. vold. niet

$\mathrm{rc}_l = f'(-1) = \left(1 - \frac{1}{2} - 1\frac{1}{2}\right)\mathrm{e}^{-1} = -\mathrm{e}^{-1} = -\dfrac{1}{\mathrm{e}}$

$$\left.\begin{array}{l} l\colon y = -\frac{1}{e}x + b \\ \text{door } \left(1\frac{1}{2}, 0\right) \end{array}\right\} \begin{array}{l} -\frac{1}{e}\cdot 1\frac{1}{2} + b = 0 \\ b = \frac{3}{2e} \end{array}$$

Dus $l\colon y = -\frac{1}{e}x + \frac{3}{2e}$ is de tweede gezochte raaklijn.

e Er geldt $f'(0) \cdot rc_m = -1$, dus $-1\frac{1}{2} \cdot rc_m = -1$

$$rc_m = \tfrac{2}{3}$$

Dus $m\colon y = \frac{2}{3}x$.

Voer in $y_1 = \left(x^2 - 1\frac{1}{2}x\right)e^x$ en $y_2 = \frac{2}{3}x$.
Intersect geeft $x = 0$ en $x = 1{,}630...$ en $y = 1{,}087...$
Dus $B(1{,}63;\ 1{,}09)$.

23 **a** $f_2(x) = 2\sqrt{x} - \ln(x)$

$$f_2'(x) = \frac{2}{2\sqrt{x}} - \frac{1}{x} = \frac{1}{\sqrt{x}} - \frac{1}{x} = x^{-\frac{1}{2}} - x^{-1}$$

$$f_2''(x) = -\tfrac{1}{2}x^{-1\frac{1}{2}} + x^{-2} = \frac{-1}{2x\sqrt{x}} + \frac{1}{x^2}$$

$f_2''(x) = 0$ geeft $\frac{-1}{2x\sqrt{x}} + \frac{1}{x^2} = 0$

$$\frac{1}{x^2} = \frac{1}{2x\sqrt{x}}$$

$$x^2 = 2x\sqrt{x}$$

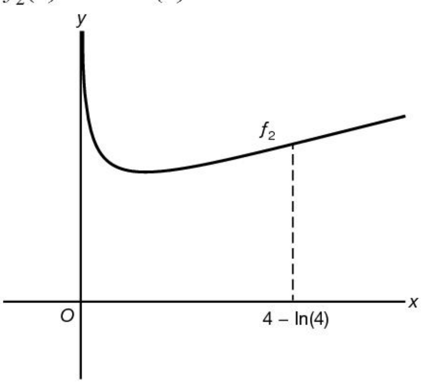
$$x\sqrt{x} \cdot \sqrt{x} = 2x\sqrt{x}$$

$$x\sqrt{x} = 0 \vee \sqrt{x} = 2$$

$$x = 0 \quad \vee\ x = 4$$

vold. niet vold.

$f_2(4) = 4 - \ln(4)$



Het buigpunt is $(4,\ 4 - \ln(4))$.

b $f_p(x) = p\sqrt{x} - \ln(x)$

$$f_p'(x) = \frac{p}{2\sqrt{x}} - \frac{1}{x} = \frac{p\sqrt{x}}{2x} - \frac{2}{2x} = \frac{p\sqrt{x} - 2}{2x}$$

$f_p'(x) = 0$ geeft $p\sqrt{x} - 2 = 0$

$$p\sqrt{x} = 2$$

$$\sqrt{x} = \frac{2}{p}$$

$$x = \frac{4}{p^2}$$

$$f_p\left(\frac{4}{p^2}\right) = p\sqrt{\frac{4}{p^2}} - \ln\left(\frac{4}{p^2}\right) = 2 - \ln\left(\frac{4}{p^2}\right)$$

Het punt $\left(\frac{4}{p^2},\ 2 - \ln\left(\frac{4}{p^2}\right)\right)$ op de cirkel met middelpunt O en straal 3 geeft

$x^2 + y^2 = 9$

$\left(\frac{4}{p^2}\right)^2 + \left(2 - \ln\left(\frac{4}{p^2}\right)\right)^2 = 9$

Voer in $y_1 = \left(\frac{4}{x^2}\right)^2 + \left(2 - \ln\left(\frac{4}{x^2}\right)\right)^2$ en $y_2 = 9$.

Intersect geeft $x = 1{,}185...$ en $x = 3{,}258...$

Dus $p \approx 1{,}19$ of $p \approx 3{,}26$.

24 **a** $f(x) = \mathrm{e}^{-\frac{1}{2}x}$ geeft $f'(x) = -\frac{1}{2}\mathrm{e}^{-\frac{1}{2}x}$ en $g(x) = p\sqrt{x}$ geeft $g'(x) = \frac{p}{2\sqrt{x}}$

De grafieken van f en g_p raken als $f(x) = g_p(x) \wedge f'(x) = g_p'(x)$

$$\mathrm{e}^{-\frac{1}{2}x} = p\sqrt{x} \wedge -\frac{1}{2}\mathrm{e}^{-\frac{1}{2}x} = \frac{p}{2\sqrt{x}}$$

$$\frac{\mathrm{e}^{-\frac{1}{2}x}}{\sqrt{x}} = p \wedge -\mathrm{e}^{-\frac{1}{2}x} \cdot \sqrt{x} = p$$

Dit stelsel heeft geen oplossingen omdat $\frac{\mathrm{e}^{-\frac{1}{2}x}}{\sqrt{x}} > 0$ en $-\mathrm{e}^{-\frac{1}{2}x} \cdot \sqrt{x} < 0$.

Er is dus geen waarde van p waarvoor de grafieken elkaar raken.

b De grafieken van f en g_p snijden elkaar loodrecht als $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$

$$\mathrm{e}^{-\frac{1}{2}x} = p\sqrt{x} \wedge -\frac{1}{2}\mathrm{e}^{-\frac{1}{2}x} \cdot \frac{p}{2\sqrt{x}} = -1$$

$$p = \frac{\mathrm{e}^{-\frac{1}{2}x}}{\sqrt{x}} \wedge p = \frac{4\sqrt{x}}{\mathrm{e}^{-\frac{1}{2}x}}$$

Voer in $y_1 = \frac{\mathrm{e}^{-\frac{1}{2}x}}{\sqrt{x}} \wedge y_2 = \frac{4\sqrt{x}}{\mathrm{e}^{-\frac{1}{2}x}}$.

Intersect geeft $x = 0{,}203...$ en $y = 2$.

Dus voor $p = 2$ snijden de grafieken elkaar loodrecht.

Alternatieve uitwerking

De grafieken van f en g_p snijden elkaar loodrecht als $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$

$$\mathrm{e}^{-\frac{1}{2}x} = p\sqrt{x} \wedge -\frac{1}{2}\mathrm{e}^{-\frac{1}{2}x} \cdot \frac{p}{2\sqrt{x}} = -1$$

Substitutie van $\mathrm{e}^{-\frac{1}{2}x} = p\sqrt{x}$ in $-\frac{1}{2}\mathrm{e}^{-\frac{1}{2}x} \cdot \frac{p}{2\sqrt{x}} = -1$ geeft $-\frac{1}{2}p\sqrt{x} \cdot \frac{p}{2\sqrt{x}} = -1$

$$\frac{-p^2}{4} = -1$$

$$p^2 = 4$$

$$p = 2 \vee p = -2$$

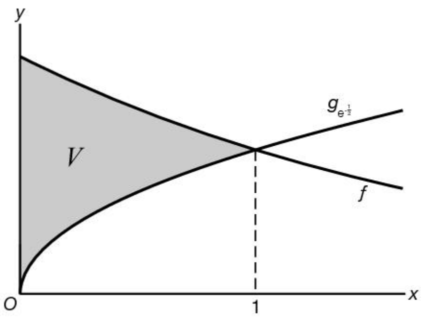
vold. vold. niet

Dus voor $p = 2$ snijden de grafieken elkaar loodrecht.

c De grafieken snijden elkaar voor $x = 1$, dus $f(1) = g_p(1)$

$$\mathrm{e}^{-\frac{1}{2}} = p\sqrt{1}$$

$$p = \mathrm{e}^{-\frac{1}{2}}$$

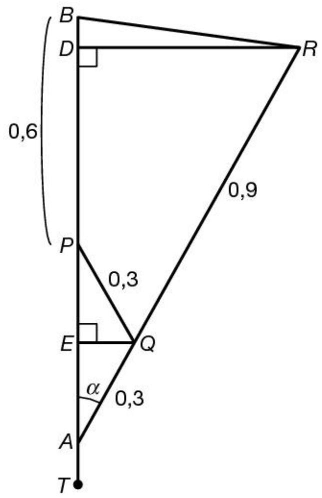


$$O(V) = \int_0^1 (f(x) - g_{\mathrm{e}^{-\frac{1}{2}}}(x))\,\mathrm{d}x = \int_0^1 \left(\mathrm{e}^{-\frac{1}{2}x} - \mathrm{e}^{-\frac{1}{2}} \cdot x^{\frac{1}{2}}\right)\mathrm{d}x = \left[-2\mathrm{e}^{-\frac{1}{2}x} - \frac{2}{3}\mathrm{e}^{-\frac{1}{2}} \cdot x^{1\frac{1}{2}}\right]_0^1$$

$$= -2\mathrm{e}^{-\frac{1}{2}} - \frac{2}{3}\mathrm{e}^{-\frac{1}{2}} \cdot 1^{1\frac{1}{2}} - \left(-2\mathrm{e}^{0} - \frac{2}{3}\mathrm{e}^{-\frac{1}{2}} \cdot 0^{1\frac{1}{2}}\right) = -2\mathrm{e}^{-\frac{1}{2}} - \frac{2}{3}\mathrm{e}^{-\frac{1}{2}} + 2 + 0 = 2 - 2\frac{2}{3}\mathrm{e}^{-\frac{1}{2}}$$

Bladzijde 229

25 **a**



In $\triangle ADR$ is $\sin(\alpha) = \dfrac{DR}{AR}$ $\qquad \cos(\alpha) = \dfrac{AD}{AR}$

$DR = AR\sin(\alpha)$ $\qquad AD = AR\cos(\alpha)$

$DR = 1{,}2\sin(\alpha)$ $\qquad AD = 1{,}2\cos(\alpha)$

In $\triangle AEQ$ is $\cos(\alpha) = \dfrac{AE}{AQ}$

$AE = AQ\cos(\alpha)$

$AE = 0{,}3\cos(\alpha)$

$AP = 2 \cdot 0{,}3\cos(\alpha) = 0{,}6\cos(\alpha)$

$BD = AB - AD = 0{,}6 + 0{,}6\cos(\alpha) - 1{,}2\cos(\alpha) = 0{,}6 - 0{,}6\cos(\alpha)$

In $\triangle BDR$ is $BR^2 = BD^2 + DR^2$

$BR^2 = (0{,}6 - 0{,}6\cos(\alpha))^2 + (1{,}2\sin(\alpha))^2$

$BR^2 = 0{,}36 - 0{,}72\cos(\alpha) + 0{,}36\cos^2(\alpha) + 1{,}44\sin^2(\alpha)$

$BR^2 = 0{,}36 - 0{,}72\cos(\alpha) + 0{,}36\cos^2(\alpha) + 1{,}44(1 - \cos^2(\alpha))$

$BR^2 = 0{,}36 - 0{,}72\cos(\alpha) + 0{,}36\cos^2(\alpha) + 1{,}44 - 1{,}44\cos^2(\alpha)$

$BR^2 = 1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)$

Dus $BR = \sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)}$.

b $BR > 1$ geeft $\sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)} > 1$

Voer in $y_1 = \sqrt{1{,}8 - 0{,}72\cos(x) - 1{,}08\cos^2(x)}$ en $y_2 = 1$.

Intersect geeft $x \approx 0{,}94$.

Dus de lengte van het uitgerolde doek is meer dan 1 meter bij een hoek van 0,94 rad of groter.

c $y = 1{,}08\cos^2(\alpha)$ geeft $\dfrac{dy}{d\alpha} = 2{,}16\cos(\alpha) \cdot {}^-\sin(\alpha) = {}^-2{,}16\sin(\alpha)\cos(\alpha)$

$L = BR = \sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)}$ geeft

$$\frac{dL}{d\alpha} = \frac{1}{2\sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)}} \cdot (0{,}72\sin(\alpha) - 2{,}6\cos(\alpha) \cdot {}^-\sin(\alpha))$$

$$= \frac{0{,}36\sin(\alpha) + 1{,}08\sin(\alpha)\cos(\alpha)}{\sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)}}$$

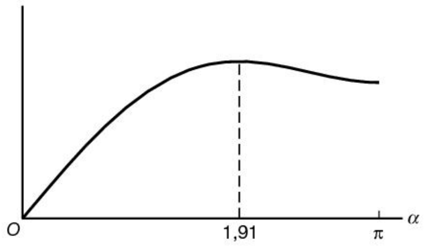
$\dfrac{dy}{d\alpha} = 0$ geeft $0{,}36\sin(\alpha) + 1{,}08\sin(\alpha)\cos(\alpha) = 0$

$0{,}36\sin(\alpha)(1 + 3\cos(\alpha)) = 0$

$0{,}36\sin(\alpha) = 0 \vee 1 + 3\cos(\alpha) = 0$

$\sin(\alpha) = 0 \vee \cos(\alpha) = {}^-\frac{1}{3}$

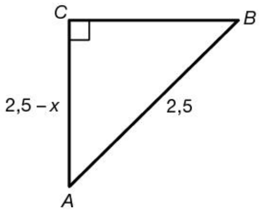
$\alpha = k \cdot \pi \vee \alpha \approx 1{,}91 + k \cdot 2\pi \vee \alpha \approx {}^-1{,}91 + k \cdot 2\pi$



Dus BR is maximaal bij een hoek van ongeveer 1,91 rad.

Bladzijde 230

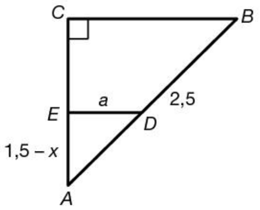
26 **a**



$x + AC = 2{,}5$, dus $AC = 2{,}5 - x$

In $\triangle ABC$ is $AC^2 + BC^2 = AB^2$

$$(2{,}5 - x)^2 + BC^2 = 2{,}5^2$$
$$6{,}25 - 5x + x^2 + BC^2 = 6{,}25$$
$$BC^2 = 5x - x^2$$
$$BC = \sqrt{5x - x^2}$$



$\triangle AED \backsim \triangle ACB$

$$\frac{AE}{AC} = \frac{ED}{CB}$$

$$\frac{1{,}5 - x}{2{,}5 - x} = \frac{a}{\sqrt{5x - x^2}}$$

$$a(2{,}5 - x) = (1{,}5 - x)\sqrt{5x - x^2}$$

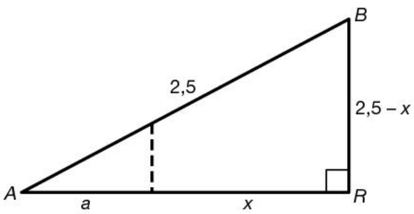
$$a = \frac{(1{,}5 - x)\sqrt{5x - x^2}}{2{,}5 - x}$$

b Voer in $y_1 = \dfrac{(1{,}5 - x)\sqrt{5x - x^2}}{2{,}5 - x}$.
De optie maximum geeft $x \approx 0{,}66$ en $y \approx 0{,}77$.
Dus de maximale waarde van a is ongeveer 0,77 m.
De onderkant van de deur is dan ongeveer 66 cm omhoog geschoven.

c Het dak van een 1,5 meter hoge auto moet minimaal 77 cm van de deur verwijderd zijn om de garagedeur te kunnen openen of sluiten.

Bladzijde 231

27



$AQ + BR = 2{,}5$, dus $BR = 2{,}5 - AQ = 2{,}5 - x$

In $\triangle ARB$ is $AR^2 + BR^2 = AB^2$

$$(a + x)^2 + (2{,}5 - x)^2 = 2{,}5^2$$
$$(a + x)^2 + 6{,}25 - 5x + x^2 = 6{,}25$$
$$(a + x)^2 = 5x - x^2$$
$$a + x = \sqrt{5x - x^2}$$
$$a = -x + \sqrt{5x - x^2}$$

$$\frac{da}{dx} = -1 + \frac{1}{2\sqrt{5x - x^2}} \cdot (5 - 2x) = -1 + \frac{5 - 2x}{2\sqrt{5x - x^2}}$$

$$\frac{da}{dx} = 0 \text{ geeft } -1 + \frac{5 - 2x}{2\sqrt{5x - x^2}} = 0$$

$$\frac{5 - 2x}{2\sqrt{5x - x^2}} = 1$$

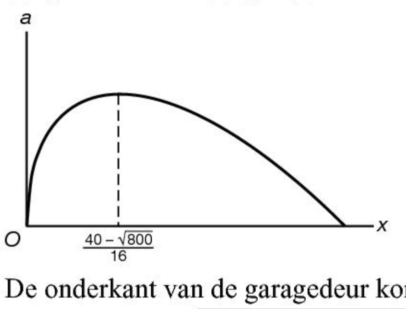
$$2\sqrt{5x - x^2} = 5 - 2x$$

kwadrateren geeft

$$4(5x - x^2) = 25 - 20x + 4x^2$$
$$20x - 4x^2 = 25 - 20x + 4x^2$$
$$8x^2 - 40x + 25 = 0$$
$$D = (-40)^2 - 4 \cdot 8 \cdot 25 = 1600 - 800 = 800$$

$$x = \frac{40 - \sqrt{800}}{16} \vee x = \frac{40 + \sqrt{800}}{16}$$

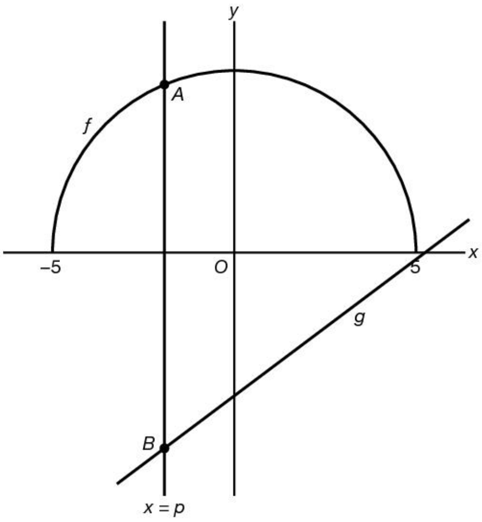
vold. vold. niet



De onderkant van de garagedeur komt maximaal

$$-\frac{40 - \sqrt{800}}{16} + \sqrt{5 \cdot \frac{40 - \sqrt{800}}{16} - \left(\frac{40 - \sqrt{800}}{16}\right)^2} \approx 1{,}04 \text{ meter naar buiten.}$$

28 **a**

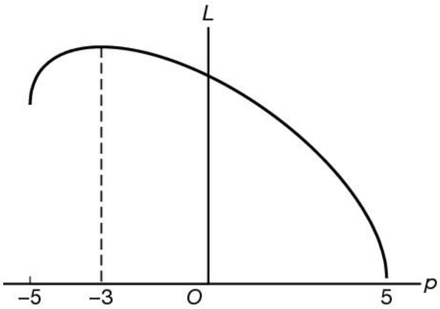


$L = f(p) - g(p) = \sqrt{25 - p^2} - \left(\frac{3}{4}p - 4\right) = \sqrt{25 - p^2} - \frac{3}{4}p + 4$

$\frac{dL}{dp} = \frac{1}{2\sqrt{25 - p^2}} \cdot -2p - \frac{3}{4} = -\frac{p}{\sqrt{25 - p^2}} - \frac{3}{4}$

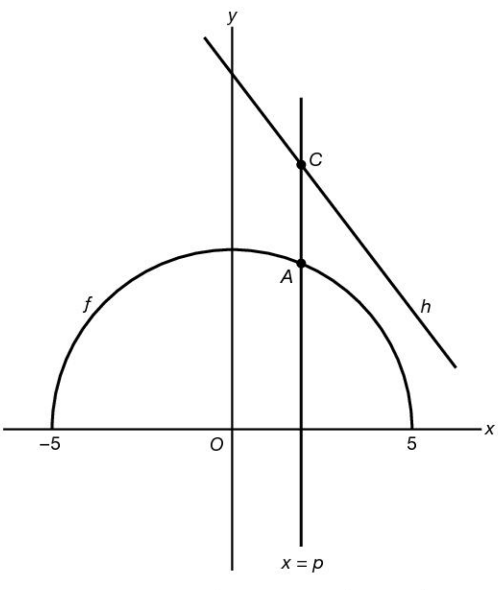
$\frac{dL}{dp} = 0$ geeft $\frac{p}{\sqrt{25 - p^2}} = -\frac{3}{4}$

$4p = -3\sqrt{25 - p^2}$
kwadrateren geeft
$16p^2 = 9(25 - p^2)$
$16p^2 = 225 - 9p^2$
$25p^2 = 225$
$p^2 = 9$
$p = -3 \vee p = 3$
vold. vold. niet



De maximale lengte is $\sqrt{25 - (-3)^2} - \frac{3}{4} \cdot -3 + 4 = 10\frac{1}{4}$.

b

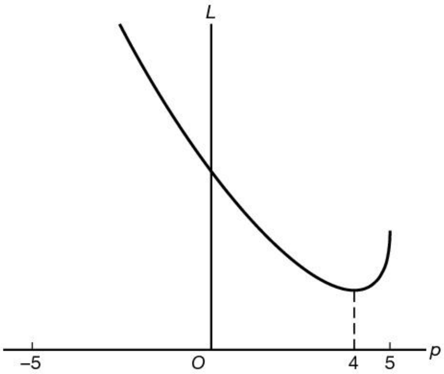


$L = h(p) - f(p) = -\frac{4}{3}p + 10 - \sqrt{25 - p^2}$

$\frac{dL}{dp} = -\frac{4}{3} - \frac{1}{2\sqrt{25-p^2}} \cdot -2p = -\frac{4}{3} + \frac{p}{\sqrt{25-p^2}}$

$\frac{dL}{dp} = 0$ geeft $\frac{p}{\sqrt{25-p^2}} = \frac{4}{3}$

$3p = 4\sqrt{25-p^2}$
kwadrateren geeft
$9p^2 = 16(25-p^2)$
$9p^2 = 400 - 16p^2$
$25p^2 = 400$
$p^2 = 16$
$p = -4 \vee p = 4$
vold. niet vold.



De minimale lengte is $-\frac{4}{3} \cdot 4 + 10 - \sqrt{25-4^2} = 1\frac{2}{3}$.

29 **a** $f(x) = -3 + \sqrt{2x+3}$ geeft $f'(x) = \frac{1}{2\sqrt{2x+3}} \cdot 2 = \frac{1}{\sqrt{2x+3}}$
$g_p(x) = p \cdot (x-3)$ geeft $g_p'(x) = p$, dus $g_1'(x) = 1$.
$f'(3) = \frac{1}{\sqrt{6+3}} = \frac{1}{3}$

$\tan(\alpha) = \frac{1}{3}$ geeft $\alpha = 18{,}43...°$
$\tan(\beta) = 1$ geeft $\beta = 45°$
$\beta - \alpha = 45° - 18{,}43...° = 26{,}56...°$
Dus de hoek tussen de grafieken is ongeveer 26,6°.

b De grafiek van f gaat door het punt (3, 0) en de grafiek van g_p gaat voor elke p door (3, 0).
De grafieken van f en g_p snijden elkaar loodrecht als $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$

$x = 3 \wedge \frac{1}{\sqrt{2x+3}} \cdot p = -1$

$x = 3 \wedge p = -\sqrt{2x+3}$
$x = 3 \wedge p = -\sqrt{6+3}$
$x = 3 \wedge p = -3$

Dus voor $p = -3$ snijden de grafieken elkaar loodrecht.

c $f'(3) = \frac{1}{3}$
$\tan(\alpha) = \frac{1}{3}$ geeft $\alpha = 18{,}434...°$
$p = \tan(18{,}434...° + 50°) = \tan(68{,}434...°) = 2{,}530...$ of
$p = \tan(18{,}534...° - 50°) = \tan(-31{,}565...°) = -0{,}614...$
Dus $p \approx 2{,}53$ of $p \approx -0{,}61$.

d I Als de grafiek van f tussen 0 en 3 boven de grafiek van g_p ligt.

$$O(V) = \int_0^3 (f(x) - g(x))\,dx = \int_0^3 (-3 + (2x+3)^{\frac{1}{2}} - p(x-3))\,dx$$
$$= \left[-3x + \tfrac{2}{3} \cdot \tfrac{1}{2}(2x+3)^{1\frac{1}{2}} - \tfrac{1}{2}p(x-3)^2\right]_0^3 = -9 + \tfrac{1}{3} \cdot 9^{1\frac{1}{2}} - \tfrac{1}{2}p \cdot 0 - \left(0 + \tfrac{1}{3} \cdot 3^{1\frac{1}{2}} - \tfrac{1}{2}p \cdot 9\right)$$
$$= -9 + 9 - 0 - \sqrt{3} + 4\tfrac{1}{2}p = 4\tfrac{1}{2}p - \sqrt{3}$$

$O(V) = 4\sqrt{3}$ geeft $4\frac{1}{2}p - \sqrt{3} = 4\sqrt{3}$

$4\frac{1}{2}p = 5\sqrt{3}$

$p = 1\frac{1}{9}\sqrt{3}$

II Als de grafiek van f tussen 0 en 3 onder de grafiek van g_p ligt.

$O(V) = \int_0^3 (g(x) - f(x))\,dx = -4\frac{1}{2}p + \sqrt{3}$

$O(V) = 4\sqrt{3}$ geeft $-4\frac{1}{2}p + \sqrt{3} = 4\sqrt{3}$

$-4\frac{1}{2}p = 3\sqrt{3}$

$p = -\frac{2}{3}\sqrt{3}$

Dus de oppervlakte van V is gelijk aan $4\sqrt{3}$ voor $p = 1\frac{1}{9}\sqrt{3} \vee p = -\frac{2}{3}\sqrt{3}$.

30 **a** De hypotenusa van de driehoek is de grafiek van de functie $g_4(x) = -x + 4$.
Noem het lichaam dat ontstaat bij wentelen van W om de x-as M.

$f(x) = 0$ geeft $x^2 - x = 0$

$x(x - 1) = 0$

$x = 0 \vee x = 1$

$f(x) = g_4(x)$ geeft $x^2 - x = -x + 4$

$x^2 = 4$

$x = -2 \vee x = 2$

vold. niet vold.

$$I(L) = I(\text{kegel}) - I(M) = \frac{1}{3}\pi \cdot 4^2 \cdot 4 - \left(\pi\int_1^2 (x^2 - x)^2\,dx + \frac{1}{3}\pi \cdot 2^2 \cdot 2\right)$$

$$= 21\frac{1}{3}\pi - \pi\int_1^2 (x^4 - 2x^3 + x^2)\,dx - \frac{8}{3}\pi$$

$$= 21\frac{1}{3}\pi - \pi\left[\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{1}{3}x^3\right]_1^2 - 2\frac{2}{3}\pi$$

$$= 18\frac{2}{3}\pi - \pi\left(\frac{32}{5} - \frac{16}{2} + \frac{8}{3} - \left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right)\right)$$

$$= 18\frac{2}{3}\pi - \pi\left(\frac{16}{15} - \frac{1}{30}\right) = 17\frac{19}{30}\pi$$

b $f(x) = g_p(x)$ geeft $x^2 - x = -x + p$

$x^2 = p$

$x = \sqrt{p} \vee x = -\sqrt{p}$

vold. vold. niet

$O(V) = O(W)$

$O(W) = \frac{1}{2} \cdot O(V + W)$

$O(W) = \frac{1}{2} \cdot \frac{1}{2}p \cdot p$

$\int_1^{\sqrt{p}} (x^2 - x)\,dx + \int_{\sqrt{p}}^{p} (-x + p)\,dx = \frac{1}{4}p^2$

$\left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_1^{\sqrt{p}} + \left[-\frac{1}{2}x^2 + px\right]_{\sqrt{p}}^{p} = \frac{1}{4}p^2$

$\frac{1}{3}p\sqrt{p} - \frac{1}{2}p - \left(\frac{1}{3} - \frac{1}{2}\right) + -\frac{1}{2}p^2 + p^2 - \left(-\frac{1}{2}p + p\sqrt{p}\right) = \frac{1}{4}p^2$

$\frac{1}{3}p\sqrt{p} - \frac{1}{2}p + \frac{1}{6} + \frac{1}{2}p^2 + \frac{1}{2}p - p\sqrt{p} = \frac{1}{4}p^2$

$-\frac{2}{3}p\sqrt{p} + \frac{1}{4}p^2 + \frac{1}{6} = 0$

Voer in $y_1 = -\frac{2}{3}x\sqrt{x} + \frac{1}{4}x^2 + \frac{1}{6}$.

De optie nulpunt geeft $x = 0{,}48565...$ en $x = 6{,}91700...$
Dus de vlakdelen V en W hebben gelijke oppervlakte voor $p \approx 6{,}9170$.

Bladzijde 232

31 **a** $f(x) = \frac{1}{x} = x^{-1}$ geeft $f'(x) = -x^{-2} = -\frac{1}{x^2}$

Stel $y = ax + b$ met $a = f'(2) = -\frac{1}{2^2} = -\frac{1}{4}$.

$\left.\begin{array}{l} y = -\frac{1}{4}x + b \\ f(2) = \frac{1}{2} \end{array}\right\}$ $-\frac{1}{4} \cdot 2 + b = \frac{1}{2}$

$-\frac{1}{2} + b = \frac{1}{2}$

$b = 1$

Dus $\left.\begin{array}{l} y = -\frac{1}{4}x + 1 \\ A(4, 0) \end{array}\right\}$ $\begin{array}{l} -\frac{1}{4} \cdot 4 + 1 = 0 \\ -1 + 1 = 0 \text{ klopt} \end{array}$

Dus de raaklijn in het punt met x-coördinaat 2 gaat door A.

b $O(V) = \frac{1}{4} \cdot 4 + \int_{\frac{1}{4}}^{4} f(x)\,\mathrm{d}x = 1 + \int_{\frac{1}{4}}^{4} \frac{1}{x}\,\mathrm{d}x = 1 + \left[\ln|x|\right]_{\frac{1}{4}}^{4} = 1 + \ln(4) - \ln\left(\frac{1}{4}\right) = 1 + \ln(4) - \ln(4^{-1})$

$= 1 + \ln(4) + \ln(4) = 1 + 2\ln(4) = \ln(\mathrm{e}) + \ln(16) = \ln(16\mathrm{e})$

c $\mathrm{rc}_{AC} = -1$, dus $f'(x) = -1$

$-\frac{1}{x^2} = -1$

$x^2 = 1$, dus $x = 1$.

$\left.\begin{array}{l} y = -x + b \\ f(1) = 1 \end{array}\right\}$ $\begin{array}{l} -1 + b = 1 \\ b = 2 \end{array}$

Dus de formule van de raaklijn is $y = -x + 2$.

Het snijpunt van de raaklijn met de y-as is $(0, 2)$, dus $a = 2$.

16 Examentraining

Bladzijde 233

32 $y = 0$ geeft $\cos(2t) = 0$

$2t = \frac{1}{2}\pi + k \cdot \pi$

$t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

De eerste keer, dus $t = \frac{1}{4}\pi$.

$\vec{r}(t) = \begin{pmatrix} \cos(3t) \\ \cos(2t) \end{pmatrix}$

$\vec{v}\left(\frac{1}{4}\pi\right) = \begin{pmatrix} -3\sin\left(\frac{3}{4}\pi\right) \\ -2\sin\left(\frac{1}{2}\pi\right) \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2}\sqrt{2} \\ -2 \end{pmatrix}$

$v\left(\frac{1}{4}\pi\right) = \sqrt{\left(-1\frac{1}{2}\sqrt{2}\right)^2 + (-2)^2} = \sqrt{8\frac{1}{2}} = \frac{1}{2}\sqrt{34}$

33 $\cos^2\left(\frac{1}{2}x\right) - \cos\left(\frac{1}{2}x\right) - 2 = 0$

$\left(\cos\left(\frac{1}{2}x\right) + 1\right)\left(\cos\left(\frac{1}{2}x\right) - 2\right) = 0$

$\cos\left(\frac{1}{2}x\right) = -1 \vee \cos\left(\frac{1}{2}x\right) = 2$

$\frac{1}{2}x = \pi + k \cdot 2\pi$ geen opl.

$x = 2\pi + k \cdot 4\pi$

34 $\vec{r}(t) = \begin{pmatrix} t^2 - \frac{1}{2}t + 1 \\ t^2 - 2t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 2t - \frac{1}{2} \\ 2t - 2 \end{pmatrix}$

$v(t) = \sqrt{\left(2t - \frac{1}{2}\right)^2 + (2t - 2)^2} = \sqrt{4t^2 - 2t + \frac{1}{4} + 4t^2 - 8t + 4} = \sqrt{8t^2 - 10t + 4\frac{1}{4}}$

$v(t)$ is minimaal als $8t^2 - 10t + 4\frac{1}{4}$ minimaal is. Dat is voor $t = -\frac{b}{2a} = \frac{10}{16} = \frac{5}{8}$.

$t = \frac{5}{8}$ geeft het punt $\left(1\frac{5}{64}, -\frac{55}{64}\right)$.

35 Raken, dus $f(x) = g(x) \wedge f'(x) = g'(x)$

$\sqrt{5 - x^2} = ax + 5 \wedge \frac{1}{2\sqrt{5 - x^2}} \cdot -2x = a$

$\sqrt{5 - x^2} = ax + 5 \wedge \frac{-x}{\sqrt{5 - x^2}} = a$

Substitutie van $a = \dfrac{-x}{\sqrt{5-x^2}}$ in $\sqrt{5-x^2} = ax + 5$ geeft $\sqrt{5-x^2} = \dfrac{-x}{\sqrt{5-x^2}} \cdot x + 5$

$$5 - x^2 = -x^2 + 5\sqrt{5-x^2}$$
$$5 = 5\sqrt{5-x^2}$$
$$1 = \sqrt{5-x^2}$$
$$1 = 5 - x^2$$
$$x^2 = 4$$
$$x = -2 \vee x = 2$$

$x = -2$ geeft $a = \dfrac{2}{\sqrt{5-(-2)^2}} = \dfrac{2}{\sqrt{5-4}} = 2$ en $x = 2$ geeft $a = \dfrac{-2}{\sqrt{5-2^2}} = \dfrac{-2}{\sqrt{5-4}} = -2$

Dus de grafieken van f en g raken elkaar voor $a = 2$ en voor $a = -2$.

36 ${}^3\log(x+1) = 4 + {}^{\frac{1}{3}}\log(x-1)$

${}^3\log(x+1) = 4 + \dfrac{{}^3\log(x-1)}{{}^3\log\left(\frac{1}{3}\right)}$

${}^3\log(x+1) = 4 + \dfrac{{}^3\log(x-1)}{-1}$

${}^3\log(x+1) = 4 - {}^3\log(x-1)$

${}^3\log(x+1) + {}^3\log(x-1) = 4$

${}^3\log(x+1)(x-1) = 4$

${}^3\log(x^2-1) = 4$

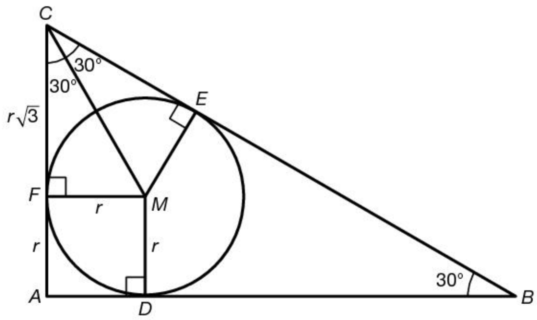
$x^2 - 1 = 3^4$

$x^2 = 82$

$x = \sqrt{82} \vee x = -\sqrt{82}$

vold. vold. niet

37 Stel r de straal van c.



Dan is $AF = r$, $FC = r\sqrt{3}$, $AC = r + r\sqrt{3}$ en $AB = (r + r\sqrt{3}) \cdot \sqrt{3} = r\sqrt{3} + 3r$.

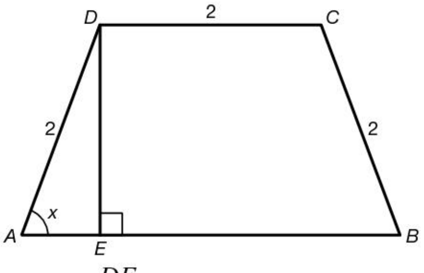
$O(\text{cirkel}) = \pi r^2$

$\frac{1}{2}O(\triangle ABC) = \frac{1}{2} \cdot \frac{1}{2} \cdot AB \cdot AC = \frac{1}{4}(r\sqrt{3} + 3r)(r + r\sqrt{3}) = \frac{1}{4}(\sqrt{3} + 3)(1 + \sqrt{3}) \cdot r^2$

$\frac{1}{4}(\sqrt{3} + 3)(1 + \sqrt{3}) = 3{,}232...$ en $\pi = 3{,}141...$

Dus de oppervlakte van de cirkel is minder dan de helft van de oppervlakte van $\triangle ABC$.

38 **a**



$\sin(x) = \dfrac{DE}{AD}$ geeft $DE = AD \cdot \sin(x) = 2\sin(x)$

$\cos(x) = \dfrac{AE}{AD}$ geeft $AE = AD \cdot \cos(x) = 2\cos(x)$

$AB = 2 + 2 \cdot AE = 2 + 2 \cdot 2\cos(x) = 2 + 4\cos(x)$

$O(\text{trapezium}) = \frac{1}{2}(AB + CD) \cdot DE = \frac{1}{2}(2 + 4\cos(x) + 2) \cdot 2\sin(x) = \sin(x)(4 + 4\cos(x))$

b $O'(x) = \cos(x)(4 + 4\cos(x)) + \sin(x) \cdot -4\sin(x) = 4\cos(x) + 4\cos^2(x) - 4\sin^2(x)$
Voer in $y_1 = 4\cos(x) + 4\cos^2(x) - 4\sin^2(x)$.
De optie nulpunt geeft $x = 1{,}047...$
$x = 1{,}047...$ geeft $O(\text{trapezium}) = 5{,}196...$
Dus de maximale oppervlakte is 5,20.

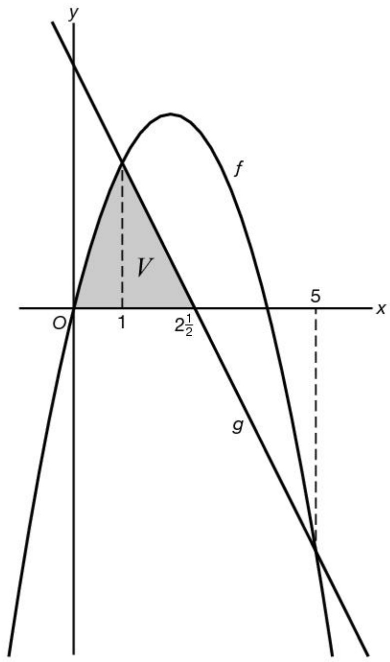
39 $\cos\left(2x - \frac{1}{2}\pi\right) = \cos\left(x + \frac{1}{3}\pi\right)$

$2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = -x - \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{5}{6}\pi + k \cdot 2\pi \vee 3x = \frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$

40 $f(x) = g(x)$ geeft $4x - x^2 = 5 - 2x$
$x^2 - 6x + 5 = 0$
$(x - 1)(x - 5) = 0$
$x = 1 \vee x = 5$
$g(x) = 0$ geeft $5 - 2x = 0$ ofwel $x = 2\frac{1}{2}$



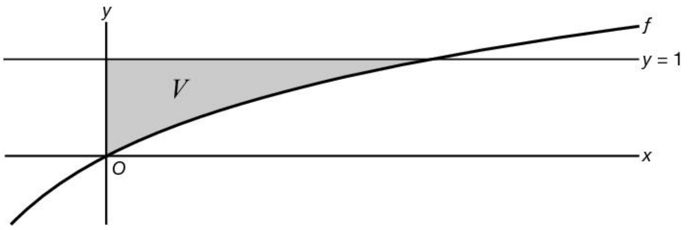
$$I(L) = \pi \int_0^1 (4x - x^2)^2 \, dx + \pi \int_1^{2\frac{1}{2}} (5 - 2x)^2 \, dx = \pi \int_0^1 (16x^2 - 8x^3 + x^4) \, dx + \pi \int_1^{2\frac{1}{2}} (25 - 20x + 4x^2) \, dx$$

$$= \pi \left[\tfrac{16}{3}x^3 - 2x^4 + \tfrac{1}{5}x^5\right]_0^1 + \pi\left[25x - 10x^2 + \tfrac{4}{3}x^3\right]_1^{2\frac{1}{2}}$$

$$= \pi\left(\tfrac{16}{3} - 2 + \tfrac{1}{5} - 0\right) + \pi\left(25 \cdot 2\tfrac{1}{2} - 10 \cdot \left(2\tfrac{1}{2}\right)^2 + \tfrac{4}{3} \cdot \left(2\tfrac{1}{2}\right)^3\right) - \pi\left(25 - 10 + \tfrac{4}{3}\right)$$

$$= \tfrac{53}{15}\pi + \tfrac{125}{6}\pi - \tfrac{49}{3}\pi = \tfrac{241}{30}\pi = 6\tfrac{1}{30}\pi$$

41 $y = f(x)$ geeft $\ln\left(\frac{1}{2}x + 1\right) = y$
$\frac{1}{2}x + 1 = e^y$
$\frac{1}{2}x = e^y - 1$
$x = 2e^y - 2$



$$I(L) = \pi \int_0^1 x^2 \, \mathrm{d}y = \pi \int_0^1 (2\,\mathrm{e}^y - 2)^2 \, \mathrm{d}y = \pi \int_0^1 (4\,\mathrm{e}^{2y} - 8\,\mathrm{e}^y + 4) \, \mathrm{d}y = \pi \left[2\,\mathrm{e}^{2y} - 8\,\mathrm{e}^y + 4y \right]_0^1$$

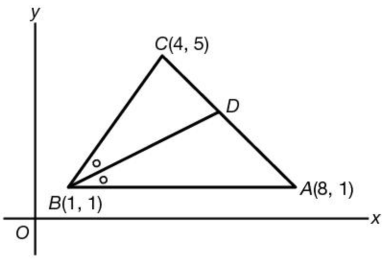
$$= \pi(2\mathrm{e}^2 - 8\mathrm{e} + 4) - \pi(2 - 8 + 0) = (2\mathrm{e}^2 - 8\mathrm{e} + 10)\pi$$

42 $f(x) = \dfrac{16}{(4x + 10)^5} = 16(4x + 5)^{-5}$ geeft $F(x) = 16 \cdot \frac{1}{4} \cdot {}^{-}\frac{1}{4}(4x + 10)^{-4} + c = \dfrac{-1}{(4x + 10)^4} + c$

$g(x) = \cos(\pi(x - 3))$ geeft $G(x) = \dfrac{1}{\pi} \sin(\pi(x - 3)) + c$

Bladzijde 234

43



$\overrightarrow{BC} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ en $\overrightarrow{BA} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\vec{r}_{BD} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$BD\colon \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$A(8, 1)$ en $B(4, 5)$ geeft $AB\colon x + y = 9$ $\Big\}$ $1 + 2\lambda + 1 + \lambda = 9$

$3\lambda = 7$

$\lambda = \frac{7}{3}$

$D\left(1 + 2 \cdot \frac{7}{3}, 1 + \frac{7}{3}\right)$ ofwel $D\left(5\frac{2}{3}, 3\frac{1}{3}\right)$

$AD = \sqrt{\left(8 - 5\frac{2}{3}\right)^2 + \left(1 - 3\frac{1}{3}\right)^2} = \sqrt{\left(2\frac{1}{3}\right)^2 + \left({}^{-}2\frac{1}{3}\right)^2} = 2\frac{1}{3}\sqrt{2}$

44 Loodrecht snijden, dus $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = {}^{-}1$

$\ln(x) = x^2 + px \wedge \dfrac{1}{x} \cdot (2x + p) = {}^{-}1$

$\ln(x) = x^2 + px \wedge 2 + \dfrac{p}{x} = {}^{-}1$

$\ln(x) = x^2 + px \wedge \dfrac{p}{x} = {}^{-}3$

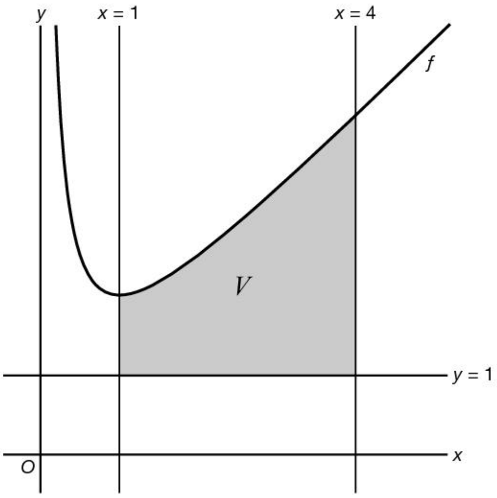
$\ln(x) = x^2 + px \wedge p = {}^{-}3x$

Substitutie van $p = {}^{-}3x$ in $\ln(x) = x^2 + px$ geeft $\ln(x) = x^2 + {}^{-}3x \cdot x$

$\ln(x) = {}^{-}2x^2$

Voer in $y_1 = \ln(x)$ en $y_2 = {}^{-}2x^2$.
Intersect geeft $x = 0{,}5482...$
Dus $p = {}^{-}3x \approx {}^{-}1{,}64$.

45 **a**



$$I(L) = \pi\int_1^4 \left(\frac{x^2+1}{x}\right)^2 \mathrm{d}x - \pi \cdot 1^2 \cdot 3 = \pi\int_1^4 \left(x + \frac{1}{x}\right)^2 \mathrm{d}x - 3\pi = \pi\int_1^4 \left(x^2 + 2 + \frac{1}{x^2}\right) \mathrm{d}x - 3\pi$$

$$= \pi\int_1^4 (x^2 + 2 + x^{-2})\,\mathrm{d}x - 3\pi = \pi\left[\tfrac{1}{3}x^3 + 2x - x^{-1}\right]_1^4 - 3\pi$$

$$= \pi\left(\tfrac{1}{3}\cdot 4^3 + 2\cdot 4 - 4^{-1} - \left(\tfrac{1}{3} + 2 - 1\right)\right) - 3\pi = 27\tfrac{3}{4}\pi - 3\pi = 24\tfrac{3}{4}\pi$$

b Verschuif de grafiek van f 1 omlaag, dit geeft de grafiek van $g(x) = x + \dfrac{1}{x} - 1$.

$$I(M) = \pi\int_1^4 (g(x))^2\,\mathrm{d}x = \pi\int_1^4 \left(\frac{x^2+1}{x} - 1\right)^2 \mathrm{d}x = \pi\int_1^4 \left(\left(\frac{x^2+1}{x}\right)^2 - 2\cdot\frac{x^2+1}{x} + 1\right)\mathrm{d}x$$

$$= \pi\int_1^4 \left(x^2 + 2 + \frac{1}{x^2} - 2\left(x + \frac{1}{x}\right) + 1\right)\mathrm{d}x = \pi\int_1^4 \left(x^2 + 2 + \frac{1}{x^2} - 2x - \frac{2}{x} + 1\right)\mathrm{d}x$$

$$= \pi\int_1^4 \left(x^2 - 2x + 3 - \frac{2}{x} + x^{-2}\right)\mathrm{d}x = \pi\left[\tfrac{1}{3}x^3 - x^2 + 3x - 2\ln|x| - x^{-1}\right]_1^4$$

$$= \pi\left(\tfrac{1}{3}\cdot 4^3 - 4^2 + 3\cdot 4 - 2\ln|4| - \tfrac{1}{4} - \left(\tfrac{1}{3} - 1 + 3 - 2\ln(1) - 1\right)\right)$$

$$= \pi\left(21\tfrac{1}{3} - 16 + 12 - 2\ln|4| - \tfrac{1}{4} - \left(\tfrac{1}{3} - 1 + 3 - 0 - 1\right)\right) = 15\tfrac{3}{4}\pi - 2\pi\ln|4|$$

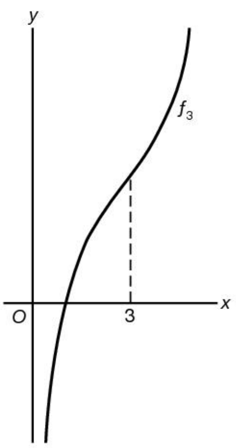
46 $f_a(x) = (x + a)\ln(x)$

$f_a'(x) = 1 \cdot \ln(x) + (x + a) \cdot \dfrac{1}{x} = \ln(x) + 1 + \dfrac{a}{x} = \ln(x) + 1 + ax^{-1}$

$f_a''(x) = \dfrac{1}{x} - ax^{-2} = \dfrac{1}{x} - \dfrac{a}{x^2}$

$f_a''(3) = 0$ geeft $\tfrac{1}{3} - \dfrac{a}{9} = 0$

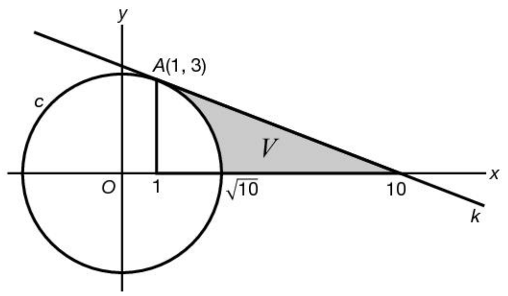
$a = 3$



De grafiek van f_a gaat voor $a = 3$ over van afnemend stijgend naar toenemend stijgend.

47 $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, dus k: $x + 3y = c$ door $A(1, 3)$ $\Big\}$ $c = 1 + 3 \cdot 3 = 10$

$\left.\begin{array}{l} k: x + 3y = 10 \\ y = 0 \end{array}\right\} x = 10$



$$I(L) = \tfrac{1}{3}\pi \cdot 3^2 \cdot 9 - \pi \int_1^{\sqrt{10}} y^2 \mathrm{d}x = 27\pi - \pi \int_1^{\sqrt{10}} (10 - x^2)\mathrm{d}x = 27\pi - \pi\Big[10x - \tfrac{1}{3}x^3\Big]_1^{\sqrt{10}}$$

$$= 27\pi - \pi\left(10\sqrt{10} - \tfrac{1}{3} \cdot 10\sqrt{10}\right) + \pi\left(10 - \tfrac{1}{3}\right) = 27\pi - \pi \cdot 6\tfrac{2}{3}\sqrt{10} + \pi \cdot 9\tfrac{2}{3} = \pi\left(36\tfrac{2}{3} - 6\tfrac{2}{3}\sqrt{10}\right)$$

48 $\left.\begin{array}{l} 2 + \dfrac{3}{x+4} = \dfrac{c}{y} \\ x = 3 \text{ en } y = 4 \end{array}\right\}$ $\dfrac{c}{4} = 2 + \dfrac{3}{3+4}$

$c = 8 + \frac{12}{7} = 9\frac{5}{7}$

$$2 + \frac{3}{x+4} = \frac{9\frac{5}{7}}{y}$$

$$\frac{2x+11}{x+4} = \frac{68}{7y}$$

$$7y = \frac{68(x+4)}{2x+11}$$

$$y = \frac{68x+272}{14x+77}$$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{68x+272}{14x+77} = \lim_{x\to\infty} \frac{68 + \frac{272}{x}}{14 + \frac{77}{x}} = \frac{68+0}{14+0} = 4\tfrac{6}{7}$$

$$2 + \frac{3}{x+4} = \frac{9\frac{5}{7}}{y}$$

$$\frac{3}{x+4} = \frac{68}{7y} - 2$$

$$\frac{3}{x+4} = \frac{68-14y}{7y}$$

$$x + 4 = \frac{21y}{68-14y}$$

$$x = \frac{21y}{68-14y} - 4$$

$$\lim_{y\to\infty} x = \lim_{y\to\infty} \left(\frac{21y}{68-14y} - 4\right) = \lim_{y\to\infty} \left(\frac{21}{\frac{68}{y} - 14} - 4\right) = \frac{21}{0-14} - 4 = -5\tfrac{1}{2}$$

49 $y = 2 + 3^{0,2x-1}$

$3^{0,2x-1} = y - 2$

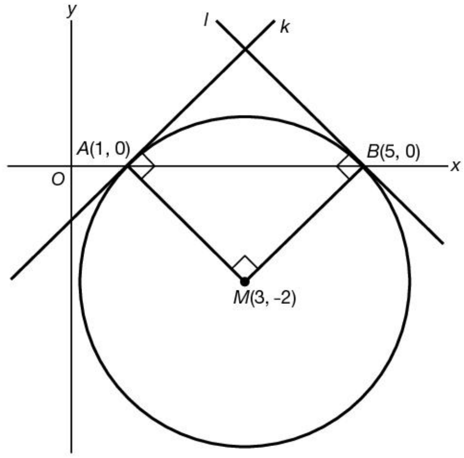
$0,2x - 1 = {}^3\log(y-2)$

$0,2x = 1 + {}^3\log(y-2)$

$x = 5 + 5 \cdot {}^3\log(y-2)$

50 $\left.\begin{array}{l} x^2 + y^2 - 6x + 4y + 5 = 0 \\ y = 0 \end{array}\right\} \begin{array}{l} x^2 - 6x + 5 = 0 \\ (x-1)(x-5) = 0 \\ x = 1 \vee x = 5 \end{array}$

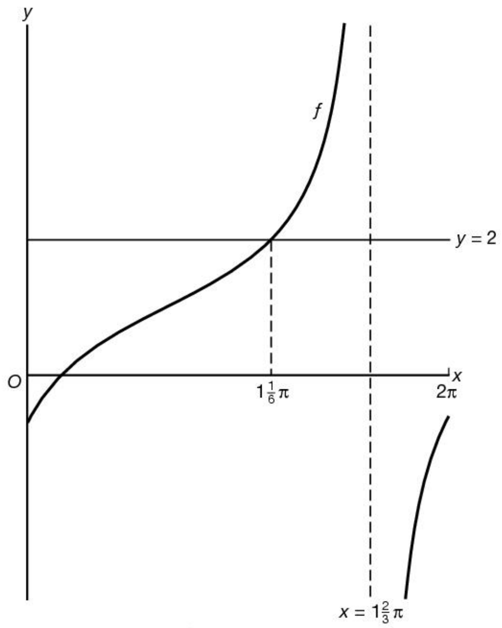
Dus $A(1, 0)$ en $B(5, 0)$.
$x^2 + y^2 - 6x + 4y + 5 = 0$
$x^2 - 6x + y^2 + 4y + 5 = 0$
$(x-3)^2 - 9 + (y+2)^2 - 4 + 5 = 0$
$(x-3)^2 + (y+2)^2 = 8$
Dus $M(3, {}^-2)$.



$\overrightarrow{MA} = \begin{pmatrix} {}^-2 \\ 2 \end{pmatrix}$ staat loodrecht op $\overrightarrow{MB} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.
Dus de gevraagde hoek is 90°.

51 $f\left(\frac{1}{2}\pi - p\right) = 2 + \left(\frac{1}{2}\pi - p - \frac{1}{2}\pi\right)\cos\left(\frac{1}{2}\pi - p\right) = 2 - p\cos\left(\frac{1}{2}\pi - p\right) = 2 - p\cos\left(p - \frac{1}{2}\pi\right)$
$f\left(\frac{1}{2}\pi + p\right) = 2 + \left(\frac{1}{2}\pi + p - \frac{1}{2}\pi\right)\cos\left(\frac{1}{2}\pi + p\right) = 2 + p\cos\left(\frac{1}{2}\pi + p\right) = 2 - p\cos\left(p - \frac{1}{2}\pi\right)$
Voor elke p is $f\left(\frac{1}{2}\pi - p\right) = f\left(\frac{1}{2}\pi + p\right)$. Dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{2}\pi$.

52 $1 + \tan\left(\frac{1}{2}x - \frac{1}{3}\pi\right) = 2$
$\tan\left(\frac{1}{2}x - \frac{1}{3}\pi\right) = 1$
$\frac{1}{2}x - \frac{1}{3}\pi = \frac{1}{4}\pi + k \cdot \pi$
$\frac{1}{2}x = \frac{7}{12}\pi + k \cdot \pi$
$x = 1\frac{1}{6}\pi + k \cdot 2\pi$
verticale asymptoot:
$\frac{1}{2}x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$
$\frac{1}{2}x = \frac{5}{6}\pi + k \cdot \pi$
$x = \frac{5}{3}\pi + k \cdot 2\pi$



$f(x) > 2$ geeft $1\frac{1}{6}\pi < x < 1\frac{2}{3}\pi$

53 $\begin{cases} x(t) = \frac{2}{3}t^3 - 2t^2 - 6t \\ y(t) = t^2 - 4 \end{cases}$ geeft $x'(t) = 2t^2 - 4t - 6$ en $y'(t) = 2t$

Raaklijn evenwijdig aan de x-as als $y'(t) = 0 \wedge x'(t) \neq 0$

$2t = 0 \wedge 2t^2 - 4t - 6 \neq 0$

$t = 0$

$t = 0$ geeft het punt $(0, -4)$.

Raaklijn evenwijdig aan de y-as als $x'(t) = 0 \wedge y'(t) \neq 0$

$2t^2 - 4t - 6 = 0 \wedge 2t \neq 0$

$t^2 - 2t - 3 = 0 \wedge t \neq 0$

$(t + 1)(t - 3) = 0 \wedge t \neq 0$

$t = -1 \vee t = 3$

$t = -1$ geeft het punt $\left(3\frac{1}{3}, -3\right)$ en $t = 3$ geeft het punt $(-18, 5)$.

54 $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ en $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ geeft $\cos(\angle(AB, AC)) = \dfrac{\left|\begin{pmatrix} -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix}\right|}{\left|\begin{pmatrix} -3 \\ 4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ 6 \end{pmatrix}\right|} = \dfrac{-6 + 24}{5 \cdot \sqrt{40}} = \dfrac{18}{5\sqrt{40}}$

$\angle(AB, AC) \approx 55{,}3°$

Bladzijde 235

55 $x^2 + y^2 + 4x - 6y + 8 = 0$

$x^2 + 4x + y^2 - 6y + 8 = 0$

$(x + 2)^2 - 4 + (y - 3)^2 - 9 + 8 = 0$

$(x + 2)^2 + (y - 3)^2 = 5$

Dus middelpunt $(-2, 3)$ en straal $\sqrt{5}$.

$d(A, c) = d(A, M) - r = \sqrt{(2 - -2)^2 + (11 - 3)^2} - \sqrt{5} = \sqrt{16 + 64} - \sqrt{5} = \sqrt{80} - \sqrt{5} = 4\sqrt{5} - \sqrt{5} = 3\sqrt{5}$

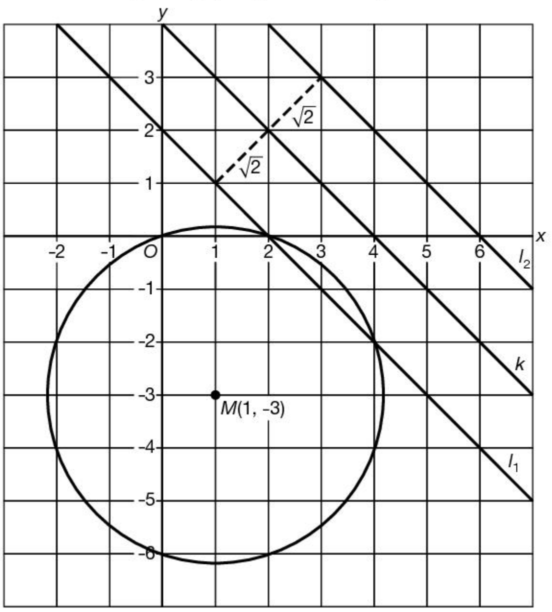
56 $x^2 + y^2 - 2x + 6y = 0$

$x^2 - 2x + y^2 + 6y = 0$

$(x - 1)^2 - 1 + (y + 3)^2 - 9 = 0$

$(x - 1)^2 + (y + 3)^2 = 10$

Dus middelpunt $(1, -3)$ en straal $\sqrt{10}$.



De enige lijn op afstand $\sqrt{2}$ van k die de cirkel snijdt is de lijn l_1: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

l_1 snijden met c geeft $(1 + \lambda)^2 + (1 - \lambda)^2 - 2(1 + \lambda) + 6(1 - \lambda) = 0$

$1 + 2\lambda + \lambda^2 + 1 - 2\lambda + \lambda^2 - 2 - 2\lambda + 6 - 6\lambda = 0$

$2\lambda^2 - 8\lambda + 6 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 1)(\lambda - 3) = 0$

$\lambda = 1 \vee \lambda = 3$

$\lambda = 1$ geeft $x = 2$ en $y = 0$ en $\lambda = 3$ geeft $x = 4$ en $y = -2$.

Dus de gevraagde punten zijn $(2, 0)$ en $(4, -2)$.

57 Substitutie van $x = \cos(t)$ en $y = 2\sin\left(2t - \frac{1}{2}\pi\right)$ in $y = 2 - 4x^2$ geeft $2\sin\left(2t - \frac{1}{2}\pi\right) = 2 - 4\cos^2(t)$

$$\sin\left(2t - \tfrac{1}{2}\pi\right) = 1 - 2\cos^2(t)$$
$$-\cos\left(2t - \tfrac{1}{2}\pi + \tfrac{1}{2}\pi\right) = -\cos(2t)$$
$$-\cos(2t) = -\cos(2t)$$

Dit klopt en omdat $x = \cos(t)$ is $-1 \leq x \leq 1$, hoort de parametervoorstelling dus bij de parabool.

58 $\overrightarrow{AB} = \begin{pmatrix} b-a \\ c \end{pmatrix}$

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AB}_L = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -c \\ b-a \end{pmatrix} = \begin{pmatrix} a-c \\ b-a \end{pmatrix}$

D op de y-as geeft $a - c = 0$ ofwel $c = a$.

$\overrightarrow{BM} = \overrightarrow{BC} + \overrightarrow{CM} = \overrightarrow{AB}_L - \frac{1}{2}\overrightarrow{AB} = \begin{pmatrix} -c \\ b-a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} b-a \\ c \end{pmatrix} = \begin{pmatrix} -c - \frac{1}{2}b + \frac{1}{2}a \\ b - a - \frac{1}{2}c \end{pmatrix}$

BM horizontaal geeft $b - a - \frac{1}{2}c = 0$.

Substitutie van $c = a$ in $b - a - \frac{1}{2}c = 0$ geeft $b - a - \frac{1}{2}a = 0$ ofwel $b = 1\frac{1}{2}a$.

$\left.\begin{matrix} \overrightarrow{BM} = \begin{pmatrix} -c - \frac{1}{2}b + \frac{1}{2}a \\ 0 \end{pmatrix} \\ c = a \text{ en } b = 1\frac{1}{2}a \end{matrix}\right\} \overrightarrow{BM} = \begin{pmatrix} -a - \frac{3}{4}a + \frac{1}{2}a \\ 0 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{4}a \\ 0 \end{pmatrix}$

$BM = 3$ geeft $\left|1\frac{1}{4}a\right| = 3$, dus $a = 2\frac{2}{5}$.

Dus $a = 2\frac{2}{5}$, $b = 3\frac{3}{5}$ en $c = 2\frac{2}{5}$.

59 k: $\dfrac{x}{2p} + \dfrac{y}{p} = 1$ ofwel k: $x + 2y = 2p$

Door (6, 1) geeft $6 + 2 = 2p$

$p = 4$

Dus voor $p = 4$ gaat de lijn door (6, 1).

60 Stel $AB = x$.

Dan is $BC = \frac{1}{2}x$, $AC = \frac{1}{2}x\sqrt{3}$, $CE = \frac{1}{2}x\sqrt{3}$ en $CD = \dfrac{\frac{1}{2}x\sqrt{3}}{\sqrt{2}} = \frac{1}{4}x\sqrt{6}$

De omtrek is $2 \cdot x + 2 \cdot \frac{1}{2}x + 4 \cdot \frac{1}{4}x\sqrt{6} = 3x + x\sqrt{6}$.

Omtrek = 12 geeft $3x + x\sqrt{6} = 12$

$$x(3 + \sqrt{6}) = 12$$
$$x = \frac{12}{3+\sqrt{6}} \cdot \frac{3-\sqrt{6}}{3-\sqrt{6}} = \frac{12\left(3-\sqrt{6}\right)}{9-6} = 4\left(3-\sqrt{6}\right) = 12 - 4\sqrt{6}$$

Dus de lengte van AB is $12 - 4\sqrt{6}$.

61 $g_{103\text{ jaar}} = 0{,}5$

$g_{\text{jaar}} = 0{,}5^{\frac{1}{103}} = 0{,}99329...$

Dus per jaar is de afname 0,67%.

Bladzijde 236

62 $\left.\begin{matrix} \text{Stel } k\text{: } y = ax + b \\ \text{door } A(-1, 1) \end{matrix}\right\} \begin{matrix} -a + b = 1 \\ b = a + 1 \end{matrix}$

Dus k: $y = ax + a + 1$ ofwel k: $ax - y + a + 1 = 0$

Raken, dus $d(M, k) = r$

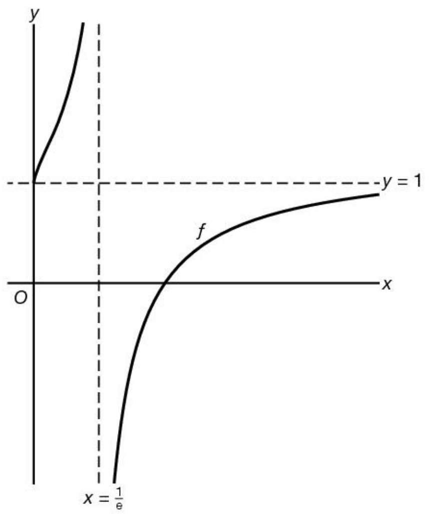
$$\frac{|3a + 1 + a + 1|}{\sqrt{a^2 + 1}} = \sqrt{10}$$
$$|4a + 2| = \sqrt{10a^2 + 10}$$
$$16a^2 + 16a + 4 = 10a^2 + 10$$
$$6a^2 + 16a - 6 = 0$$

$3a^2 + 8a - 3 = 0$
$D = 8^2 - 4 \cdot 3 \cdot -3 = 64 + 36 = 100$
$a = \frac{-8 + 10}{6} \vee a = \frac{-8 - 10}{6}$
$a = \frac{1}{3} \vee a = -3$
$a = \frac{1}{3}$ geeft k_1: $y = \frac{1}{3}x + 1\frac{1}{3}$ en $a = -3$ geeft k_2: $y = -3x - 2$.

63 verticale asymptoot:
$\ln(x) + 1 = 0$
$\ln(x) = -1$
$x = e^{-1} = \frac{1}{e}$
Dus de verticale asymptoot is de lijn $x = \frac{1}{e}$.
horizontale asymptoot:

$$\lim_{x \to \infty} \frac{\ln(2x) - 1}{\ln(x) + 1} = \lim_{\ln(x) \to \infty} \frac{\ln(2) + \ln(x) - 1}{\ln(x) + 1} = \lim_{\ln(x) \to \infty} \frac{\frac{\ln(2)}{\ln(x)} + 1 - \frac{1}{\ln(x)}}{1 + \frac{1}{\ln(x)}} = \frac{0 + 1 - 0}{1 + 0} = 1$$

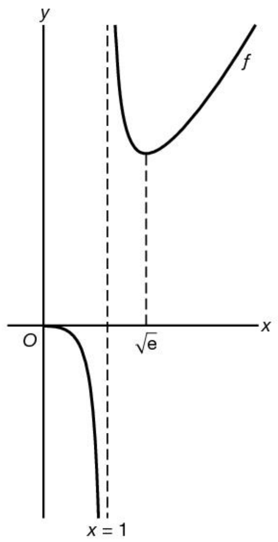
Dus de horizontale asymptoot is de lijn $y = 1$.



64 $d(A, B) = \sqrt{(2 - 2p)^2 + (p + 1 - 3)^2} = \sqrt{4 - 8p + 4p^2 + (p - 2)^2} = \sqrt{4 - 8p + 4p^2 + p^2 - 4p + 4}$
$= \sqrt{5p^2 - 12p + 8}$
Dit is minimaal als $5p^2 - 12p + 8$ minimaal is. Dus als $p = \frac{-12}{-2 \cdot 5} = 1\frac{1}{5}$.
De minimale afstand is $\sqrt{5 \cdot \left(\frac{6}{5}\right)^2 - 12 \cdot \frac{6}{5} + 8} = \sqrt{\frac{4}{5}} = \frac{4}{5}\sqrt{5}$.

65 $f(x) = \frac{x^3 + 5x^2 + 6x + 4 + \sqrt{x}}{x^2} = x + 5 + \frac{6}{x} + 4x^{-2} + x^{-1\frac{1}{2}}$ geeft
$F(x) = \frac{1}{2}x^2 + 5x + 6\ln|x| - 4x^{-1} - 2x^{-\frac{1}{2}} + c = \frac{1}{2}x^2 + 5x + 6\ln|x| - \frac{4}{x} - \frac{2}{\sqrt{x}} + c$
$= \frac{\frac{1}{2}x^3 + 5x^2 + 6x\ln|x| - 4 - 2\sqrt{x}}{x} + c$

66 $f(x) = \frac{x^2}{\ln(x)}$ geeft $f'(x) = \frac{\ln(x) \cdot 2x - x^2 \cdot \frac{1}{x}}{\ln^2(x)} = \frac{2x\ln(x) - x}{\ln^2(x)}$
$f'(x) = 0$ geeft $2x\ln(x) - x = 0$
$x(2\ln(x) - 1) = 0$
$x = 0 \quad \vee \quad \ln(x) = \frac{1}{2}$
vold. niet $\quad x = e^{\frac{1}{2}} = \sqrt{e}$



Dus min. is $f\left(\sqrt{e}\right) = \dfrac{e}{\ln\left(e^{\frac{1}{2}}\right)} = \dfrac{e}{\frac{1}{2}} = 2e$.

67 $\triangle ABC \backsim \triangle AED$ geeft $\dfrac{AB}{AE} = \dfrac{AC}{AD} = \dfrac{BC}{DE}$

$\dfrac{47}{26} = \dfrac{AC}{18} = \dfrac{21}{DE}$

$\dfrac{47}{26} = \dfrac{AC}{18}$ geeft $26 \cdot AC = 47 \cdot 18$

$AC = 32{,}53...$

$CE = AC - AE = 32{,}53... - 26 \approx 6{,}5$

$\dfrac{47}{26} = \dfrac{21}{DE}$ geeft $47 \cdot DE = 26 \cdot 21$

$DE \approx 11{,}6$

68 $f_a(x) = \dfrac{x^2 + 4x - 5}{2x + a} = \dfrac{(x-1)(x+5)}{2x+a} = \dfrac{\frac{1}{4}(2x-2)(2x+10)}{2x+a}$

De grafiek van f_a is een lijn met een perforatie als $a = -2$ of als $a = 10$.

69 Substitutie van $x = 3t + p$ en $y = t + 1$ in $x^2 + y^2 + 4x - 8y + 10 = 0$ geeft

$(3t+p)^2 + (t+1)^2 + 4(3t+p) - 8(t+1) + 10 = 0$

$9t^2 + 6pt + p^2 + t^2 + 2t + 1 + 12t + 4p - 8t - 8 + 10 = 0$

$10p^2 + (6p + 6)t + p^2 + 4p + 3 = 0$

Raken, dus $D = 0$

$(6p+6)^2 - 4 \cdot 10 \cdot (p^2 + 4p + 3) = 0$

$36p^2 + 72p + 36 - 40p^2 - 160p - 120 = 0$

$-4p^2 - 88p - 84 = 0$

$p^2 + 22p + 21 = 0$

$(p+1)(p+21) = 0$

$p = -1 \vee p = -21$

70 Voer in $y_1 = 2\cos(2x) + 3\sin(2x)$.

De optie maximum geeft $x = 0{,}491...$ en $y = 3{,}605...$

De optie minimum geeft $x = 2{,}062...$ en $y = -3{,}605...$

$b = 3{,}605...$

$u = b\cos(2t - d) = b\cos\left(2\left(t - \frac{1}{2}d\right)\right)$

$\frac{1}{2}d = 0{,}491...$

$d = 0{,}982...$

Dus $u = 3{,}61\cos(2t - 0{,}98)$.

71

	$u_1 = 3\sin(3t)$	$u_2 = 5\sin(5t)$	$u_3 = 7\sin(7t)$
in $[0, 2\pi]$	3 periodes	5 periodes	7 periodes

Dus de periode is 2π.

72 $f(x) = \ln(2x^3) + e^{2x+4} = \ln(2) + \ln(x^3) + e^{2x+4} = \ln(2) + 3\ln(x) + e^{2x+4}$
$F(x) = x\ln(2) + 3(x\ln(x) - x) + \frac{1}{2}e^{2x+4} + c = x\ln(2) + 3x\ln(x) - 3x + \frac{1}{2}e^{2x+4} + c$

73 $f(x) = \dfrac{1}{2\sin(x) - 1}$ geeft $f'(x) = \dfrac{(2\sin(x) - 1)\cdot 0 - 1\cdot 2\cos(x)}{(2\sin(x) - 1)^2} = \dfrac{-2\cos(x)}{(2\sin(x) - 1)^2}$

$f\left(\frac{1}{2}\pi\right) = \dfrac{1}{2\sin\left(\frac{1}{2}\pi\right) - 1} = \dfrac{1}{2 - 1} = 1$

Stel k: $y = ax + b$ met $a = f'\left(\frac{1}{2}\pi\right) = \dfrac{-2\cos\left(\frac{1}{2}\pi\right)}{\left(2\sin\left(\frac{1}{2}\pi\right) - 1\right)^2} = 0.$

Dus k: $y = 1$.
$2\sin(x) - 1 = 0$ geeft $\sin(x) = \frac{1}{2}$
$x = \frac{1}{6}\pi + k\cdot 2\pi \vee x = \frac{5}{6}\pi + k\cdot 2\pi$
x op $[0, 2\pi]$ geeft de verticale asymptoten de lijnen $x = \frac{1}{6}\pi$ en $x = \frac{5}{6}\pi$.

Dus $B\left(\frac{1}{6}\pi, 1\right)$ en $C\left(\frac{5}{6}\pi, 1\right)$.

Bladzijde 237

74 $f_p(x) = \dfrac{2x}{x^2 + p}$ geeft $f_p'(x) = \dfrac{(x^2 + p)\cdot 2 - 2x\cdot 2x}{(x^2 + p)^2} = \dfrac{2x^2 + 2p - 4x^2}{(x^2 + p)^2} = \dfrac{-2x^2 + 2p}{(x^2 + p)^2}$
$f_p'(x) = 0$ geeft $-2x^2 + 2p = 0$
$x^2 = p$
$x = \sqrt{p} \vee x = -\sqrt{p}$

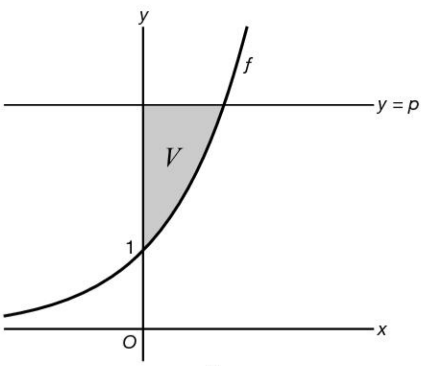
$f\left(\sqrt{p}\right) = \dfrac{2\sqrt{p}}{(\sqrt{p})^2 + p} = \dfrac{2\sqrt{p}}{2p} = \dfrac{1}{\sqrt{p}}$ en $f\left(-\sqrt{p}\right) = \dfrac{-2\sqrt{p}}{(-\sqrt{p})^2 + p} = \dfrac{-2\sqrt{p}}{2p} = -\dfrac{1}{\sqrt{p}}$

Dus $A\left(\sqrt{p}, \dfrac{1}{\sqrt{p}}\right)$ en $B\left(-\sqrt{p}, -\dfrac{1}{\sqrt{p}}\right)$.

$AB = \sqrt{(2\sqrt{p})^2 + \left(\dfrac{2}{\sqrt{p}}\right)^2} = \sqrt{4p + \dfrac{4}{p}}$

$AB = 4$ geeft $4p + \dfrac{4}{p} = 16$
$4p^2 + 4 = 16p$
$4p^2 - 16p + 4 = 0$
$p^2 - 4p + 1 = 0$
$(p - 2)^2 - 4 + 1 = 0$
$(p - 2)^2 = 3$
$p - 2 = \sqrt{3} \vee p - 2 = -\sqrt{3}$
$p = 2 + \sqrt{3} \vee p = 2 - \sqrt{3}$

75



$O(V) = \displaystyle\int_1^p x\,dy = \int_1^p \ln(y)\,dy = \Big[y\ln(y) - y\Big]_1^p = p\ln(p) - p - (\ln(1) - 1) = p\ln(p) - p + 1$
$O(V) = 1$ geeft $p\ln(p) - p + 1 = 1$
$p\ln(p) - p = 0$
$p(\ln(p) - 1) = 0$
$p = 0 \quad \vee \quad \ln(p) = 1$
vold. niet $\quad p = e$
Dus voor $p = e$.

76 $f(x) = \cos^2(4x) + \sin^2(6x) = \frac{1}{2} + \frac{1}{2}\cos(8x) + \frac{1}{2} - \frac{1}{2}\cos(12x) = 1 + \frac{1}{2}\cos(8x) - \frac{1}{2}\cos(12x)$ geeft
$F(x) = x + \frac{1}{16}\sin(8x) - \frac{1}{24}\sin(12x) + c$

77 De middelpunten liggen op de bissectrices van l en m.

$$\frac{|x + 2y - 1|}{\sqrt{5}} = \frac{|2x - y - 2|}{\sqrt{5}}$$

$|x + 2y - 1| = |2x - y - 2|$

$x + 2y - 1 = 2x - y - 2 \vee x + 2y - 1 = -2x + y + 2$

$-x + 3y = -1 \vee 3x + y = 3$

$$\begin{cases} -x + 3y = -1 \\ \;\; x + 7y = 21 \end{cases} +$$

$10y = 20$

$\left.\begin{array}{l} y = 2 \\ x - 3y = 1 \end{array}\right\} \begin{array}{l} x - 6 = 1 \\ x = 7 \end{array}$

Dus $M_1(7, 2)$.

$$\begin{cases} 3x + y = 3 \\ x + 7y = 21 \end{cases} \begin{vmatrix} 1 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} 3x + \;\;\; y = 3 \\ 3x + 21y = 63 \end{cases} -$$

$-20y = -60$

$\left.\begin{array}{l} y = 3 \\ x + 7y = 21 \end{array}\right\} \begin{array}{l} x + 21 = 21 \\ x = 0 \end{array}$

Dus $M_2(0, 3)$.

$$r_1 = d(M_1, l) = \frac{|7 + 2 \cdot 2 - 1|}{\sqrt{5}} = \frac{10}{\sqrt{5}} = 2\sqrt{5} \text{ en } r_2 = d(M_2, l) = \frac{|0 + 2 \cdot 3 - 1|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

Dus c_1: $(x - 7)^2 + (y - 2)^2 = 20$ en c_2: $x^2 + (y - 3)^2 = 5$.

78 $f(x) = g(x)$ geeft ${}^3\log(x + 2) = 1 + {}^3\log(2x - 11)$

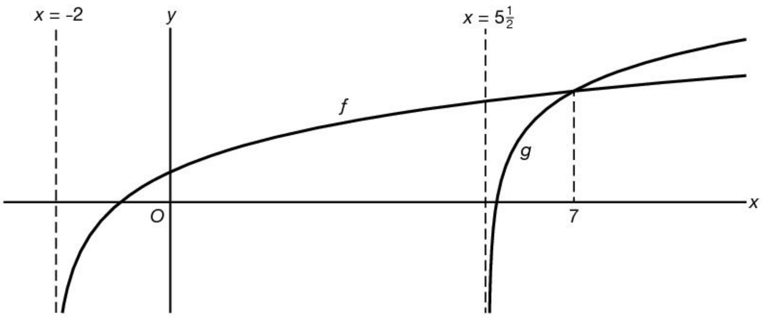
${}^3\log(x + 2) = {}^3\log(3) + {}^3\log(2x - 11)$

${}^3\log(x + 2) = {}^3\log(6x - 33)$

$x + 2 = 6x - 33$

$-5x = -35$

$x = 7$



$f(x) \geq g(x)$ geeft $5\frac{1}{2} < x \leq 7$

79 $x'(t) = \cos\left(t - \frac{1}{4}\pi\right)$ en $y'(t) = 2\cos(2t)$

$y = 0$ geeft $\sin(2t) = 0$

$2t = k \cdot \pi$

$t = k \cdot \frac{1}{2}\pi$

$t = \frac{1}{2}\pi$ geeft $x = \sin\left(\frac{1}{4}\pi\right) = \frac{1}{2}\sqrt{2}$, dus $t = \frac{1}{2}\pi$ voldoet.

$$\left[\frac{dy}{dt}\right]_{t=\frac{1}{2}\pi} = \frac{2\cos(\pi)}{\cos\left(\frac{1}{4}\pi\right)} = \frac{-2}{\frac{1}{2}\sqrt{2}} = -2\sqrt{2}$$

$\tan(\alpha) = -2\sqrt{2}$ geeft $\alpha = -70{,}52...°$

Dus de gevraagde hoek is 71°.

80 $f(x) = g(x)$ geeft $\sqrt{x+4} = 2 + \sqrt{x-4}$

kwadrateren geeft

$x + 4 = 4 + 4\sqrt{x-4} + x - 4$

$4 = 4\sqrt{x-4}$

$\sqrt{x-4} = 1$

$x - 4 = 1$

$x = 5$

vold.

$$f'(x) = \frac{1}{2\sqrt{x+4}} \text{ geeft } f'(5) = \frac{1}{2 \cdot \sqrt{9}} = \tfrac{1}{6}$$

$$g'(x) = \frac{1}{2\sqrt{x-4}} \text{ geeft } g'(5) = \frac{1}{2 \cdot \sqrt{1}} = \tfrac{1}{2}$$

$\tan(\alpha) = \frac{1}{6}$ geeft $\alpha = 9{,}462...°$

$\tan(\beta) = \frac{1}{2}$ geeft $\beta = 26{,}565...°$

$\beta - \alpha = 17{,}103...$

Dus de gevraagde hoek is 17,1°.

81 $2\cos^2(x) + \sin(x) - 2 = 0$

$2(1 - \sin^2(x)) + \sin(x) - 2 = 0$

$2 - 2\sin^2(x) + \sin(x) - 2 = 0$

$-2\sin^2(x) + \sin(x) = 0$

$\sin(x)(-2\sin(x) + 1) = 0$

$\sin(x) = 0 \vee \sin(x) = \frac{1}{2}$

$x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi$

x op $[0, 2\pi]$ geeft $x = 0 \vee x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = \pi \vee x = 2\pi$

82 $x'(t) = -t^2 + 2t + 3$ en $y'(x) = 2t - 4$

Raaklijn verticaal als $x'(t) = 0$

$t^2 - 2t - 3 = 0$

$(t + 1)(t - 3) = 0$

$t = -1 \vee t = 3$

$t = -1$ geeft het punt $\left(-1\frac{2}{3}, 9\right)$ en $t = 3$ geeft het punt $(9, 1)$.

$y = 9$ geeft $t^2 - 4t + 4 = 9$

$t^2 - 4t - 5 = 0$

$(t + 1)(t - 5) = 0$

$t = -1 \vee t = 5$

$$t = 5 \text{ geeft } \left[\frac{dy}{dx}\right]_{t=5} = \frac{6}{-25 + 10 + 3} = \frac{6}{-12} = -\tfrac{1}{2}$$

$\tan(\alpha) = -\frac{1}{2}$ geeft $\alpha = -26{,}565...°$

De gevraagde hoek is $90 - 26{,}565...° \approx 63°$.

83 De grafiek van f_a heeft een perforatie als $\ln(ax^2) + 2 = 0 \wedge \ln(x) + 4 = 0$

$\ln(ax^2) = -2 \wedge \ln(x) = -4$

$ax^2 = e^{-2} \wedge x = e^{-4}$

Substitutie van $x = e^{-4}$ in $ax^2 = e^{-2}$ geeft $a(e^{-4})^2 = e^{-2}$

$ae^{-8} = e^{-2}$

$$a = \frac{e^{-2}}{e^{-8}} = e^6$$

84 $2\sin(2x) = -\sqrt{3}$

$\sin(2x) = -\frac{1}{2}\sqrt{3}$

$2x = -\frac{1}{3}\pi + k \cdot 2\pi \vee 2x = 1\frac{1}{3}\pi + k \cdot 2\pi$

$x = -\frac{1}{6}\pi + k \cdot \pi \vee x = \frac{2}{3}\pi + k \cdot \pi$

x op $[0, 2\pi]$ geeft $x = \frac{2}{3}\pi \vee x = \frac{5}{6}\pi \vee x = 1\frac{2}{3}\pi \vee x = 1\frac{5}{6}\pi$

Bladzijde 238

85 De grafiek van f_a heeft een perforatie als $\sin^2(x) - a\cos(x) = 0 \wedge \sin(x) + \frac{1}{2} = 0$.

$\sin(x) + \frac{1}{2} = 0$ geeft $\sin(x) = -\frac{1}{2}$

$x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

x op $[0, 2\pi]$ geeft $x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$

$x = 1\frac{1}{6}\pi$ geeft $\sin^2\left(1\frac{1}{6}\pi\right) - a\cos\left(1\frac{1}{6}\pi\right) = 0$

$$\left(-\tfrac{1}{2}\right)^2 - a \cdot -\tfrac{1}{2}\sqrt{3} = 0$$
$$\tfrac{1}{4} + \tfrac{1}{2}a\sqrt{3} = 0$$
$$\tfrac{1}{2}a\sqrt{3} = -\tfrac{1}{4}$$
$$a = \frac{-\frac{1}{4}}{\frac{1}{2}\sqrt{3}} \cdot \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{-\frac{1}{2}\sqrt{3}}{3} = -\tfrac{1}{6}\sqrt{3}$$

$x = 1\frac{5}{6}\pi$ geeft $\sin^2\left(1\frac{5}{6}\pi\right) - a\cos\left(1\frac{5}{6}\pi\right) = 0$

$$\left(-\tfrac{1}{2}\right)^2 - a \cdot \tfrac{1}{2}\sqrt{3} = 0$$
$$\tfrac{1}{4} - \tfrac{1}{2}a\sqrt{3} = 0$$
$$\tfrac{1}{2}a\sqrt{3} = \tfrac{1}{4}$$
$$a = \frac{\frac{1}{4}}{\frac{1}{2}\sqrt{3}} \cdot \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{\frac{1}{2}\sqrt{3}}{3} = \tfrac{1}{6}\sqrt{3}$$

Dus de grafiek van f_a heeft een perforatie als $a = -\frac{1}{6}\sqrt{3} \vee a = \frac{1}{6}\sqrt{3}$.

86 $\text{rc}_k = \dfrac{-25 - -43}{3 - -3} = \dfrac{18}{6} = 3$

k: $y = 3x + b$
door $A(-3, -43)$ } $3 \cdot -3 + b = -43$
$-9 + b = -43$
$b = -34$

Dus k: $y = 3x - 34$.

k snijden met l geeft $-4 + 2\lambda = 3(3 + 3\lambda) - 34$
$-4 + 2\lambda = 9 + 9\lambda - 34$
$-7\lambda = -21$
$\lambda = 3$

$\lambda = 3$ geeft het snijpunt $(12, 2)$.

87 $0 \le t \le 5$
$a(t) = 0{,}012t^2$
$v(t) = 0{,}004t^3 + v(0)$
$v(0) = 0$ } $v(t) = 0{,}004t^3$
$s(t) = 0{,}001t^4$
$s(0) = 0$ } $s(t) = 0{,}001t^4$
$v(5) = 0{,}004 \cdot 5^3 = 0{,}5$
$s(5) = 0{,}001 \cdot 5^4 = 0{,}625$

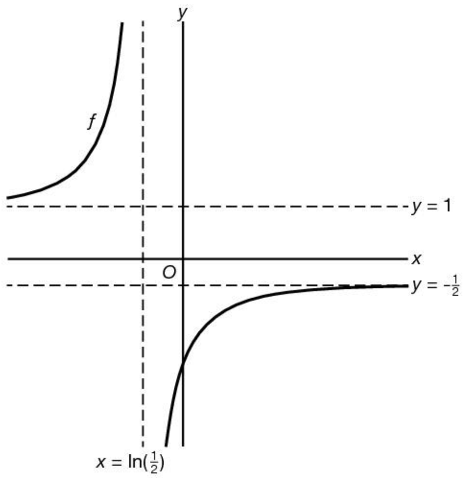
$t \ge 5$
$v(t) = 0{,}5$
$s(t) = 0{,}5(t - 5) + 0{,}625 = 0{,}5t - 1{,}875$
$s(10) = 5 - 1{,}875 = 3{,}125$
Dus er wordt in de eerste tien seconden 31,25 meter afgelegd.

88 verticale asymptoot:
$1 - 2\,e^x = 0$
$e^x = \frac{1}{2}$
$x = \ln\left(\frac{1}{2}\right)$
Dus de verticale asymptoot is de lijn $x = \ln\left(\frac{1}{2}\right)$.
horizontale asymptoot:

$$\lim_{x\to\infty} \frac{e^x + 1}{1 - 2e^x} = \lim_{x\to\infty} \frac{1 + \frac{1}{e^x}}{\frac{1}{e^x} - 2} = \frac{1 + 0}{0 - 2} = -\tfrac{1}{2}$$

$$\lim_{x\to-\infty} \frac{e^x + 1}{1 - 2e^x} = \frac{0 + 1}{1 - 2 \cdot 0} = \frac{1}{1} = 1$$

Dus de horizontale asymptoten zijn de lijnen $y = -\frac{1}{2}$ en $y = 1$.



89 $\vec{r}_k = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ geeft $\vec{n}_k = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$$\cos(\angle(k, l)) = |\cos(\angle(\vec{n}_k, \vec{n}_l))| = \frac{\left|\begin{pmatrix} 2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ 5 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right|} = \frac{|4 + 15|}{\sqrt{4 + 25} \cdot \sqrt{4 + 9}} = \frac{19}{\sqrt{29} \cdot \sqrt{13}}$$

Dus $\angle(k, l) \approx 11{,}9°$.

90 $y = \sqrt{x - 1}$
$y^2 = x - 1$
$x = y^2 + 1$

$$I(L) = \pi \int_0^q x^2 \, dy = \pi \int_0^q (y^2 + 1)^2 \, dy = \pi \int_0^q (y^4 + 2y^2 + 1) \, dy$$

$$= \pi\left[\tfrac{1}{5}y^5 + \tfrac{2}{3}y^3 + y\right]_0^q = \pi\left(\tfrac{1}{5}q^5 + \tfrac{2}{3}q^3 + q - 0\right) = \pi\left(\tfrac{1}{5}q^5 + \tfrac{2}{3}q^3 + q\right)$$

$q = \sqrt{p - 1}$ geeft $I(L) = \pi\left(\tfrac{1}{5}\left(\sqrt{p - 1}\right)^5 + \tfrac{2}{3}\left(\sqrt{p - 1}\right)^3 + \sqrt{p - 1}\right)$

$$I(M) = \pi \int_1^p y^2 \, dx = \pi \int_1^p \left(\sqrt{x - 1}\right)^2 dx = \pi \int_1^p (x - 1) \, dx$$

$$= \pi\left[\tfrac{1}{2}x^2 - x\right]_1^p = \pi\left(\tfrac{1}{2}p^2 - p - \left(\tfrac{1}{2} - 1\right)\right) = \pi\left(\tfrac{1}{2}p^2 - p + \tfrac{1}{2}\right)$$

$I(L) = 2 \cdot I(M)$ geeft $\pi\left(\tfrac{1}{5}\left(\sqrt{p - 1}\right)^5 + \tfrac{2}{3}\left(\sqrt{p - 1}\right)^3 + \sqrt{p - 1}\right) = 2 \cdot \pi\left(\tfrac{1}{2}p^2 - p + \tfrac{1}{2}\right)$

Voer in $y_1 = \pi\left(\tfrac{1}{5}\left(\sqrt{x - 1}\right)^5 + \tfrac{2}{3}\left(\sqrt{x - 1}\right)^3 + \sqrt{x - 1}\right)$ en $y_2 = 2\pi\left(\tfrac{1}{2}x^2 - x + \tfrac{1}{2}\right)$
Intersect geeft $x = 3{,}448...$
Dus $p \approx 3{,}45$.

91 $e^{2x} - 6e^{x+1} + 5e^2 = 0$
$e^{2x} - 6e \cdot e^x + 5e^2 = 0$
$(e^x - e)(e^x - 5e) = 0$
$e^x = e \vee e^x = 5e$
$x = 1 \vee x = \ln(5e)$

92 scheve asymptoot:

$$\frac{x^3 + 3x^2 - 5x + 8}{x^2 - 1} = \frac{x(x^2 - 1) + x + 3x^2 - 5x + 8}{x^2 - 1} = x + \frac{3x^2 - 4x + 8}{x^2 - 1} =$$

$$x + \frac{3(x^2 - 1) + 3 - 4x + 8}{x^2 - 1} = x + 3 + \frac{-4x + 11}{x^2 - 1}$$

$$\lim_{x \to \infty} \frac{-4x + 11}{x^2 - 1} = \lim_{x \to \infty} \frac{-\frac{4}{x} + \frac{11}{x^2}}{1 - \frac{1}{x^2}} = \frac{-0 + 0}{1 - 0} = 0 \text{ en } \lim_{x \to -\infty} \frac{-4x + 11}{x^2 - 1} = 0$$

Dus de scheve asymptoot is de lijn $y = x + 3$.
verticale asymptoot:
$x^2 - 1 = 0$ geeft $x = -1 \vee x = 1$
Dus de verticale asymptoten zijn de lijnen $x = -1$ en $x = 1$.

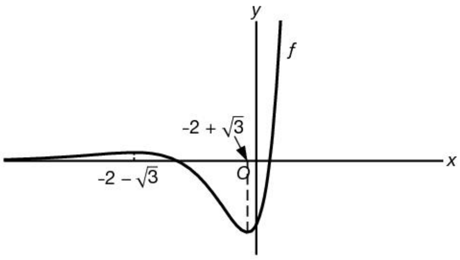
93 $f'(x) = (2x + 2) \cdot e^x + (x^2 + 2x - 1) \cdot e^x = (x^2 + 4x + 1) \cdot e^x$

$f'(x) = 0$ geeft $x^2 + 4x + 1 = 0$

$$D = 4^2 - 4 \cdot 1 \cdot 1 = 12$$

$$x = \frac{-4 - 2\sqrt{3}}{2} \vee x = \frac{-4 + 2\sqrt{3}}{2}$$

$$x = -2 - \sqrt{3} \vee x = -2 + \sqrt{3}$$



max. is $f(-2 - \sqrt{3}) = ((-2 - \sqrt{3})^2 + 2(-2 - \sqrt{3}) - 1)e^{-2 - \sqrt{3}} = (4 + 4\sqrt{3} + 3 - 4 - 2\sqrt{3} - 1)e^{-2 - \sqrt{3}}$

$= (2 + 2\sqrt{3})e^{-2 - \sqrt{3}}$

min. is $f(-2 + \sqrt{3}) = ((-2 + \sqrt{3})^2 + 2(-2 + \sqrt{3}) - 1)e^{-2 + \sqrt{3}} = (4 - 4\sqrt{3} + 3 - 4 + 2\sqrt{3} - 1)e^{-2 + \sqrt{3}}$

$= (2 - 2\sqrt{3})e^{-2 + \sqrt{3}}$

Dus de vergelijking $f(x) = p$ heeft precies twee oplossingen voor

$(2 - 2\sqrt{3})e^{-2 + \sqrt{3}} < p \leq 0 \vee p = (2 + 2\sqrt{3})e^{-2 - \sqrt{3}}$.

Bladzijde 239

94 $O(V) = \int_0^p x^2 \cdot \sqrt{x}\, dx = \int_0^p x^{2\frac{1}{2}} dx = \left[\frac{2}{7}x^{3\frac{1}{2}}\right]_0^p = \frac{2}{7}p^{3\frac{1}{2}} - 0 = \frac{2}{7}p^{3\frac{1}{2}}$

$O(V) = 10$ geeft $\frac{2}{7}p^{3\frac{1}{2}} = 10$

$$p^{3\frac{1}{2}} = 35$$

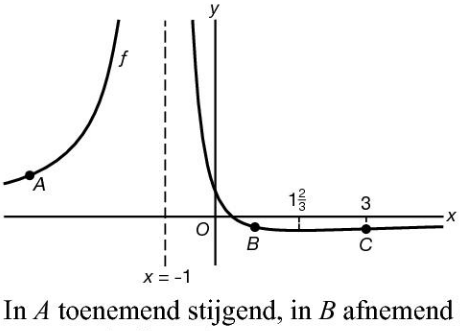
$$p = 35^{\frac{2}{7}} = \sqrt[7]{35^2} = \sqrt[7]{1225}$$

95 $f(x) = \dfrac{1 - 3x}{(x + 1)^2}$

$$f'(x) = \frac{(x + 1)^2 \cdot -3 - (1 - 3x) \cdot 2(x + 1)}{(x + 1)^4} = \frac{-3(x + 1) - 2(1 - 3x)}{(x + 1)^3} = \frac{-3x - 3 - 2 + 6x}{(x + 1)^3} = \frac{3x - 5}{(x + 1)^3}$$

$$f''(x) = \frac{(x + 1)^3 \cdot 3 - (3x - 5) \cdot 3(x + 1)^2}{(x + 1)^6} = \frac{3(x + 1) - 3(3x - 5)}{(x + 1)^4} = \frac{3x + 3 - 9x + 15}{(x + 1)^4} = \frac{-6x + 18}{(x + 1)^4}$$

$f'(x) = 0$ geeft $3x - 5 = 0$ ofwel $3x = 5$ ofwel $x = 1\frac{2}{3}$ en $f''(x) = 0$ geeft $-6x + 18 = 0$ ofwel $6x = 18$, dus $x = 3$.



In A toenemend stijgend, in B afnemend dalend en in C overgaand van toenemend stijgend naar afnemend stijgend.

96 $y = \sin(x)$

↓ verm. y-as, 2

$y = \sin(\frac{1}{2}x)$

↓ translatie $(-2, \frac{1}{4}\pi)$

$y = \sin(\frac{1}{2}(x + 2)) + \frac{1}{4}\pi$

Dus $f(x) = \sin(\frac{1}{2}x + 1) + \frac{1}{4}\pi$.

97 $\vec{r}(t) = \begin{pmatrix} -\frac{1}{3}t^3 + t^2 + 3t \\ t^2 - 2t + 1 \end{pmatrix}$

$\vec{v}(t) = \begin{pmatrix} -t^2 + 2t + 3 \\ 2t - 2 \end{pmatrix}$

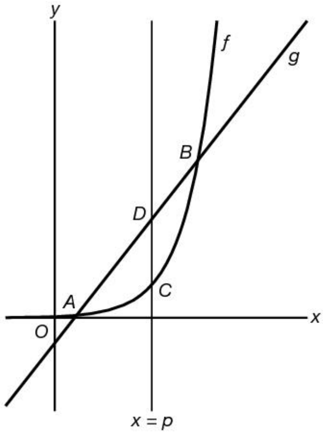
$\vec{a}(t) = \begin{pmatrix} -2t + 2 \\ 2 \end{pmatrix}$

$y = 0$ geeft $t^2 - 2t + 1 = 0$
$(t-1)^2 = 0$
$t = 1$

$v(1) = \sqrt{(-1+2+3)^2 + (2-2)^2} = \sqrt{16+0} = 4$

$$a_b(1) = \frac{\vec{v}(1) \cdot \vec{a}(1)}{|\vec{v}(1)|} = \frac{\begin{pmatrix} 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix}}{4} = 0$$

98



$L = g(p) - f(p) = 3p - 1 - e^{2p-3}$

$\frac{dL}{dp} = 3 - 2e^{2p-3}$

$\frac{dL}{dp} = 0$ geeft $2e^{2p-3} = 3$

$e^{2p-3} = 1\frac{1}{2}$

$2p - 3 = \ln(1\frac{1}{2})$

$2p = 3 + \ln(1\frac{1}{2})$

$p = 1\frac{1}{2} + \frac{1}{2}\ln(1\frac{1}{2})$

Dus voor $p = 1\frac{1}{2} + \frac{1}{2}\ln(1\frac{1}{2})$ is de lengte van CD maximaal.

De lengte is dan $3(1\frac{1}{2} + \frac{1}{2}\ln(1\frac{1}{2})) - 1 - 1\frac{1}{2} = 4\frac{1}{2} + 1\frac{1}{2}\ln(1\frac{1}{2}) - 2\frac{1}{2} = 2 + 1\frac{1}{2}\ln(1\frac{1}{2})$.

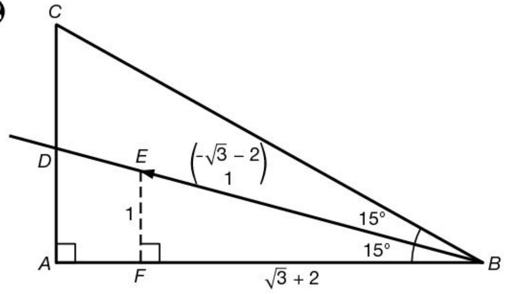
99 $y = {}^5\log(x)$
↓ translatie (3, 0)
$y = {}^5\log(x-3)$
↓ verm. y-as, $\frac{1}{2}$
$y = {}^5\log(2x-3)$
↓ translatie (0, −2)
$f(x) = -2 + {}^5\log(2x-3)$

100 $-\cos(2x + \frac{1}{3}\pi) = \cos(2x + 1\frac{1}{3}\pi) = \sin(2x + 1\frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(2x + 1\frac{5}{6}\pi)$

101 $f(x) = \frac{4x-3}{2x+1} = \frac{2(2x+1) - 2 - 3}{2x+1} = 2 - \frac{5}{2x+1}$

$F(x) = 2x - 5 \cdot \frac{1}{2}\ln|2x+1| + c = 2x - 2\frac{1}{2}\ln|2x+1| + c$

102



$\vec{r}_{BC} = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix}$ en $\vec{r}_{BA} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$, beide met lengte 2.

Dus $\vec{r}_{\text{bissectrice}} = \overrightarrow{BE} = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sqrt{3}-2 \\ 1 \end{pmatrix}$

De stelling van Pythagoras in $\triangle BEF$ geeft $BE^2 = BF^2 + EF^2 = (-\sqrt{3}-2)^2 + 1 = 3 + 4\sqrt{3} + 4 + 1 = 8 + 4\sqrt{3}$

$BE = \sqrt{8+4\sqrt{3}}$

$$\cos(15°) = \frac{BF}{BE} = \frac{\sqrt{3}+2}{\sqrt{8+4\sqrt{3}}} \cdot \frac{\sqrt{8+4\sqrt{3}}}{\sqrt{8+4\sqrt{3}}} = \frac{(\sqrt{3}+2)\sqrt{8+4\sqrt{3}}}{8+4\sqrt{3}} \cdot \frac{8-4\sqrt{3}}{8-4\sqrt{3}} =$$

$$\frac{(\sqrt{3}+2)(8-4\sqrt{3})\sqrt{8+4\sqrt{3}}}{64-48} = \frac{(8\sqrt{3}-12+16-8\sqrt{3})\sqrt{8+4\sqrt{3}}}{16} = \frac{4\sqrt{8+\sqrt{16\cdot 3}}}{16} = \tfrac{1}{4}\sqrt{8+\sqrt{48}} = \tfrac{1}{2}\sqrt{2+\sqrt{3}}$$

Alternatieve uitwerking

$\cos(2\alpha) = 2\cos^2(\alpha) - 1$

$2\cos^2(\alpha) = 1 + \cos(2\alpha)$

$\cos^2(\alpha) = \frac{1}{2} + \frac{1}{2}\cos(2\alpha)$

$\cos^2(15°) = \frac{1}{2} + \frac{1}{2}\cos(30°)$

$\cos^2(15°) = \frac{1}{2} + \frac{1}{4}\sqrt{3}$

$\cos(15°) = \sqrt{\frac{1}{2} + \frac{1}{4}\sqrt{3}} \vee \cos(15°) = -\sqrt{\frac{1}{2} + \frac{1}{4}\sqrt{3}}$

vold. vold. niet

Dus $\cos(15°) = \sqrt{\frac{1}{2} + \frac{1}{4}\sqrt{3}} = \frac{1}{2}\sqrt{2+\sqrt{3}}$.

103 Stel k: $y = 2x + b$.

$$\left.\begin{array}{l} c: x^2 + y^2 + 2x - 6y + 5 = 0 \\ y = 2x + b \end{array}\right\} \begin{array}{l} x^2 + (2x+b)^2 + 2x - 6(2x+b) + 5 = 0 \\ x^2 + 4x^2 + 4bx + b^2 + 2x - 12x - 6b + 5 = 0 \\ 5x^2 + (4b-10)x + b^2 - 6b + 5 = 0 \end{array}$$

Raken dus $D = 0$

$(4b-10)^2 - 4 \cdot 5 \cdot (b^2 - 6b + 5) = 0$

$16b^2 - 80b + 100 - 20b^2 + 120b - 100 = 0$

$-4b^2 + 40b = 0$

$-4b(b-10) = 0$

$b = 0 \vee b = 10$

Dus de raaklijnen zijn $y = 2x$ en $y = 2x + 10$.

104 $g(x) = \frac{x-4}{x-6}$, dus voor g^{inv} geldt $x = \frac{y-4}{y-6}$

$xy - 6x = y - 4$

$xy - y = 6x - 4$

$y(x-1) = 6x - 4$

$y = \frac{6x-4}{x-1} = \frac{5(x-1)+5+x-4}{x-1} = \frac{5(x-1)+x+1}{x-1} = 5 + \frac{x+1}{x-1}$

Dus g is de inverse van f voor $a = 5$.

Bladzijde 240

105 De punten B, C en D zijn de beeldpunten van A bij de rotaties om O met 90°, 180° en 270°.

Dus $B(-\frac{1}{2}\sqrt{3}, \frac{1}{2})$, $C(-\frac{1}{2}, -\frac{1}{2}\sqrt{3})$ en $D(\frac{1}{2}\sqrt{3}, -\frac{1}{2})$.

106 $\cos(2x) = 2\cos^2(x) - 1$

$2\cos^2(x) = 1 + \cos(2x)$

$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

$\cos^4(x) = \left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)^2$

$\cos^4(x) = \frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)$

$\cos^4(x) = \frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)$

$\cos^4(x) = \frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x)$

$\cos^4(x) = \frac{3}{8} + \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)$

Dus $y = \cos^4(x) = \frac{3}{8} + \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)$.

107 $f(x) = x \cdot \tan^3\left(3x - \frac{1}{2}\pi\right)$ geeft

$f'(x) = 1 \cdot \tan^3\left(3x - \frac{1}{2}\pi\right) + x \cdot 3\tan^2\left(3x - \frac{1}{2}\pi\right) \cdot 3\left(1 + \tan^2\left(3x - \frac{1}{2}\pi\right)\right)$

$= \tan^3\left(3x - \frac{1}{2}\pi\right) + 9x\tan^2\left(3x - \frac{1}{2}\pi\right) \cdot \left(1 + \tan^2\left(3x - \frac{1}{2}\pi\right)\right)$

108 evenwichtsstand $= a = \dfrac{-1 + 5}{2} = 2$

amplitude $= b = 5 - 2 = 3$

periode $2\pi - \frac{2}{3}\pi = \frac{4}{3}\pi$, dus $c = \dfrac{2\pi}{\frac{4}{3}\pi} = 1\frac{1}{2}$

top $\left(\frac{2}{3}\pi, 5\right)$ geeft $d = \frac{2}{3}\pi$

Dus $y = 2 + 3\cos\left(1\frac{1}{2}\left(x - \frac{2}{3}\pi\right)\right)$.

109 $x = 0$ geeft $4 - t^2 = 0$

$t^2 = 4$

$t = -2 \vee t = 2$

$t = -2$ geeft het punt $(0, -2)$, dus voldoet niet.

$t = 2$ geeft het punt $(0, 2)$, dus voldoet.

$\vec{r}(t) = \begin{pmatrix} 4 - t^2 \\ t^3 - 3t \end{pmatrix}$

$\vec{v}(t) = \begin{pmatrix} -2t \\ 3t^2 - 3 \end{pmatrix}$

$\vec{a}(t) = \begin{pmatrix} -2 \\ 6t \end{pmatrix}$

baansnelheid $= |\vec{v}(2)| = \left|\begin{pmatrix} -4 \\ 9 \end{pmatrix}\right| = \sqrt{16 + 81} = \sqrt{97}$

$$a_b(2) = \frac{\vec{v}(2) \cdot \vec{a}(2)}{|\vec{v}(2)|} = \frac{\begin{pmatrix} -4 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 12 \end{pmatrix}}{\sqrt{97}} = \frac{8 + 108}{\sqrt{97}} = \frac{116}{\sqrt{97}} = 1\tfrac{19}{97}\sqrt{97}$$

110 $$f'(x) = \frac{x^2 \cdot (\cos(x) + \sin(x)) - ((\sin(x) - \cos(x)) \cdot 2x}{x^4} = \frac{x(\cos(x) + \sin(x)) - 2(\sin(x) - \cos(x))}{x^3}$$

$$= \frac{(x + 2)\cos(x) + (x - 2)\sin(x)}{x^3}$$

111 $x < 0$ geeft $2 + 4\cos(2t) < 0$

$4\cos(2t) < -2$

$\cos(2t) < -\frac{1}{2}$

$\frac{2}{3}\pi + k \cdot 2\pi < 2t < 1\frac{1}{3}\pi + k \cdot 2\pi$

$\frac{1}{3}\pi + k \cdot \pi < t < \frac{2}{3}\pi + k \cdot \pi$

x op $[0, \pi]$ geeft $\frac{1}{3}\pi < t < \frac{2}{3}\pi$

$y > 1$ geeft $1 + 4\sin(2t) > 1$

$4\sin(2t) > 0$

$\sin(2t) > 0$

$0 + k \cdot 2\pi < 2t < \pi + k \cdot 2\pi$

$k \cdot \pi < t < \frac{1}{2}\pi + k \cdot \pi$

x op $[0, \pi]$ geeft $0 < t < \frac{1}{2}\pi$

$x < 0 \wedge y > 1$ geeft $\frac{1}{3}\pi < t < \frac{1}{2}\pi$

Dus P bevindt zich $\frac{1}{2}\pi - \frac{1}{3}\pi = \frac{1}{6}\pi$ seconden links van de y-as en boven de lijn $y = 1$.

Bladzijde 241

112 $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, dus $\vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

$l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 7 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$.

$$\cos(\angle(k, l)) = \frac{\left|\begin{pmatrix} 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 7 \end{pmatrix}\right|}{\left|\begin{pmatrix} 3 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -3 \\ 7 \end{pmatrix}\right|} = \frac{|-9 - 14|}{\sqrt{9+4} \cdot \sqrt{9+49}} = \frac{23}{\sqrt{13} \cdot \sqrt{58}}$$

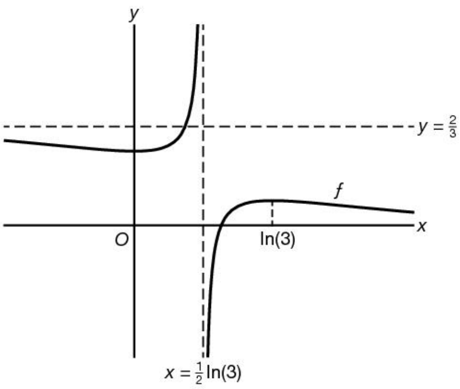
Dus $\angle(k, l) \approx 33{,}1°$.

113 $f'(x) = \frac{(e^{2x} - 3) \cdot e^x - (e^x - 2) \cdot 2e^{2x}}{(e^{2x} - 3)^2} = \frac{e^{3x} - 3e^x - 2e^{3x} + 4e^{2x}}{(e^{2x} - 3)^2} = \frac{-e^{3x} + 4e^{2x} - 3e^x}{(e^{2x} - 3)^2} = \frac{-e^x(e^{2x} - 4e^x + 3)}{(e^{2x} - 3)^2}$

$f'(x) = 0$ geeft $e^{2x} - 4e^x + 3 = 0$
$(e^x - 1)(e^x - 3) = 0$
$e^x = 1 \vee e^x = 3$
$x = 0 \vee x = \ln(3)$

$f(0) = \frac{1-2}{1-3} = \frac{1}{2}$ en $f(\ln(3)) = \frac{3-2}{9-3} = \frac{1}{6}$

$\lim_{x \to \infty} \frac{e^x - 2}{e^{2x} - 3} = \lim_{x \to \infty} \frac{\frac{1}{e^x} - \frac{2}{e^{2x}}}{1 - \frac{3}{e^{2x}}} = \frac{0-0}{1-0} = 0$ en $\lim_{x \to -\infty} \frac{e^x - 2}{e^{2x} - 3} = \frac{0-2}{0-3} = \frac{2}{3}$



Dus $B_f = \langle \leftarrow, \frac{1}{6}]$ en $[\frac{1}{2}, \rightarrow\rangle$.

114 $25^x + 6 = 5^{x+1}$
$5^{2x} - 5^{x+1} + 6 = 0$
$(5^x)^2 - 5 \cdot 5^x + 6 = 0$
$(5^x - 2)(5^x - 3) = 0$
$5^x = 2 \vee 5^x = 3$
$x = {}^5\log(2) \vee x = {}^5\log(3)$

115 $x^2 + y^2 - 6x + 4y + 8 = 0$
$x^2 - 6x + y^2 + 4y + 8 = 0$
$(x-3)^2 - 9 + (y+2)^2 - 4 + 8 = 0$
$(x-3)^2 + (y+2)^2 = 5$

Dus $\begin{cases} x_P = 3 + \sqrt{5} \cdot \cos(t) \\ y_P = -2 + \sqrt{5} \cdot \sin(t) \end{cases}$

$P(3 + \sqrt{5} \cdot \cos(t), -2 + \sqrt{5} \cdot \sin(t))$ en $A(10, 0)$ geeft

$Q\left(\frac{3 + \sqrt{5} \cdot \cos(t) + 10}{2}, \frac{-2 + \sqrt{5} \cdot \sin(t) + 0}{2}\right)$ ofwel $Q\left(1\frac{1}{2} + \frac{1}{2}\sqrt{5} \cdot \cos(t), -1 + \frac{1}{2}\sqrt{5} \cdot \sin(t)\right)$

$$\begin{cases} x_Q = 1\tfrac{1}{2} + \tfrac{1}{2}\sqrt{5} \cdot \cos(t) \\ y_Q = -1 + \tfrac{1}{2}\sqrt{5} \cdot \sin(t) \end{cases}$$

Dus Q op $\left(x - 1\tfrac{1}{2}\right)^2 + (y + 1)^2 = \left(\tfrac{1}{2}\sqrt{5}\right)^2$

$$x^2 - 3x + 2\tfrac{1}{4} + y^2 + 2y + 1 = \tfrac{1}{4} \cdot 5$$
$$x^2 + y^2 - 3x + 2y + 2 = 0$$

Dus c_2: $x^2 + y^2 - 3x + 2y + 2 = 0$.

116 Stel l: $y = px + 5$ ofwel l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ p \end{pmatrix}$.

$$\vec{r}_{BC} = \begin{pmatrix} 6 - 4 \\ 2 - 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$\angle(l, BC) = 20°$ geeft $\cos(20°) = \dfrac{\left| \begin{pmatrix} 1 \\ p \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|}{\sqrt{1 + p^2} \cdot 1}$

$$\sqrt{1 + p^2} = \frac{1}{\cos(20°)} = 1{,}064...$$
$$1 + p^2 = 1{,}132...$$
$$p^2 = 0{,}132...$$
$$p = \sqrt{0{,}132...} \vee p = -\sqrt{0{,}132...}$$
$$p = 0{,}363... \vee p = -0{,}363...$$

Dus de gevraagde lijnen zijn $y = 0{,}36x + 5$ en $y = -0{,}36x + 5$.

117 Stel k: $y = ax + 1$ met $a = \text{rc}_k = \dfrac{1 - 2}{0 - -1} = -1$.

Dus k: $y = -x + 1$ ofwel k: $x + y - 1 = 0$

$$d(A, k) = \frac{|9 + 2 - 1|}{\sqrt{1 + 1}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

118 Stel c: $(x - 8)^2 + (y - 5)^2 = r^2$

k: $\begin{cases} x = t + 2 \\ y = 3t + 1 \end{cases}$

$$(t - 6)^2 + (3t - 4)^2 = r^2$$
$$t^2 - 12t + 36 + 9t^2 - 24t + 16 - r^2 = 0$$
$$10t^2 - 36t + 52 - r^2 = 0$$

Raken dus $D = 0$

$$(-36)^2 - 4 \cdot 10 \cdot (52 - r^2) = 0$$
$$1296 - 2080 + 40r^2 = 0$$
$$40r^2 = 784$$
$$r^2 = 19{,}6$$

Dus c: $(x - 8)^2 + (y - 5)^2 = 19{,}6$.

119 $\ln^2(x) - 8\ln(x) + 12 = 0$

$(\ln(x) - 2)(\ln(x) - 6) = 0$

$\ln(x) = 2 \vee \ln(x) = 6$

$x = \mathrm{e}^2 \vee x = \mathrm{e}^6$

120 l: $\begin{cases} x = 3t - 3 \\ y = 2t + b \end{cases}$ ofwel l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ b \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\vec{r}_l = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ geeft $\vec{n}_l = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, dus l: $2x - 3y = c$
door $(-3, b)$

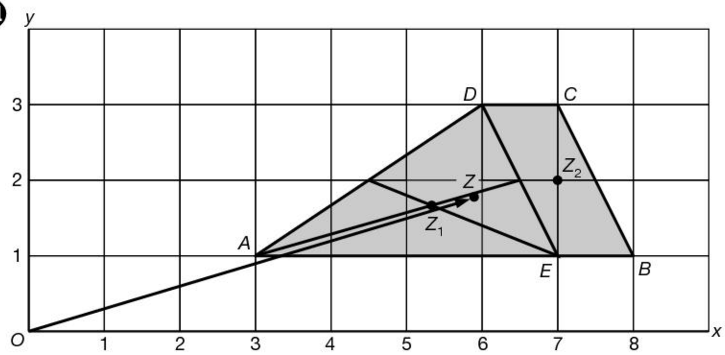
$$c = 2 \cdot -3 - 3 \cdot b = -6 - 3b$$

l: $2x - 3y = -6 - 3b$
k: $2x + ay = 12$

vallen samen als $-3 = a \wedge -6 - 3b = 12$

$$a = -3 \wedge -3b = 6$$
$$a = -3 \wedge b = -2$$

121



$\vec{z}_1 = \frac{1}{3}(\vec{a} + \vec{e} + \vec{d}) = \frac{1}{3}\left(\begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}7\\1\end{pmatrix} + \begin{pmatrix}6\\3\end{pmatrix}\right) = \frac{1}{3}\begin{pmatrix}16\\5\end{pmatrix} = \begin{pmatrix}5\frac{1}{3}\\1\frac{2}{3}\end{pmatrix}$

$\vec{z}_2 = \begin{pmatrix}7\\2\end{pmatrix}$

$m = m_1 + m_2 = \frac{1}{2}\cdot 4\cdot 2 + 1\cdot 2 = 4 + 2 = 6$

$\vec{z} = \frac{1}{6}\left(4\cdot\begin{pmatrix}5\frac{1}{3}\\1\frac{2}{3}\end{pmatrix} + 2\cdot\begin{pmatrix}7\\2\end{pmatrix}\right) = \frac{2}{3}\begin{pmatrix}5\frac{1}{3}\\1\frac{2}{3}\end{pmatrix} + \frac{1}{3}\begin{pmatrix}7\\2\end{pmatrix} = \begin{pmatrix}5\frac{8}{9}\\1\frac{7}{9}\end{pmatrix}$, dus $Z(5\frac{8}{9}, 1\frac{7}{9})$.

122 $\begin{cases}x(t) = t^2 - 4t\\ y(t) = 2t^2 + 4t\end{cases}$ geeft $\begin{cases}x'(t) = 2t - 4\\ y'(t) = 4t + 4\end{cases}$

Raaklijn horizontaal geeft $y'(t) = 0 \wedge x'(t) \neq 0$

$4t + 4 = 0 \wedge 2t - 4 \neq 0$

$4t = -4 \wedge 2t \neq 4$

$t = -1 \wedge t \neq 2$

$t = -1$ geeft het punt $(5, -2)$.

Raaklijn verticaal geeft $x'(t) = 0 \wedge y'(t) \neq 0$

$2t - 4 = 0 \wedge 4t + 4 \neq 0$

$t = 2 \wedge t \neq -1$

$t = 2$ geeft het punt $(-4, 16)$.

Dus in het punt $(5, -2)$ is de baan evenwijdig met de x-as en in het punt $(-4, 16)$ is de baan evenwijdig met de y-as.

Verantwoording

Omslagontwerp: In Ontwerp, Assen
Ontwerp binnenwerk: Ebel Kuipers, Sappemeer
Technisch tekenwerk: OKS, Delhi (India)
Lay-out: OKS, Delhi (India)

MIX
Papier van verantwoorde herkomst
FSC® C118189

2 / 18

ISBN 978-90-01-84251-2

GETAL & RUIMTE

J.H. Dijkhuis
C.J. Admiraal
J.A. Verbeek
G. de Jong
H.J. Houwing
J.D. Kuis
F. ten Klooster
S.K.A. de Waal
J. van Braak
J.H.M. Liesting-Maas
M. Wieringa
M.L.M. van Maarseveen
R.D. Hiele
J.E. Romkes
M. Haneveld
S. Voets
I. Cornelisse

Noordhoff Uitgevers

MIX
Papier van verantwoorde herkomst
FSC® C118189
www.fsc.org

ISBN 978-90-01-84251-2

9 789001 842512

www.getalenruimte.noordhoff.nl